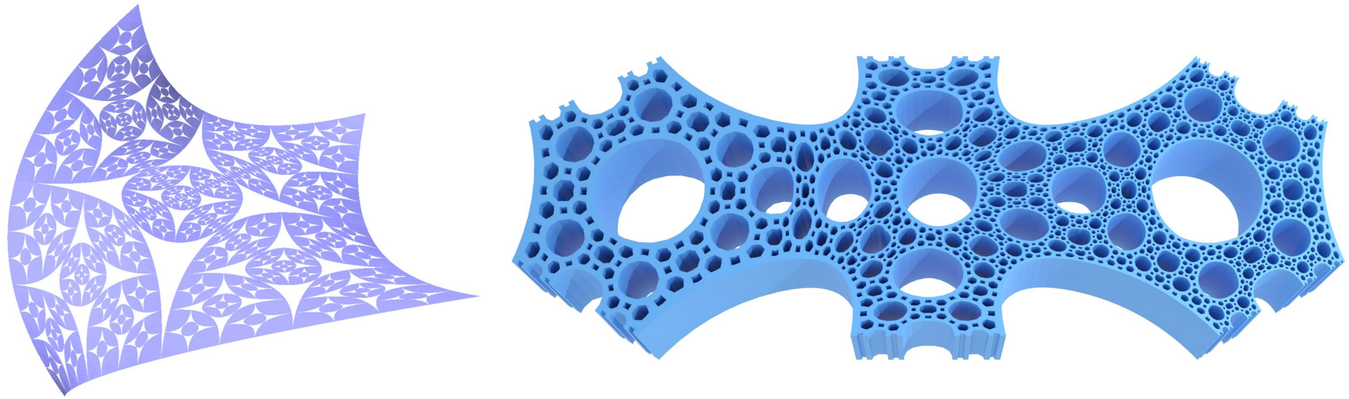
Conception automatique de structures lacunaires fractales



Boris Bordeaux[1], Christian Gentil[1], Lionel Garnier[1]
Charles-Adèle Desormeaux[1], Iván Rasskin[2]

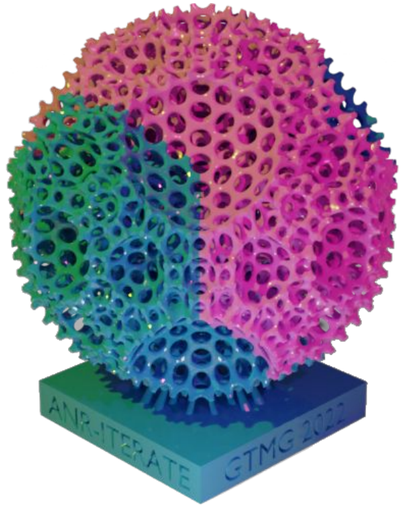
[1]LIB – Université Bourgogne Europe, Dijon, France
[2]LIS – Aix-Marseille Université, Marseille, France

ANR ITERATE – ANR-20-CE46-0011 (2021-2025)

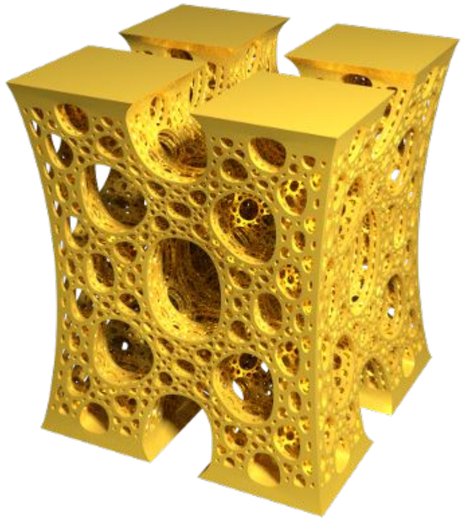
Projet ANR : ITERATE

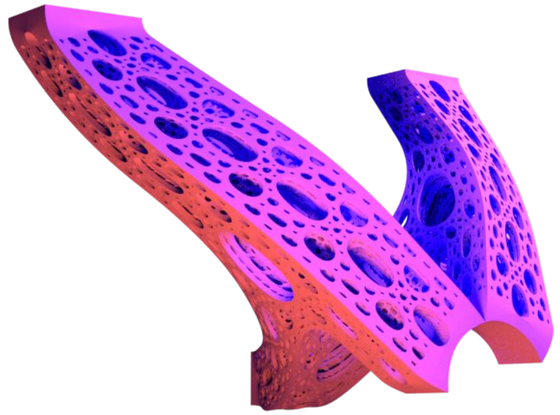
- Structures lacunaires fractales [GGS21]











Fractales : géométrie de la nature





[GGS21]





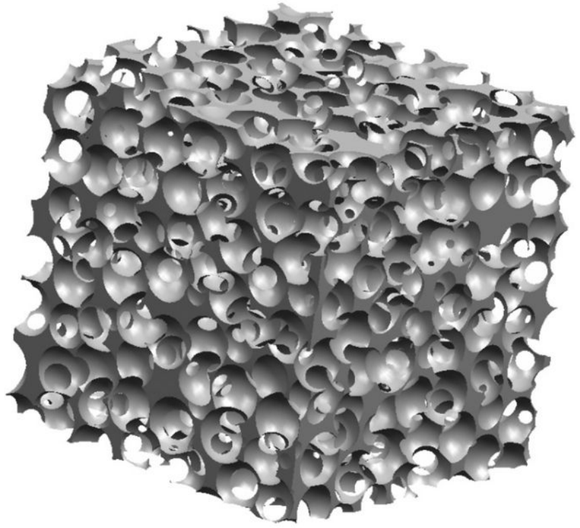
Plan

- État de l’art
- Problématique et démarche
- Conception automatique 2D à partir des bords
- Conception automatique 2D à partir de polyèdres
- Conception automatique 3D à partir de polyèdres
- Conclusion

État de l’art
Structures lacunaires

Structures lacunaires

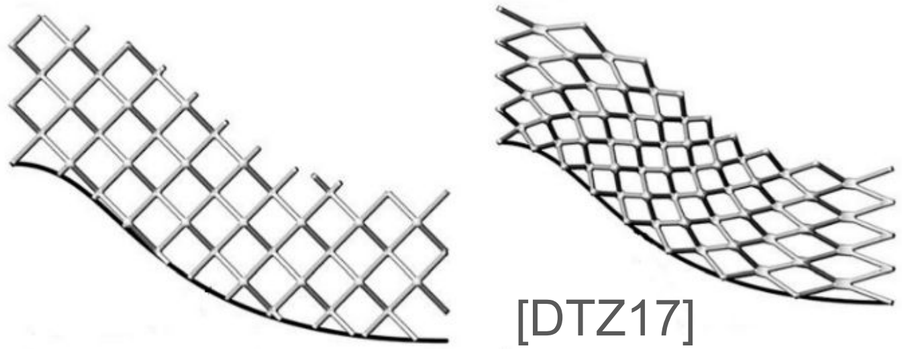
- Mousses
  - Aléatoire
- Structures lattices
  - Topologie régulière
  - Variation de géométrie
- Surfaces minimales
  - Triplement périodique
  - Géométrie contrainte



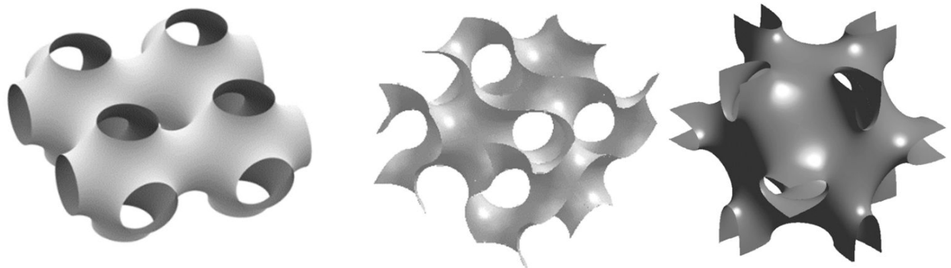
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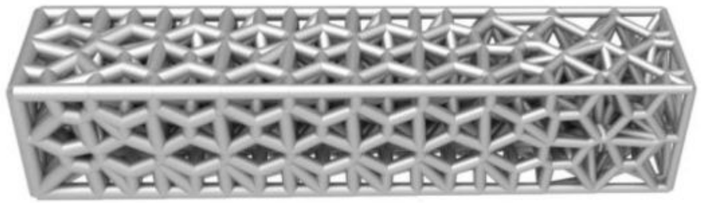
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[DTZ17]



[BPFL19]



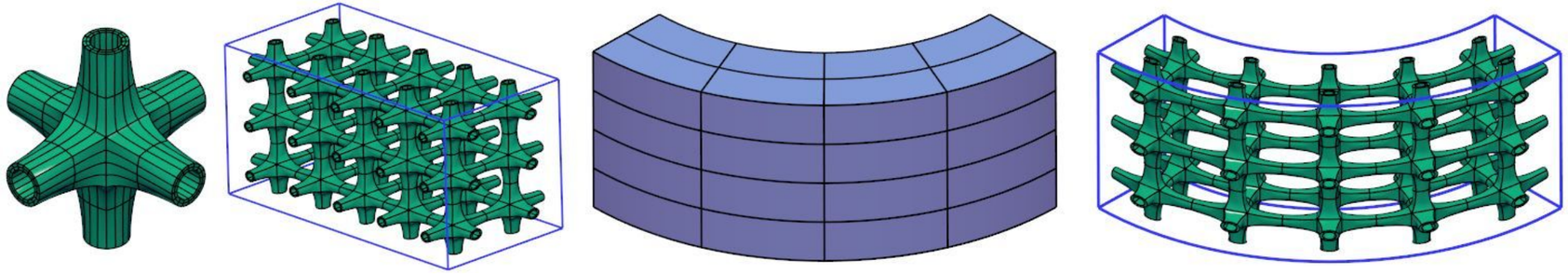
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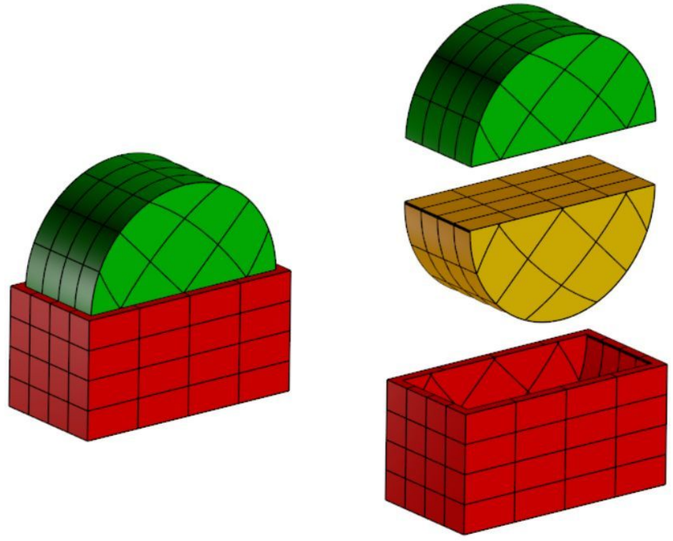
État de l’art
Méthodes récursives

Méthodes récursives

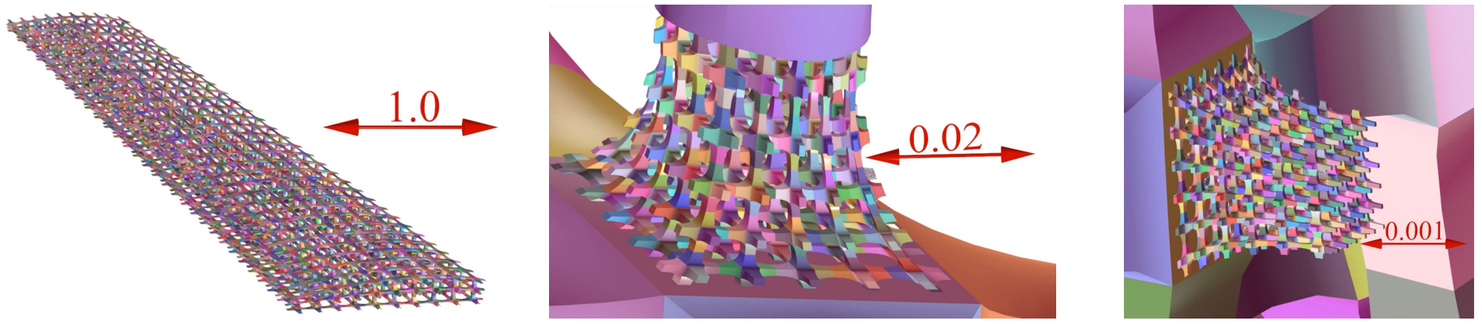
- Espace paramétrique
  - Motif répété
  - Gestion des intersections et des raccords
  - Sauts d’échelle importants



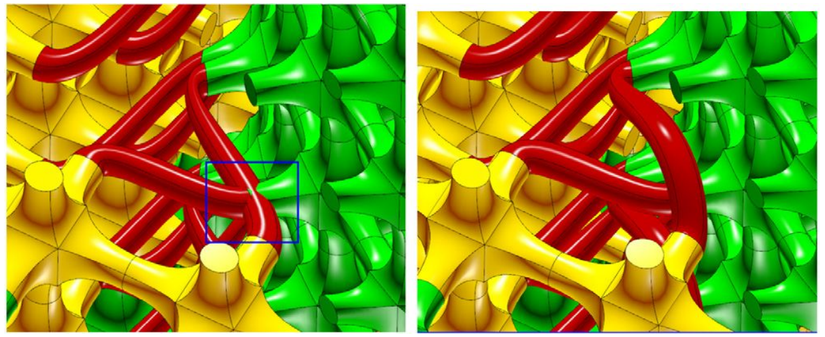
[Elb23]



[Elb23]



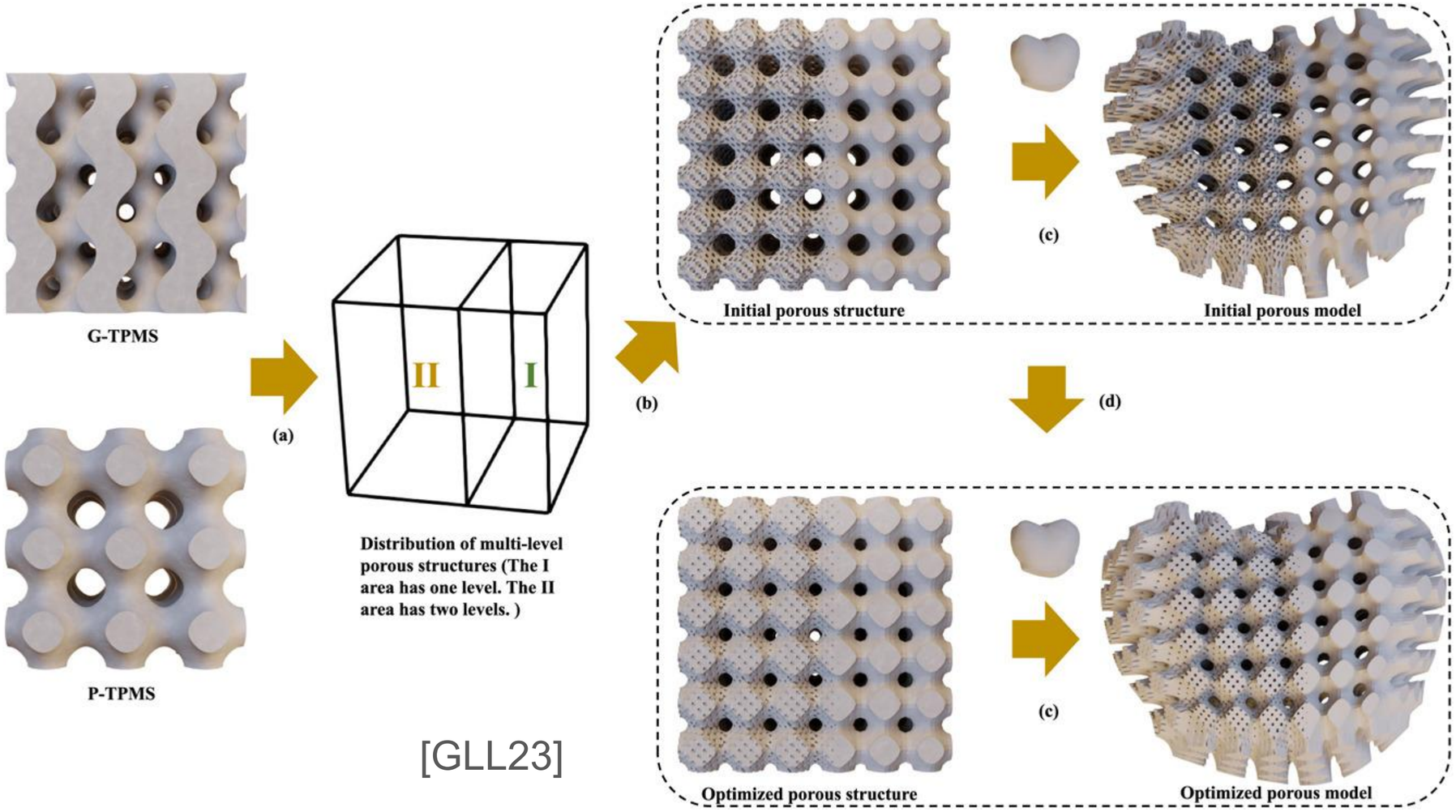
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[HE21]

Méthodes récursives

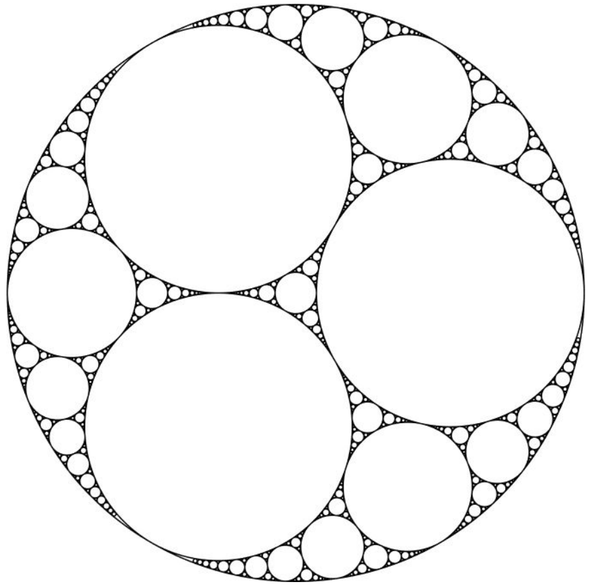
- Surfaces minimales triplement périodiques
  - Fonctions implicites
  - Paramétrables

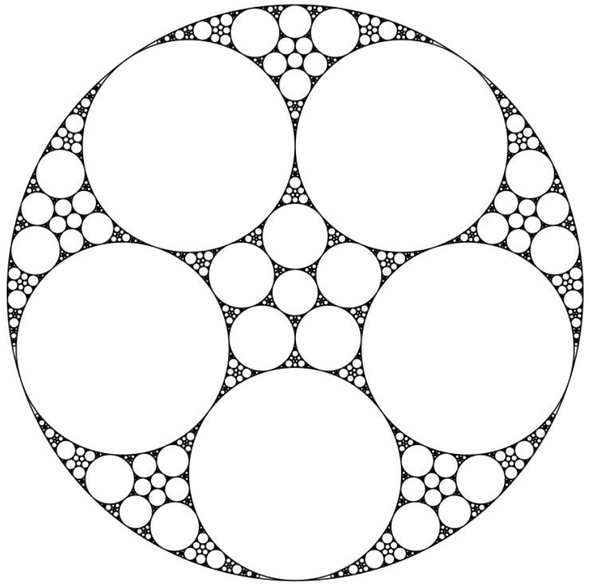


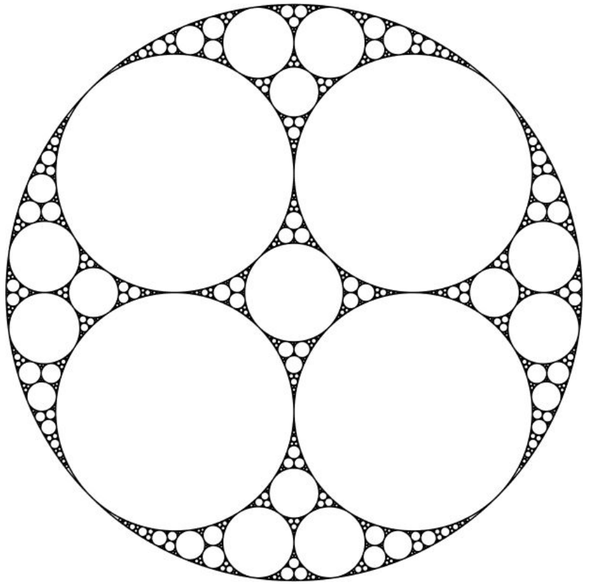
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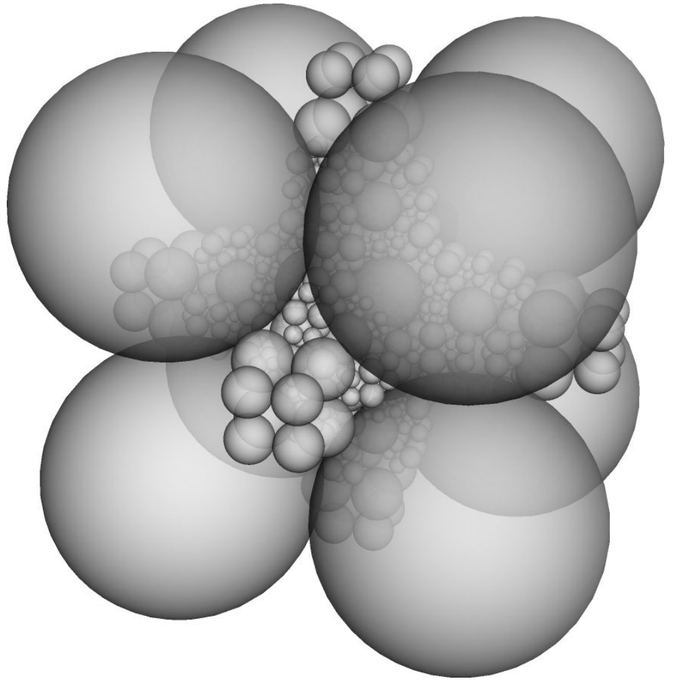
Méthodes récursives

- Empilements cristallographiques de sphères [KN19]
  - Inversions
  - Généralisable









Iván Rasskin

État de l’art
Modèles de fractales

Modèles de fractales

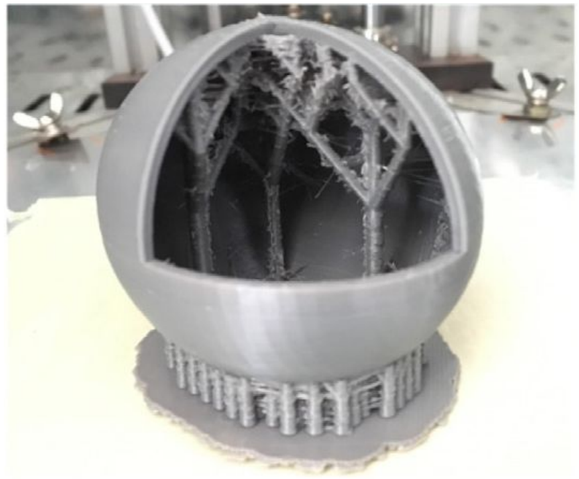
- L-Systems [Pru86]
  - Structures arborescentes



[PMKL01]



[PCFH18]



[ZLRL19]

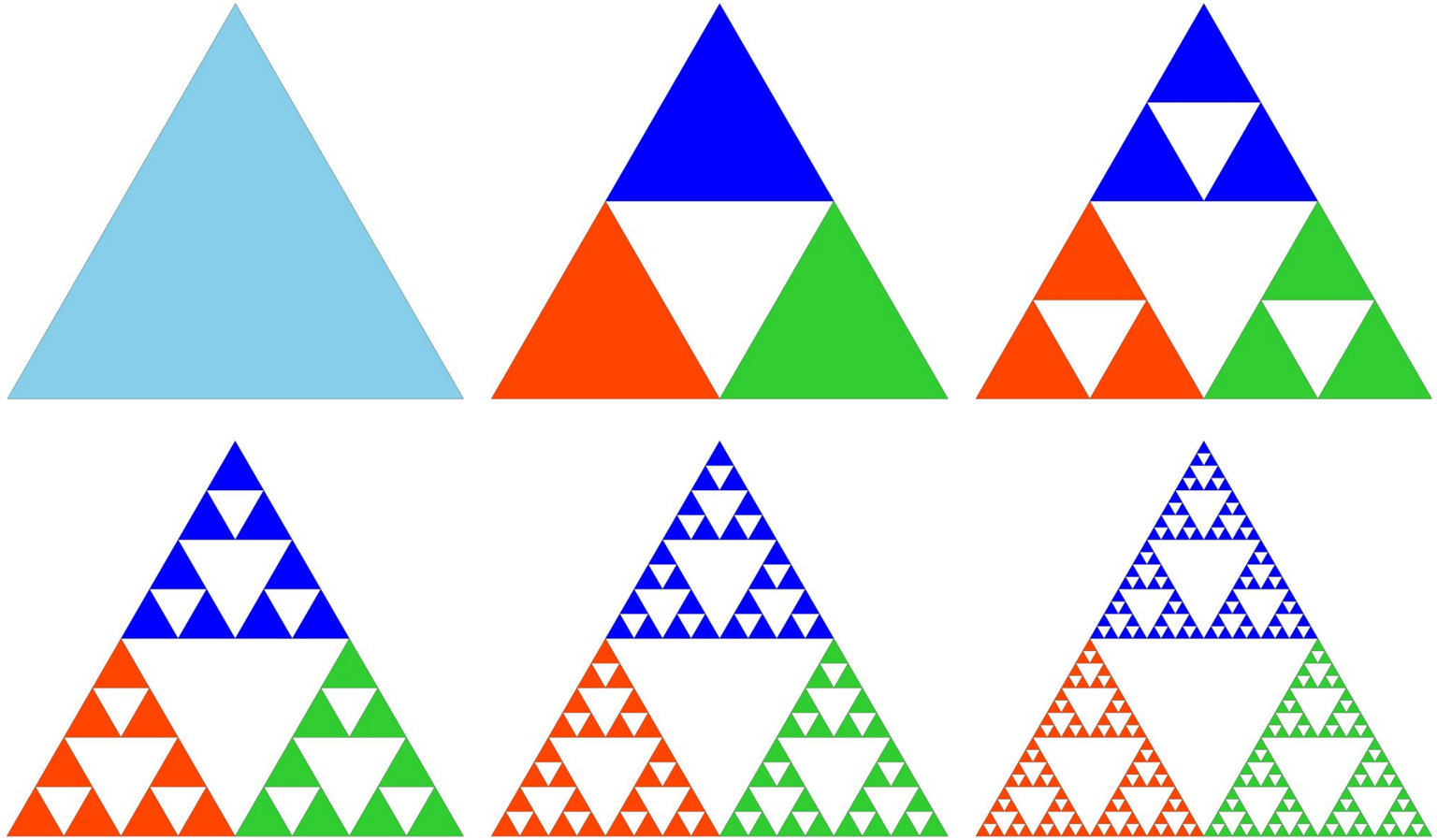
Modèles de fractales

- Iterated Function System [Hut81, BD85]
  - Transformations contractantes

$$T_2 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$T_0 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \quad T_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

$$S = T_0(S) \cup T_1(S) \cup T_2(S)$$



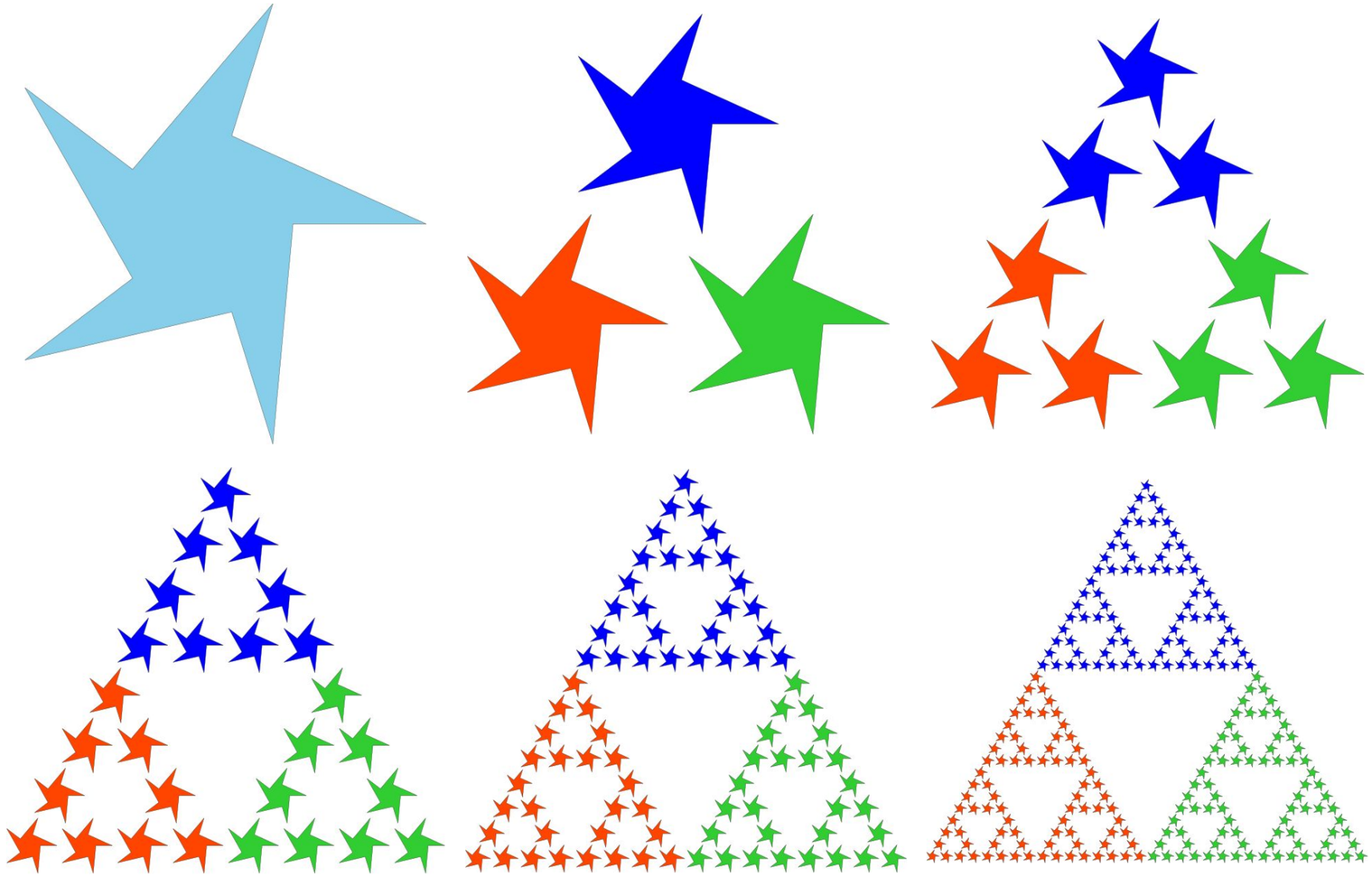
Modèles de fractales

- Iterated Function System [Hut81, BD85]
  - Transformations contractantes
  - Peu importe la primitive

$$T_2 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

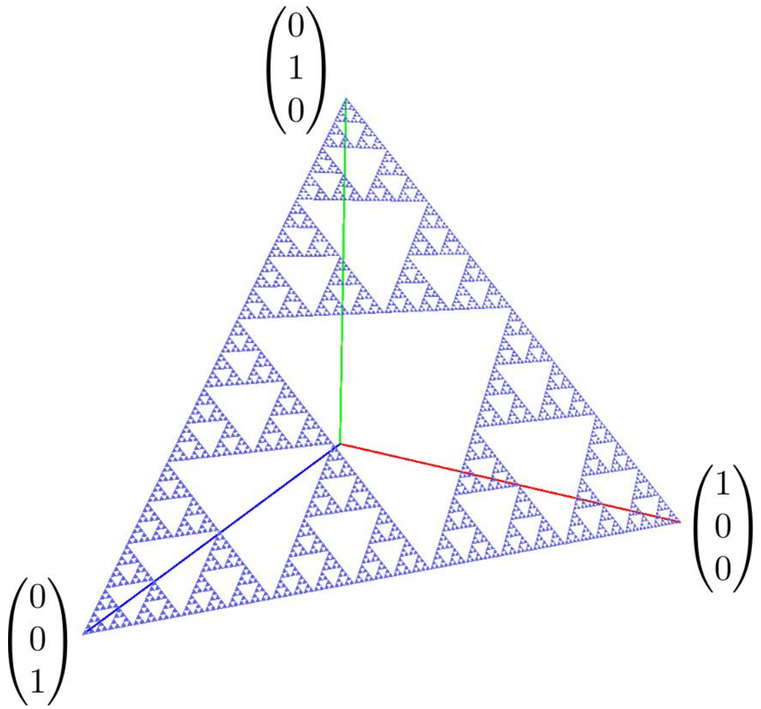
$$T_0 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \quad T_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

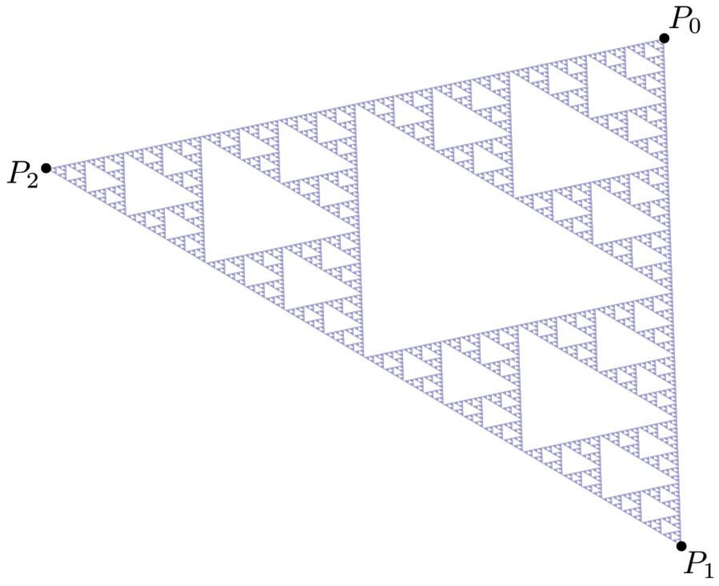
$$S = T_0(S) \cup T_1(S) \cup T_2(S)$$

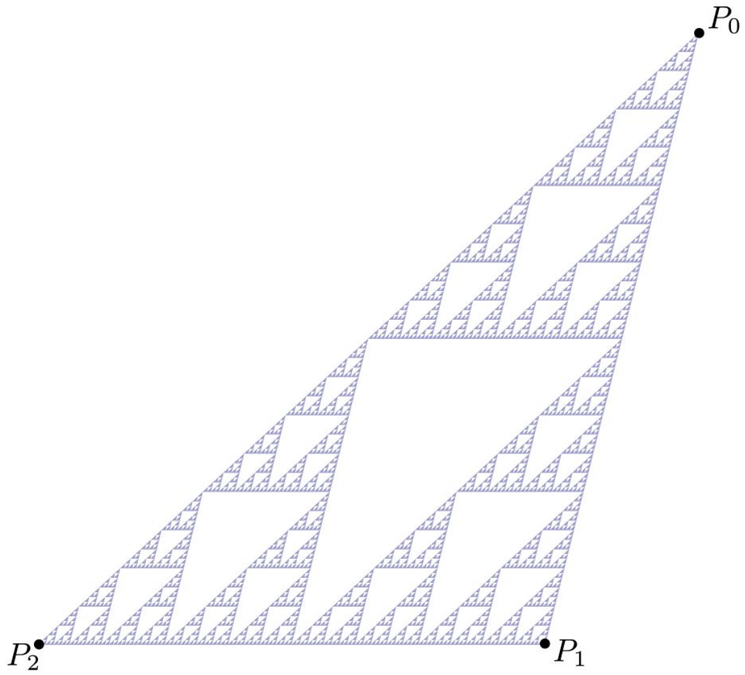


Modèles de fractales

- Iterated Function System [Hut81, BD85]
  - Transformations contractantes
  - Peu importe la primitive
  - Espace barycentrique

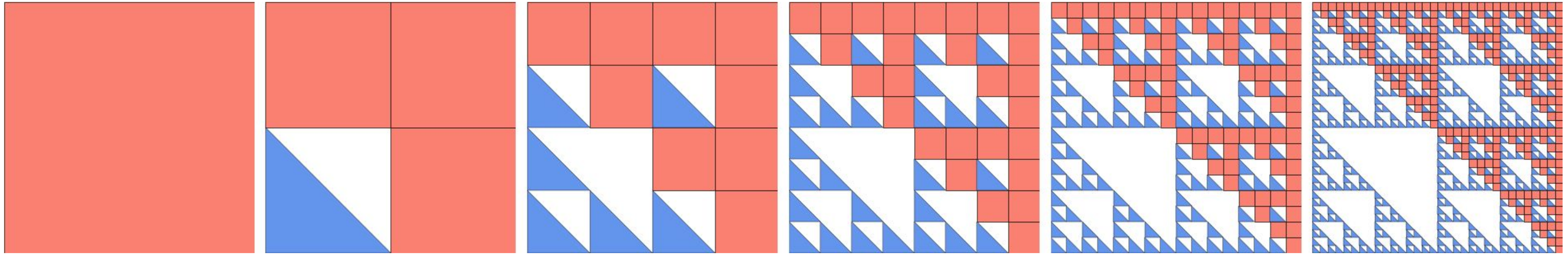


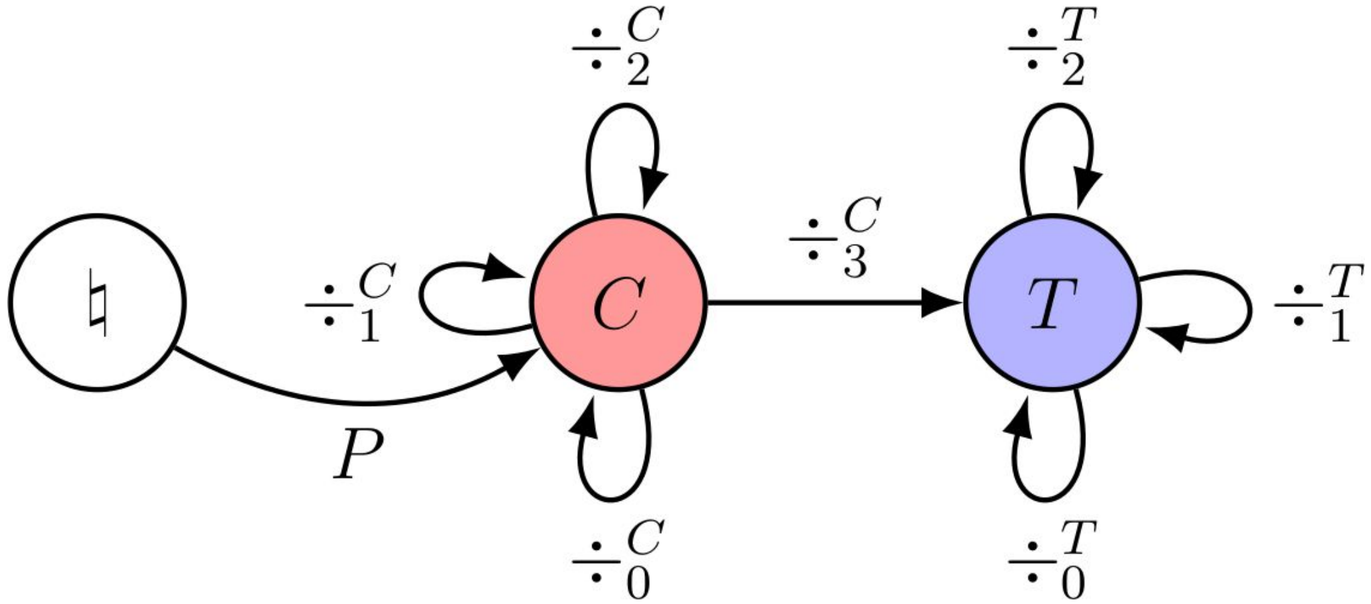




Modèles de fractales

- Controlled-IFS

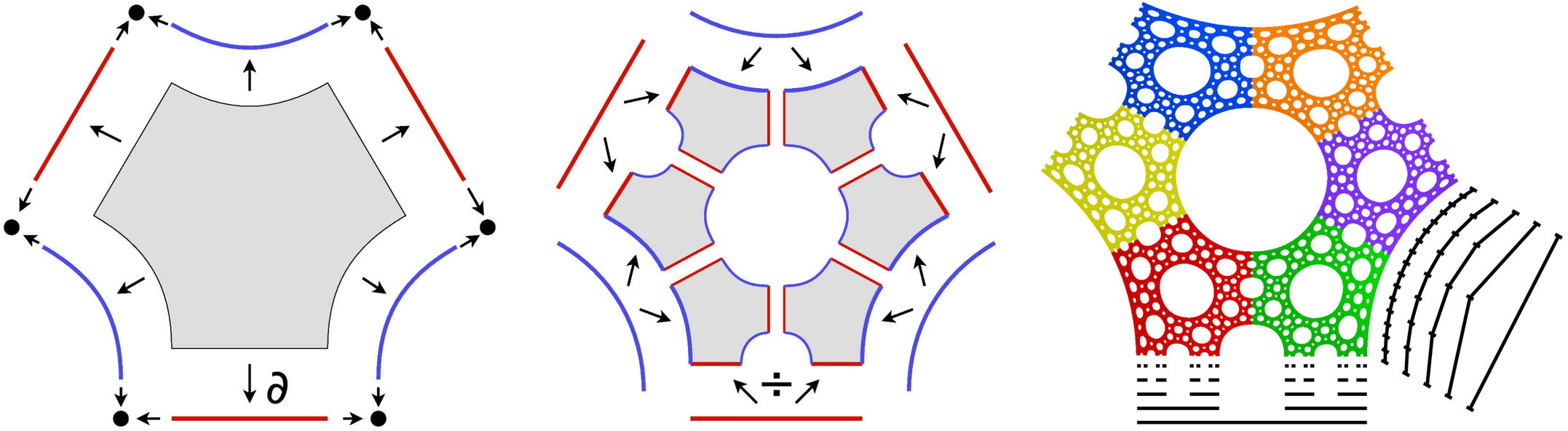




[GGS21]

Modèles de fractales

- Boundary Controlled IFS
  - Décomposition cellulaire



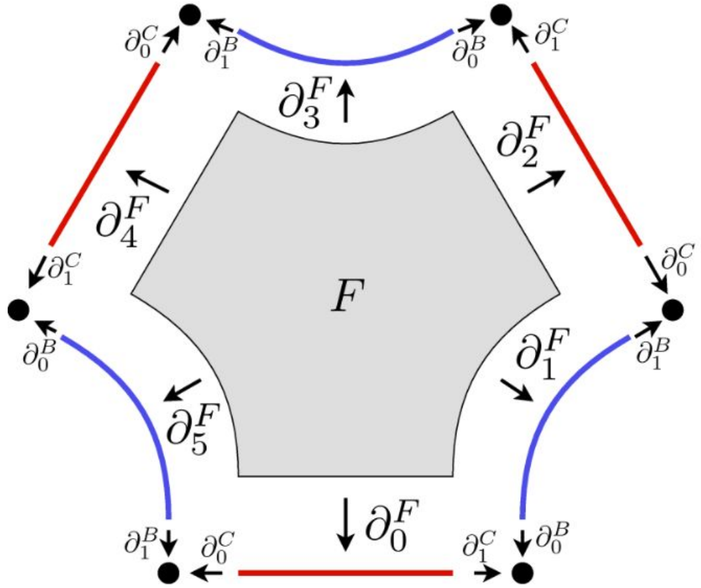
[GGS21]

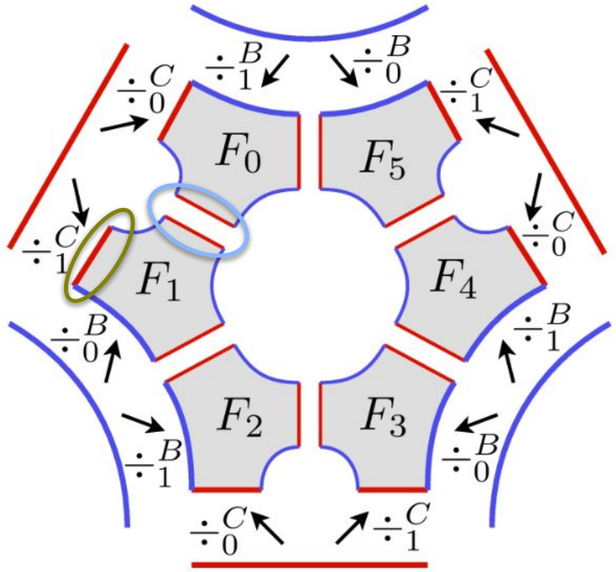
Modèles de fractales

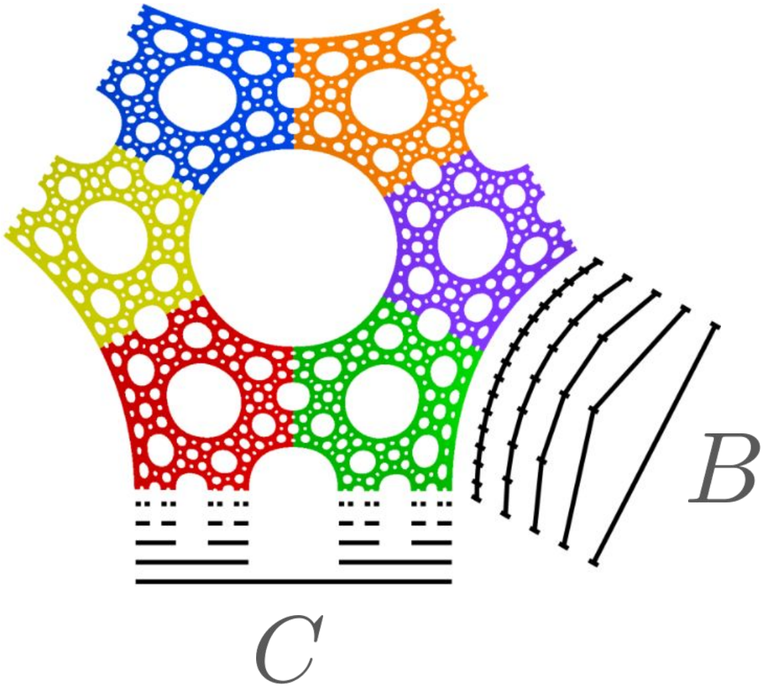
- BC-IFS
  - Décomposition cellulaire
  - Contraintes d'incidence et d'adjacence

$$\partial_4^F \div_1^C = \div_1^F \partial_0^F$$

$$\div_0^F \partial_2^F = \div_1^F \partial_4^F$$





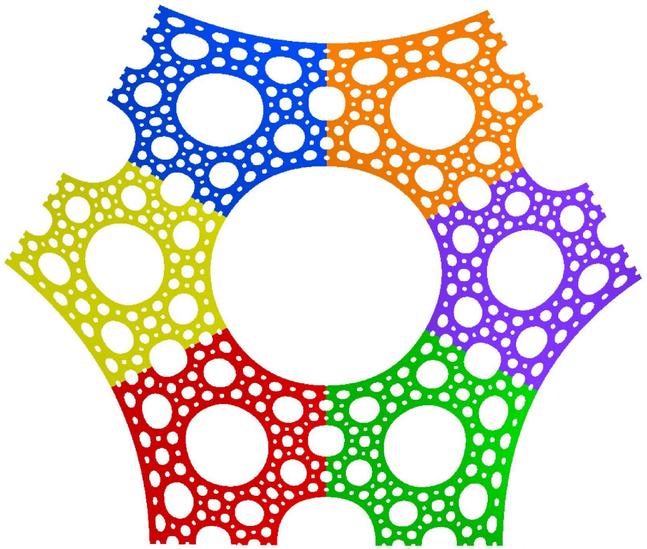


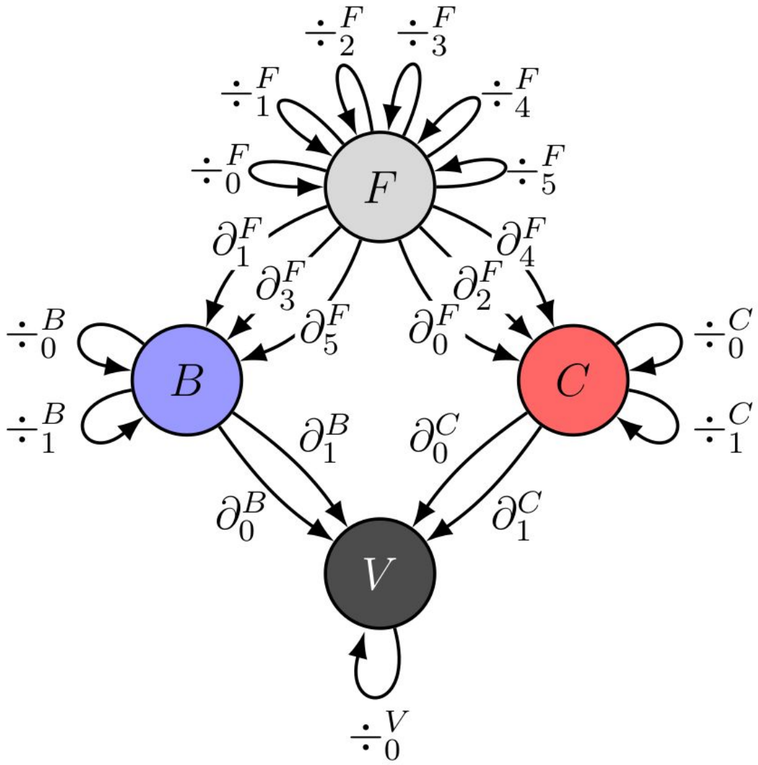
[GGS21]

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Modèles de fractales

- BC-IFS
  - Décomposition cellulaire
  - Contraintes d'incidence et d'adjacence



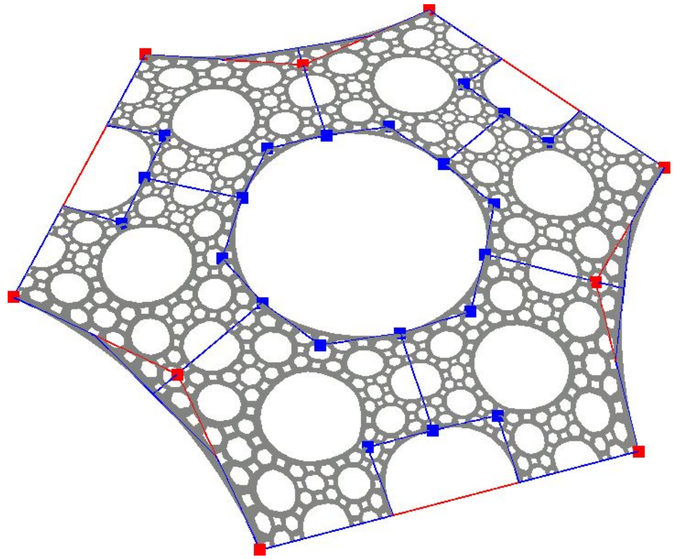


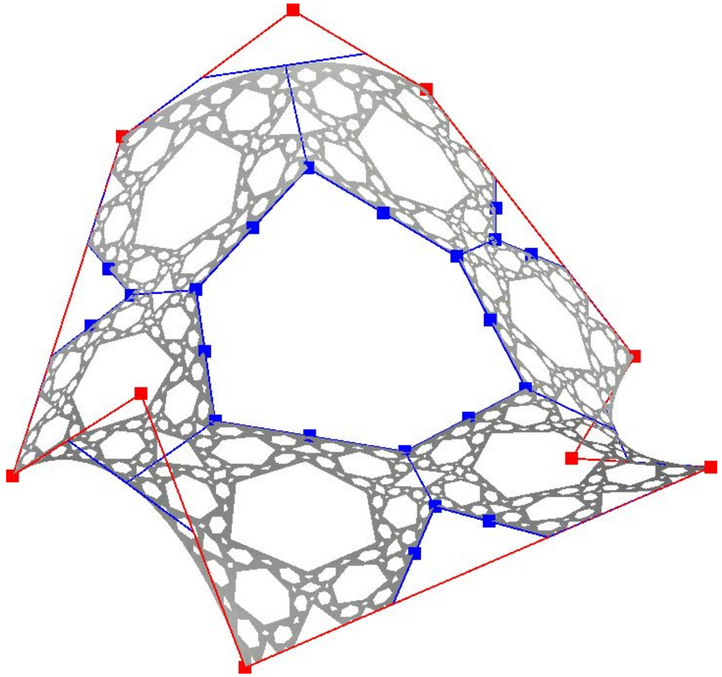
$$\partial_0^F \div_0^C = \div_2^F \partial_0^F$$
$$\partial_0^F \div_1^C = \div_3^F \partial_0^F$$
$$\partial_1^F \div_0^B = \div_3^F \partial_1^F$$
$$\partial_1^F \div_1^B = \div_4^F \partial_5^F$$
$$\partial_2^F \div_0^C = \div_4^F \partial_0^F$$
$$\partial_2^F \div_1^C = \div_5^F \partial_0^F$$
$$\partial_3^F \div_0^B = \div_5^F \partial_1^F$$
$$\partial_3^F \div_1^B = \div_0^F \partial_5^F$$
$$\partial_4^F \div_0^C = \div_0^F \partial_0^F$$
$$\partial_4^F \div_1^C = \div_1^F \partial_0^F$$
$$\partial_5^F \div_0^B = \div_1^F \partial_1^F$$
$$\partial_5^F \div_1^B = \div_2^F \partial_5^F$$

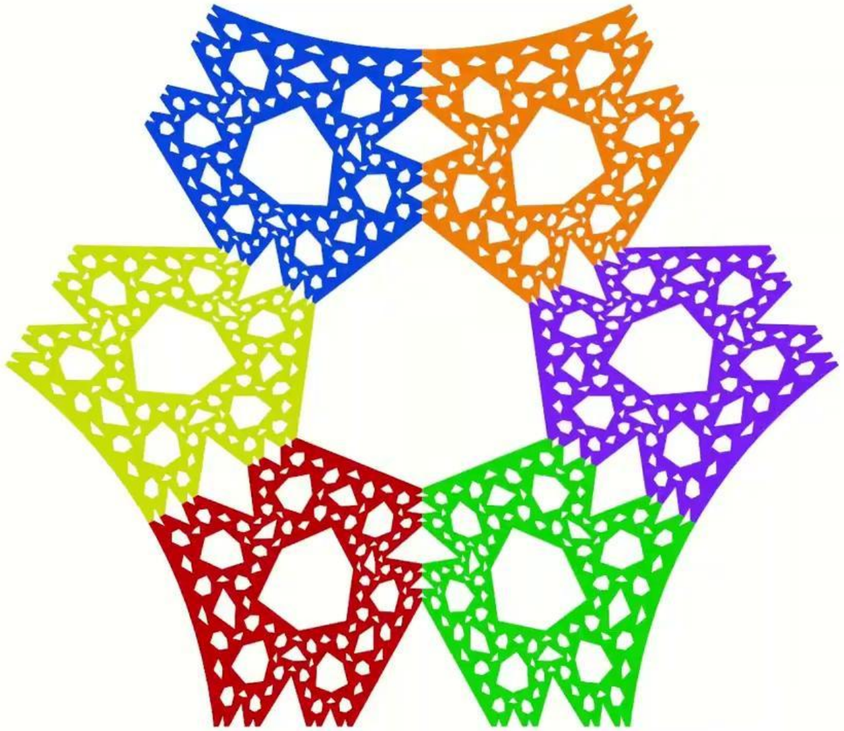
$$\div_0^F \partial_2^F = \div_1^F \partial_4^F$$
$$\div_1^F \partial_2^F = \div_2^F \partial_4^F$$
$$\div_2^F \partial_2^F = \div_3^F \partial_4^F$$
$$\div_3^F \partial_2^F = \div_4^F \partial_4^F$$
$$\div_4^F \partial_2^F = \div_5^F \partial_4^F$$
$$\div_5^F \partial_2^F = \div_0^F \partial_4^F$$

Modèles de fractales

- BC-IFS
  - Décomposition cellulaire
  - Contraintes d’incidence et d’adjacence
  - Réalisation géométrique





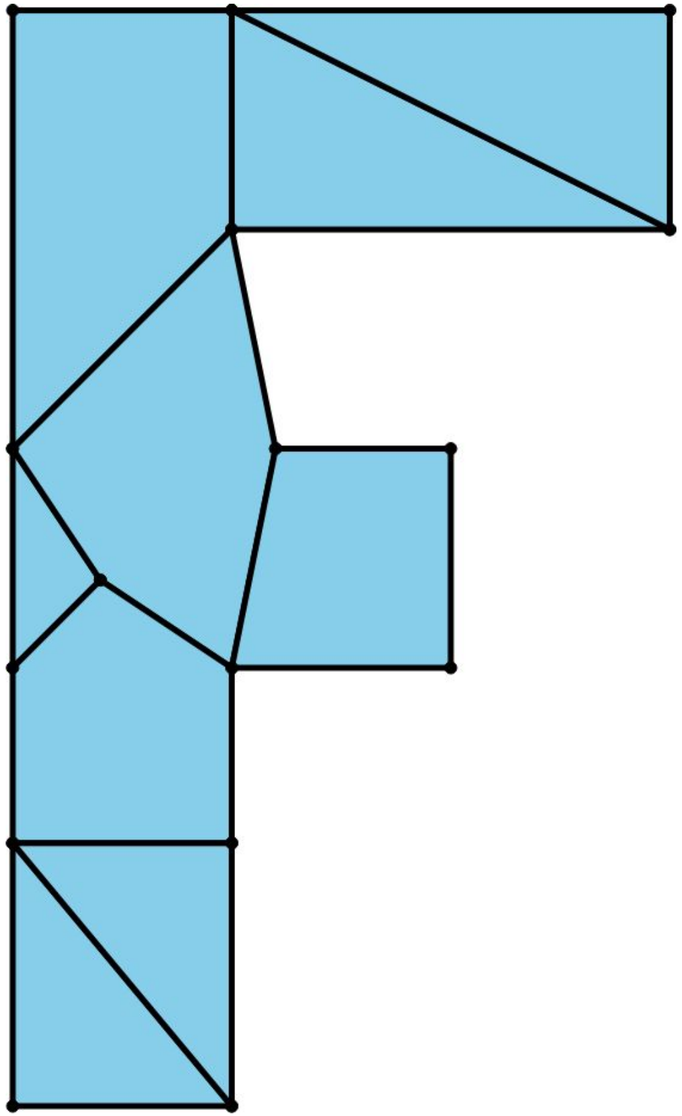


Problématique

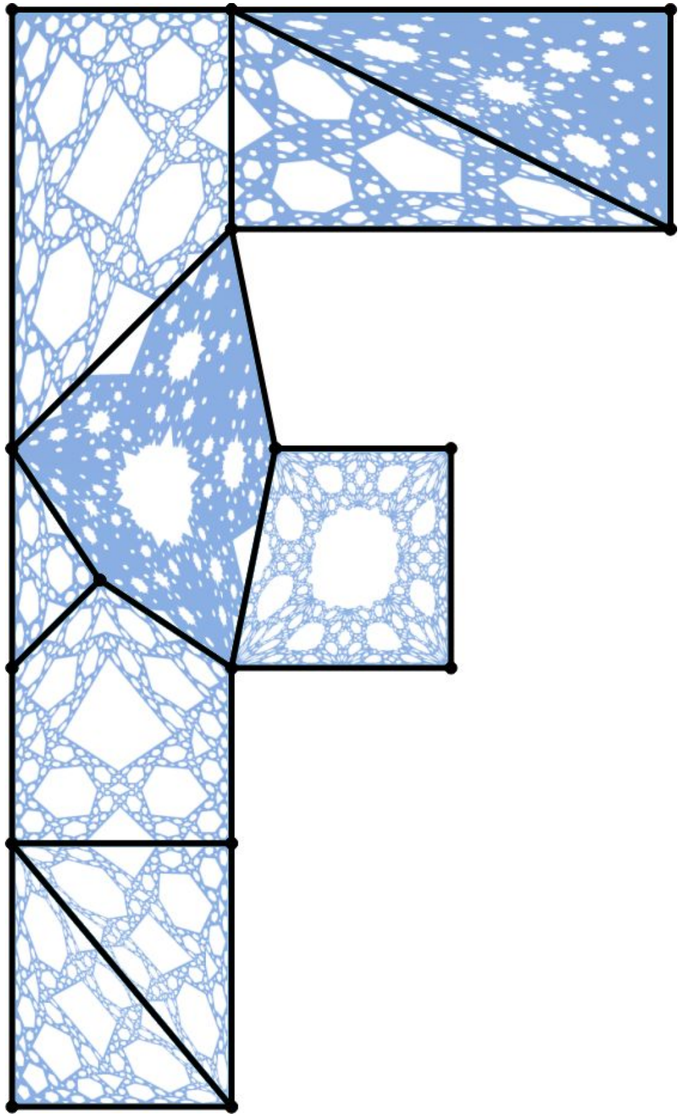
Problématique



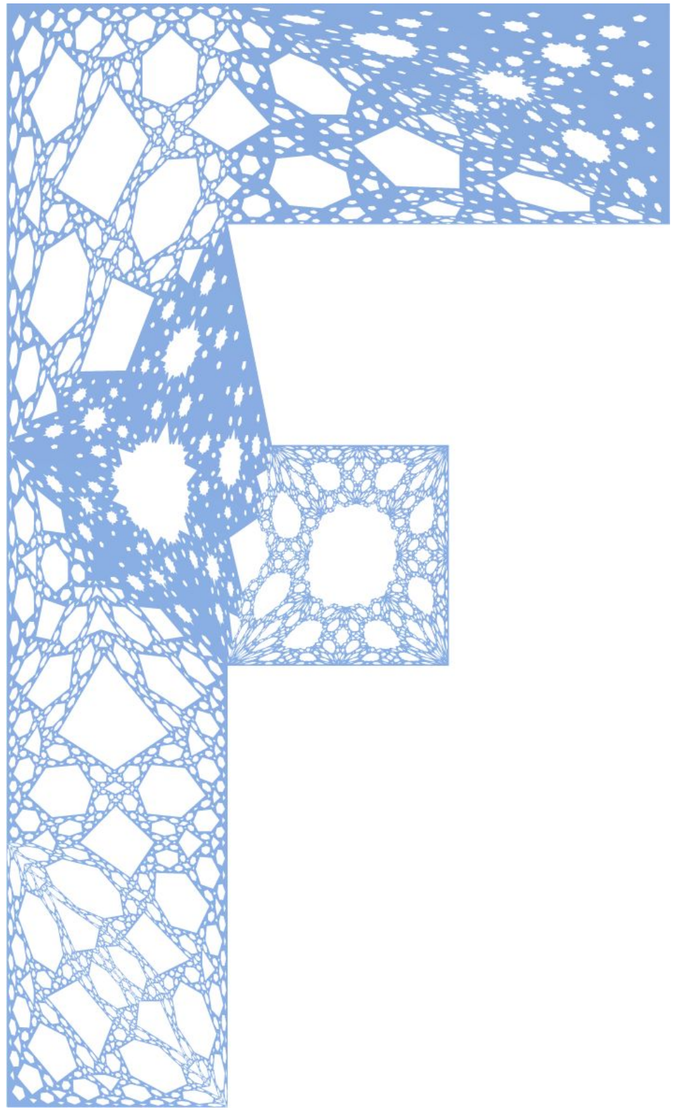
Problématique



Problématique

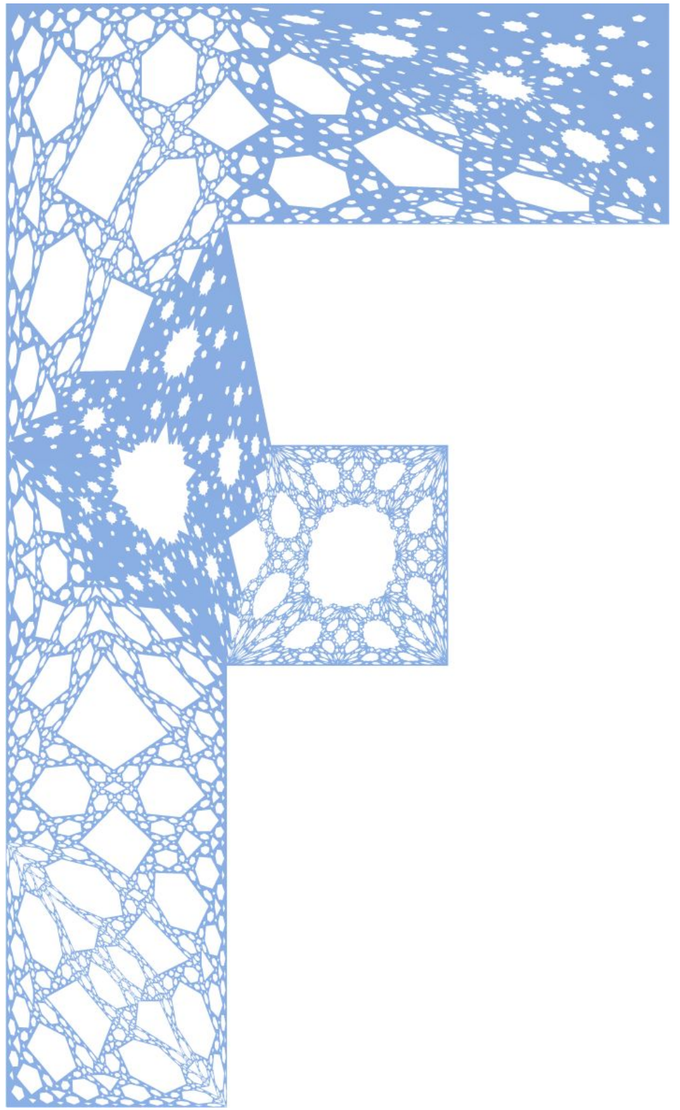


Problématique



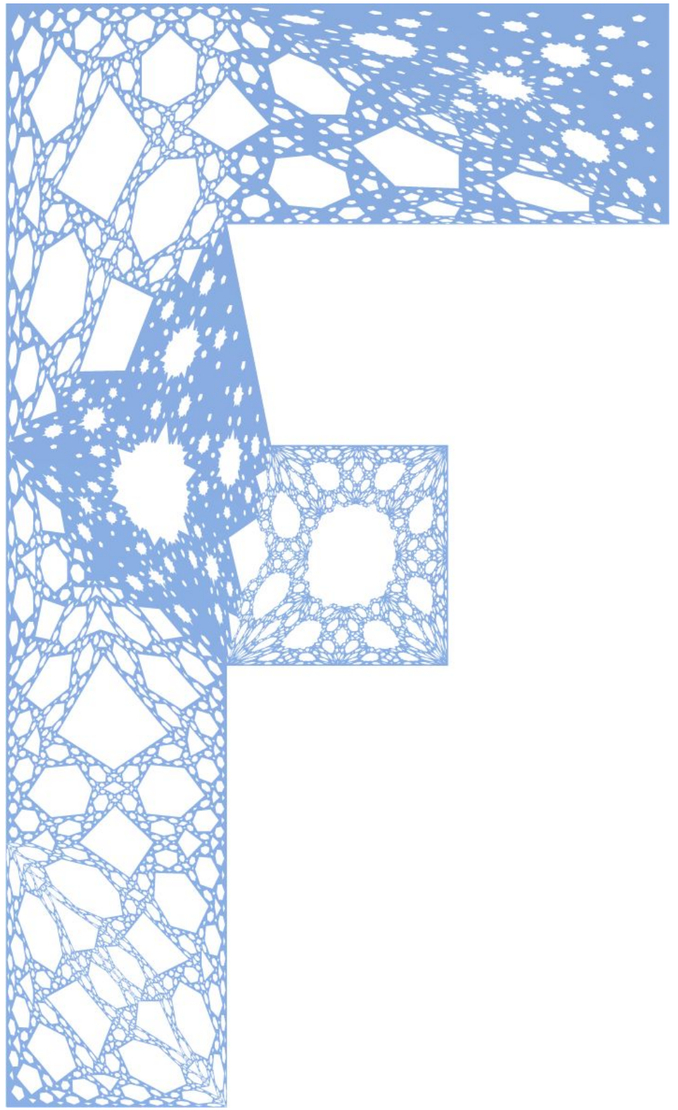
Démarche

- Utilisation du modèle BC-IFS
  - Gestion des raccords topologiques
  - Déformation avec des points de contrôle



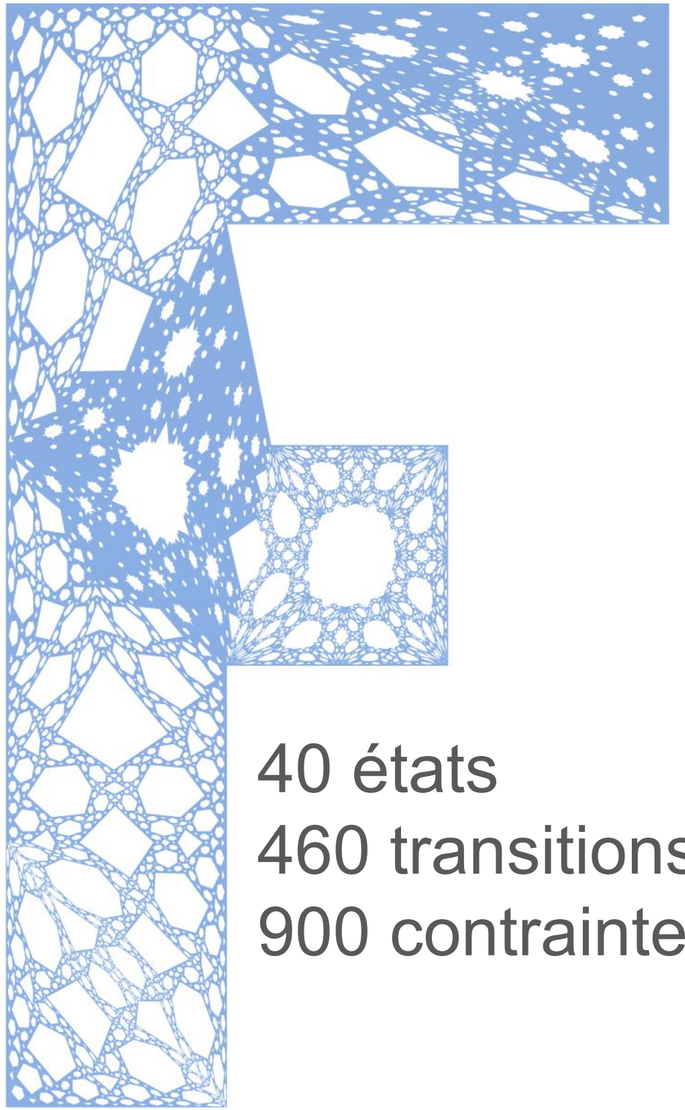
Démarche

- Utilisation du modèle BC-IFS
  - Gestion des raccords topologiques
  - Déformation avec des points de contrôle
- Modélisation fastidieuse
  - Imaginer le processus de subdivision
  - Le traduire avec un BC-IFS (automate + contraintes)



Démarche

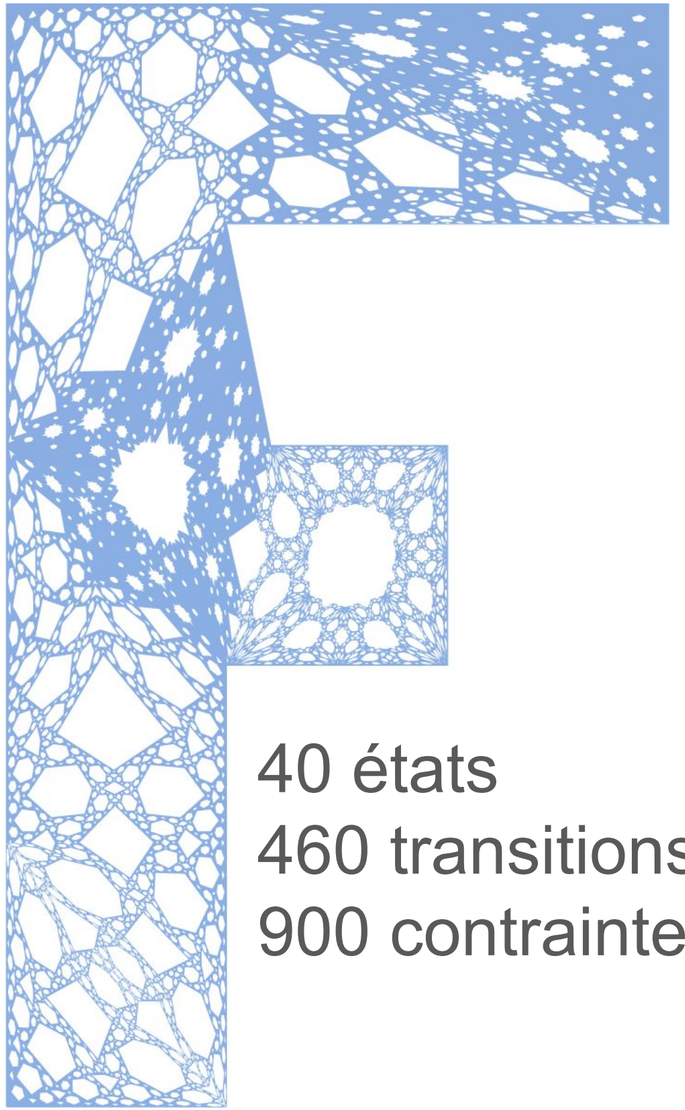
- Utilisation du modèle BC-IFS
  - Gestion des raccords topologiques
  - Déformation avec des points de contrôle
- Modélisation fastidieuse
  - Imaginer le processus de subdivision
  - Le traduire avec un BC-IFS (automate + contraintes)



40 états
460 transitions
900 contraintes

Démarche

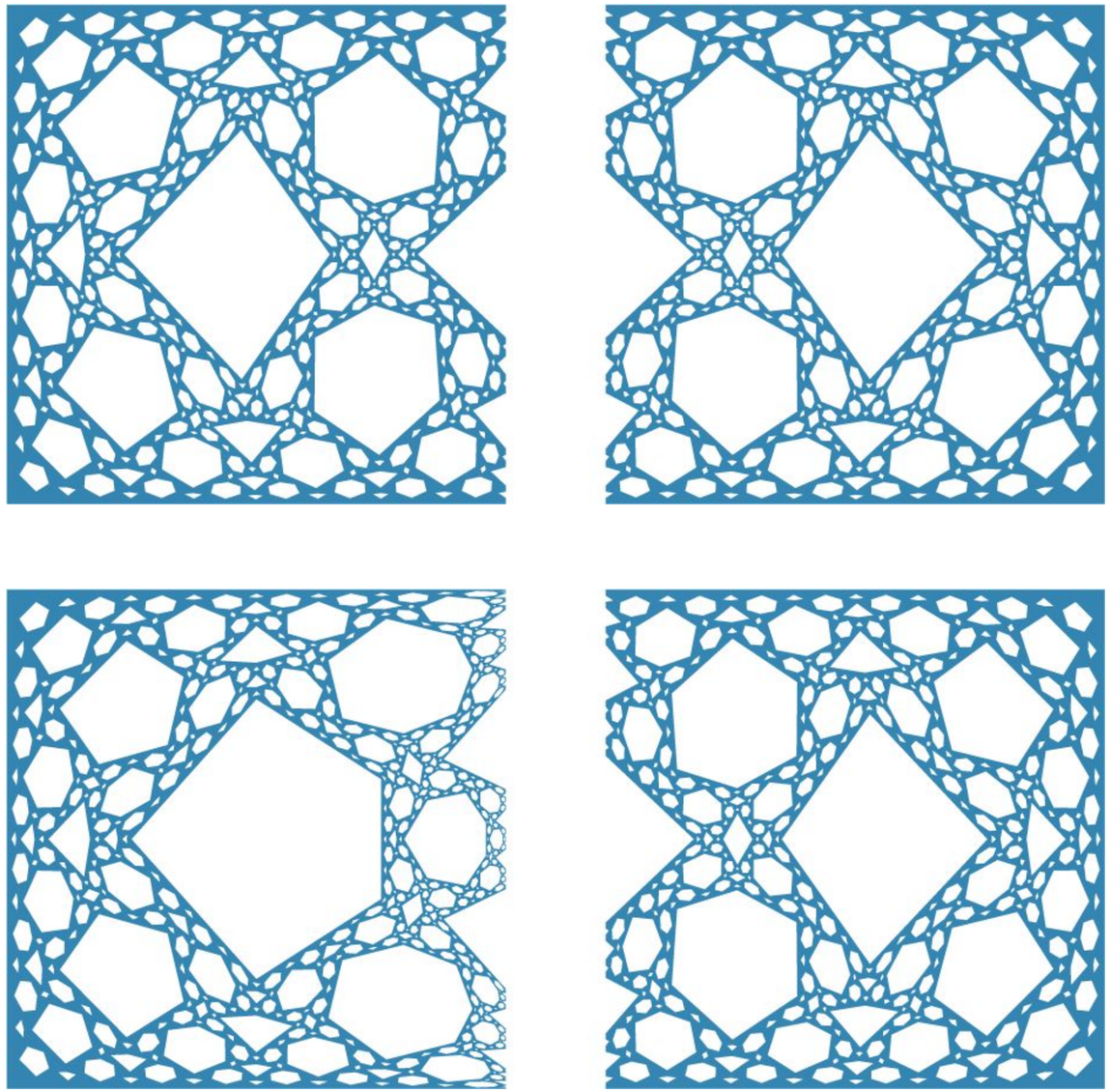
- Utilisation du modèle BC-IFS
  - Gestion des raccords topologiques
  - Déformation avec des points de contrôle
- Modélisation fastidieuse
  - Imaginer le processus de subdivision
  - Le traduire avec un BC-IFS (automate + contraintes)
- Nécessité de faciliter la conception
  - 3 méthodes de conception



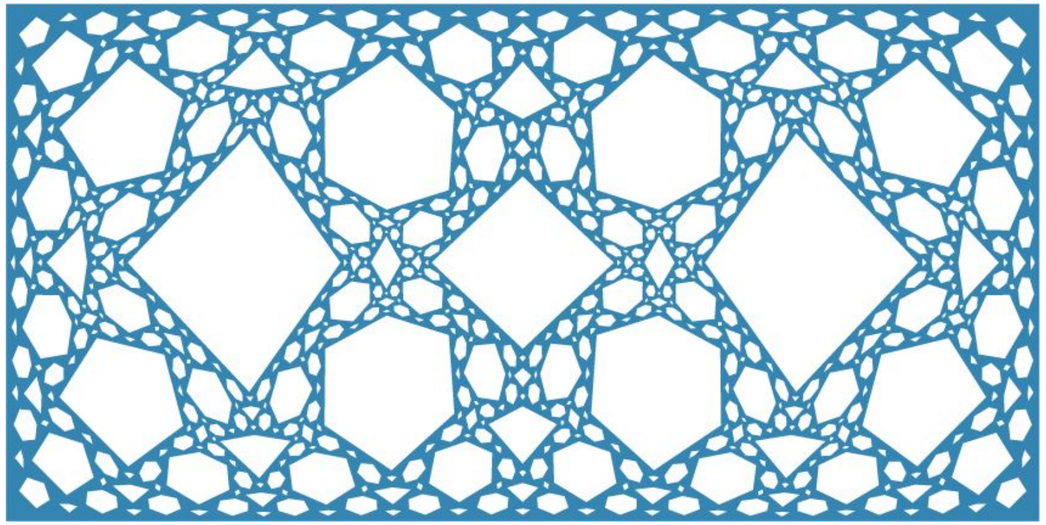
40 états
460 transitions
900 contraintes

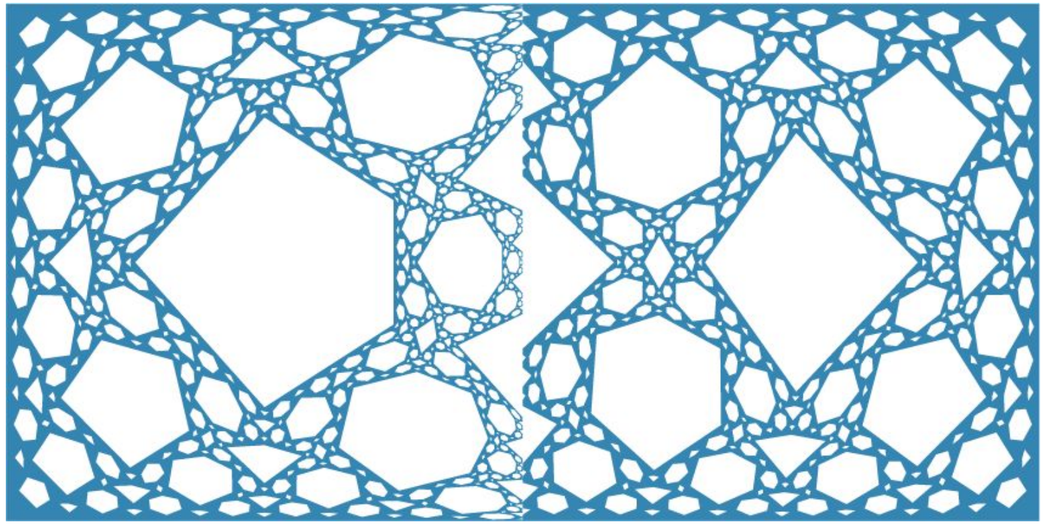
Conception automatique 2D à partir des bords

Pourquoi à partir des bords ?



Pourquoi à partir des bords ?





Les types de bords

- Bords de type Cantor
  - Subdivisions non connexes de type Cantor



C_2

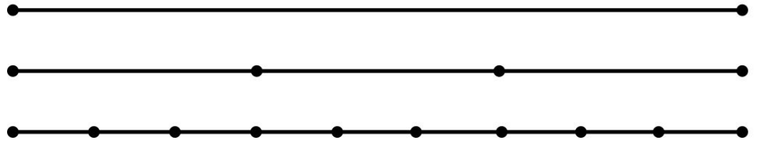


C_3

- Bords de type Courbe
  - Subdivisions connexes de type Courbe



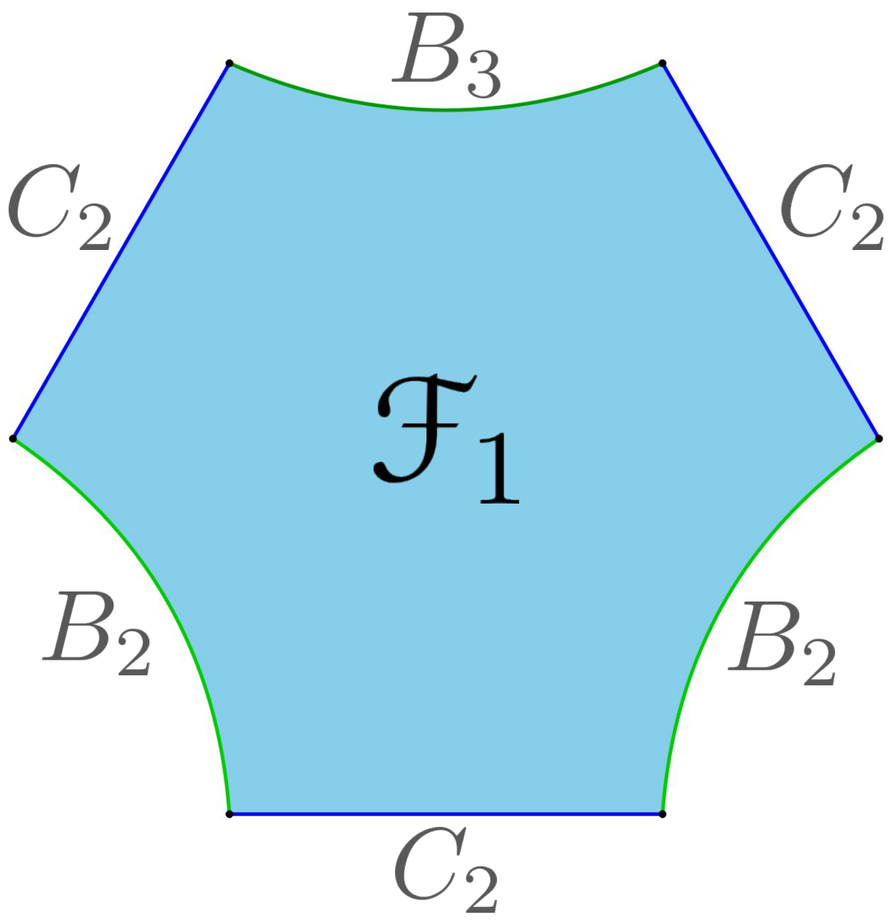
B_2



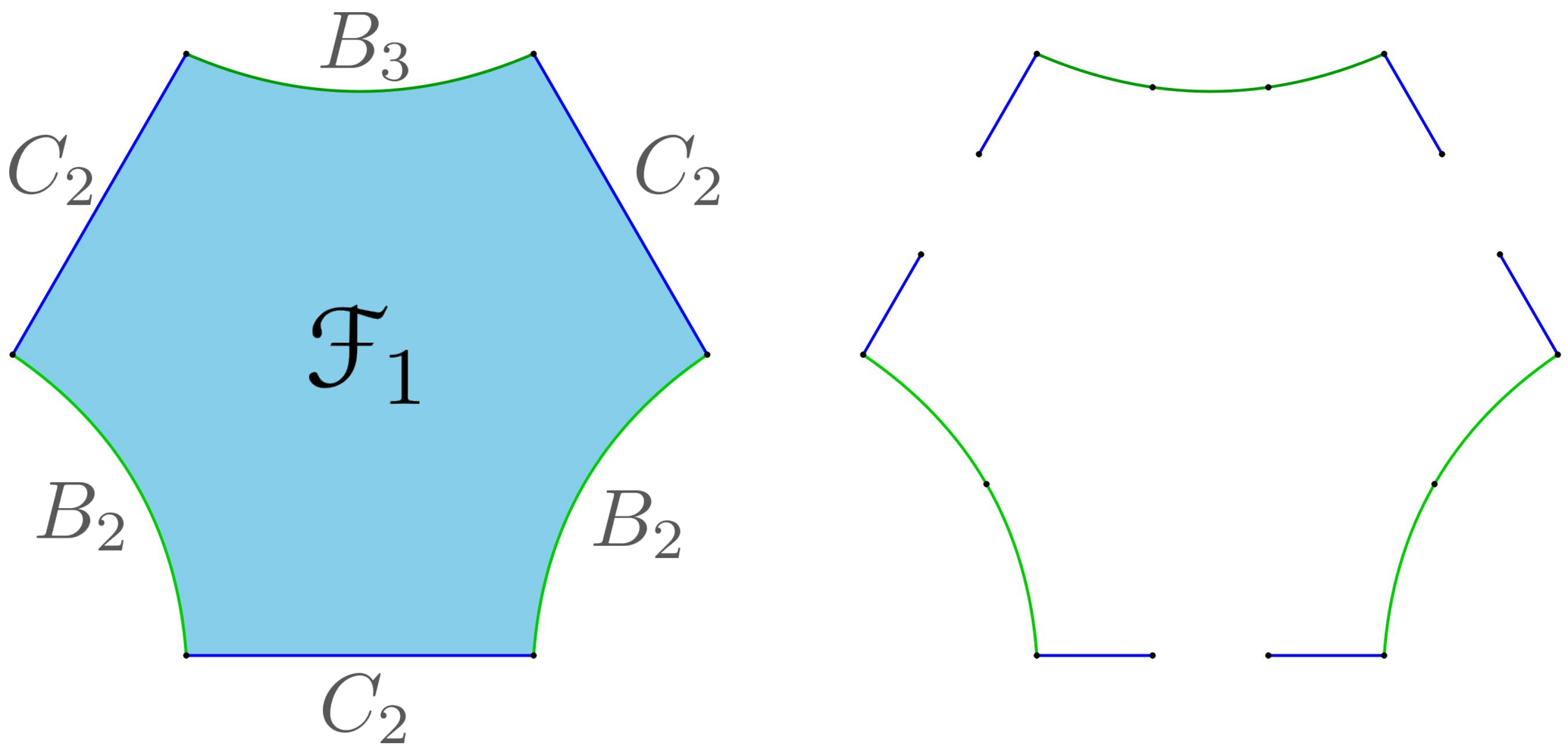
B_3

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Comment définir les règles de subdivision ?

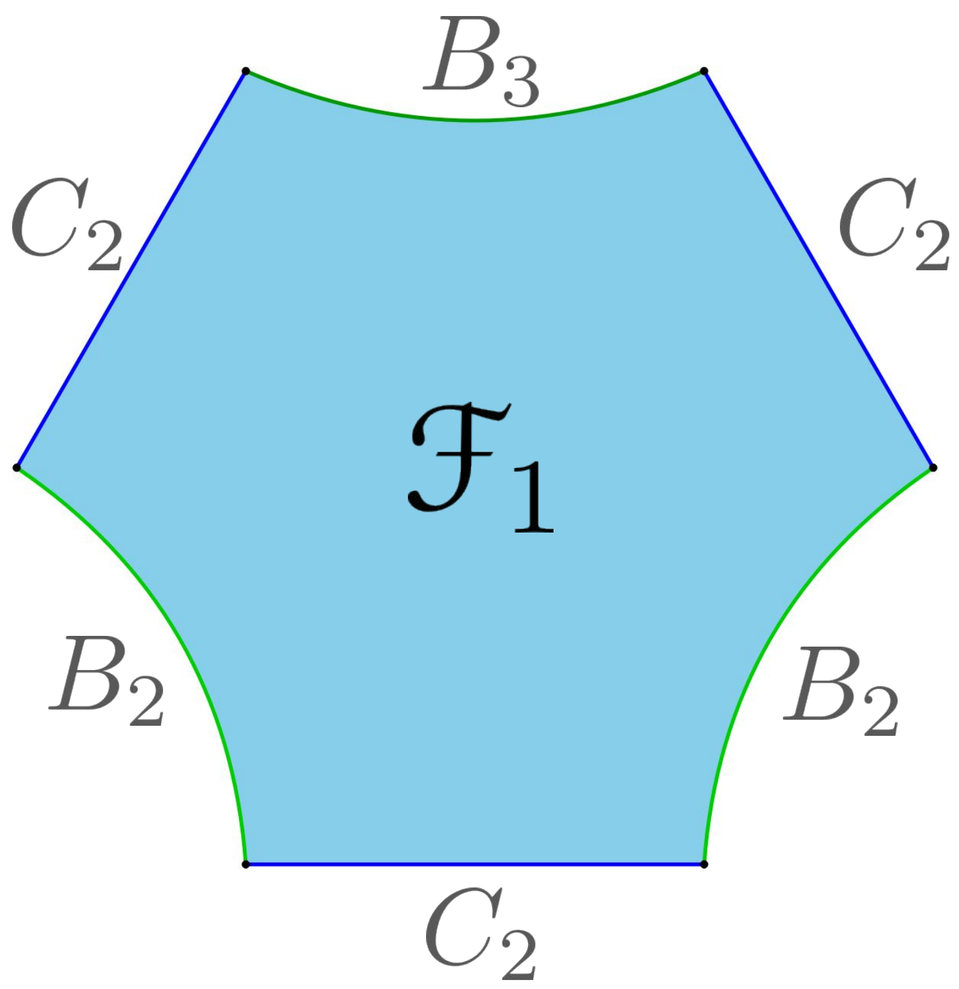


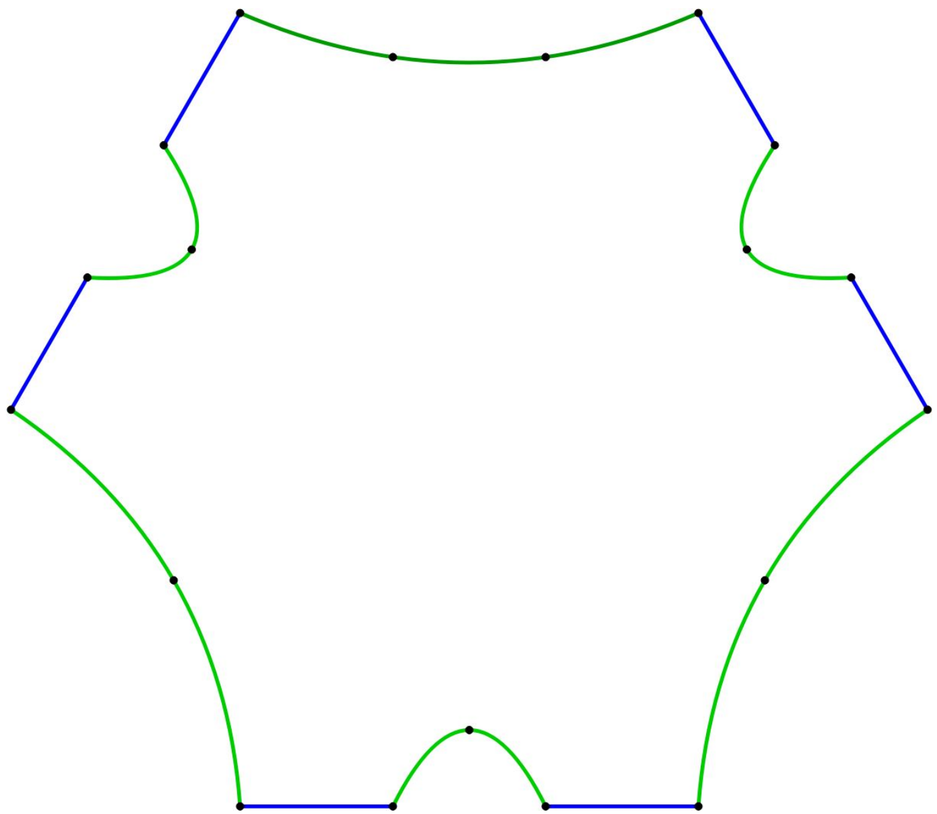
Processus de subdivision



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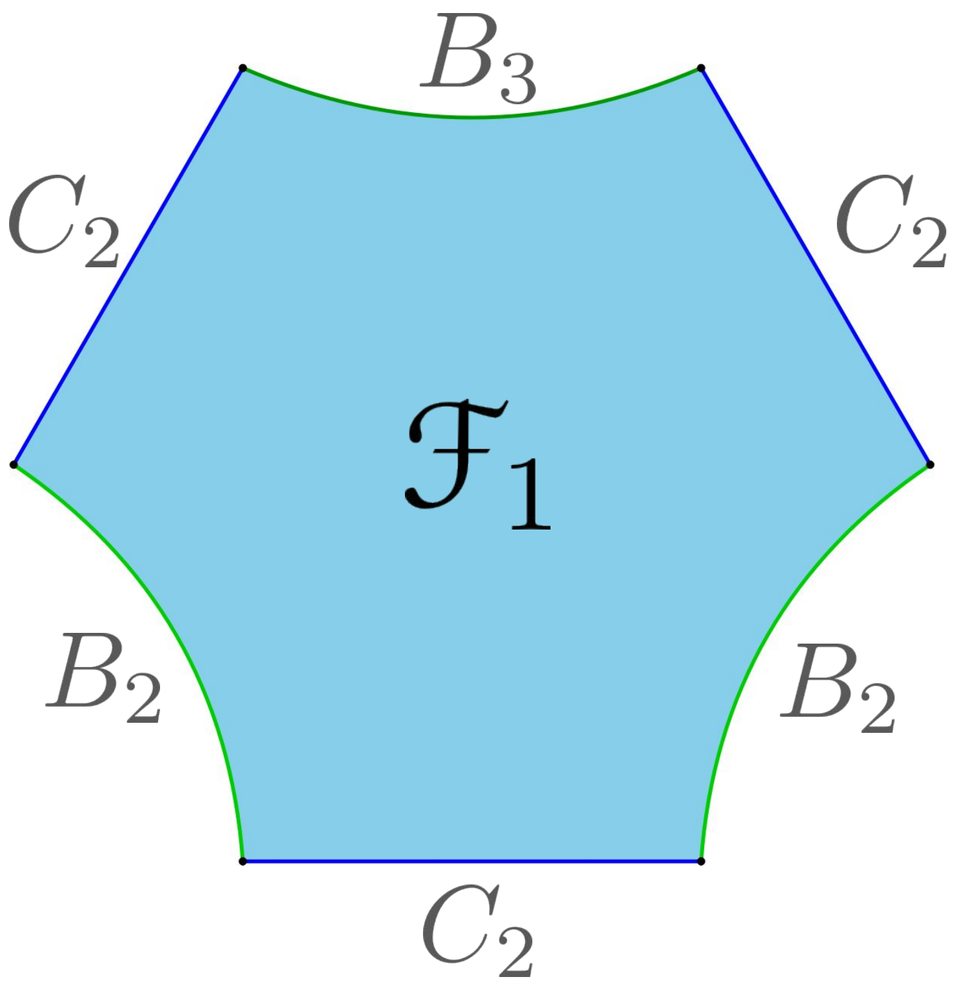
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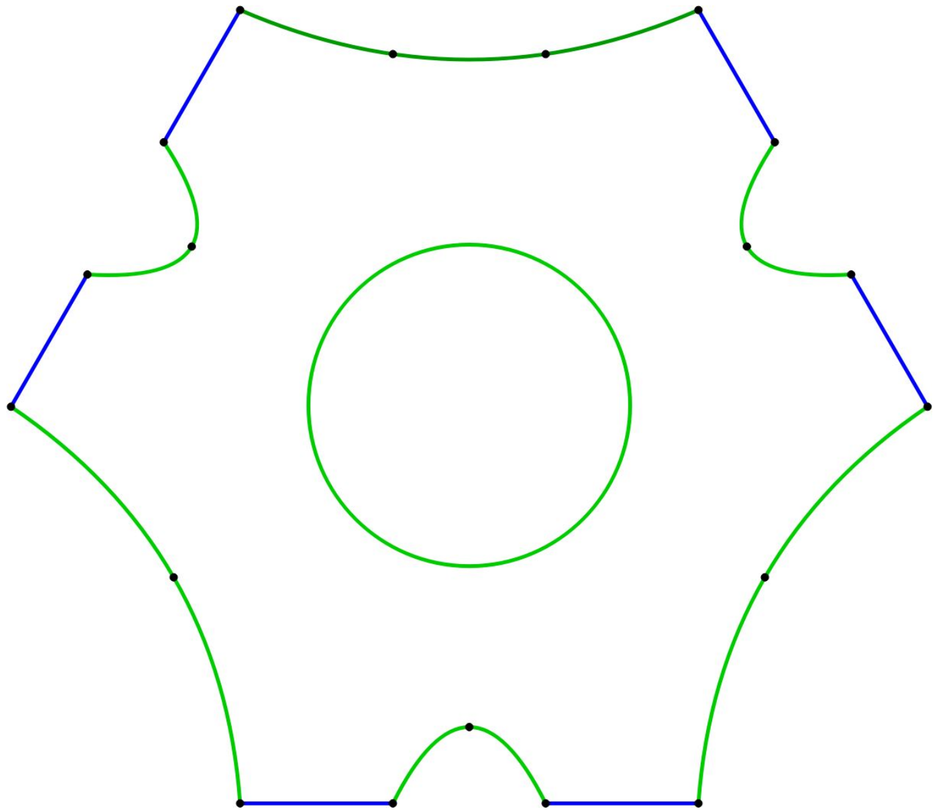




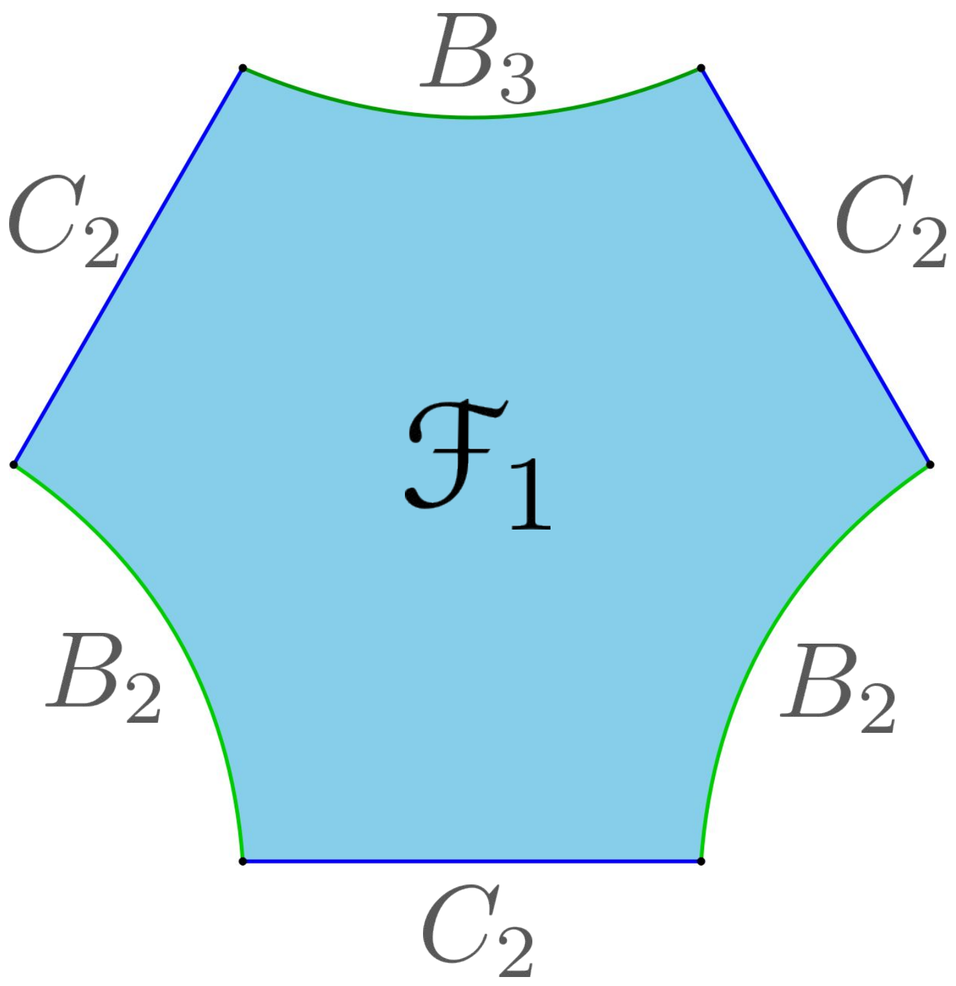
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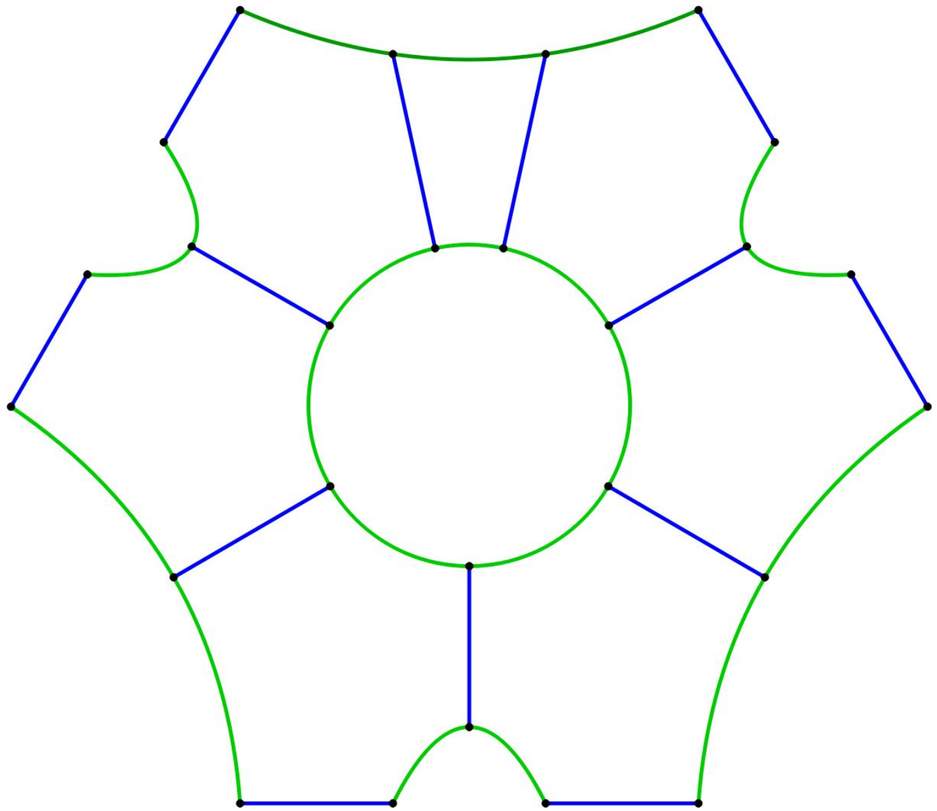
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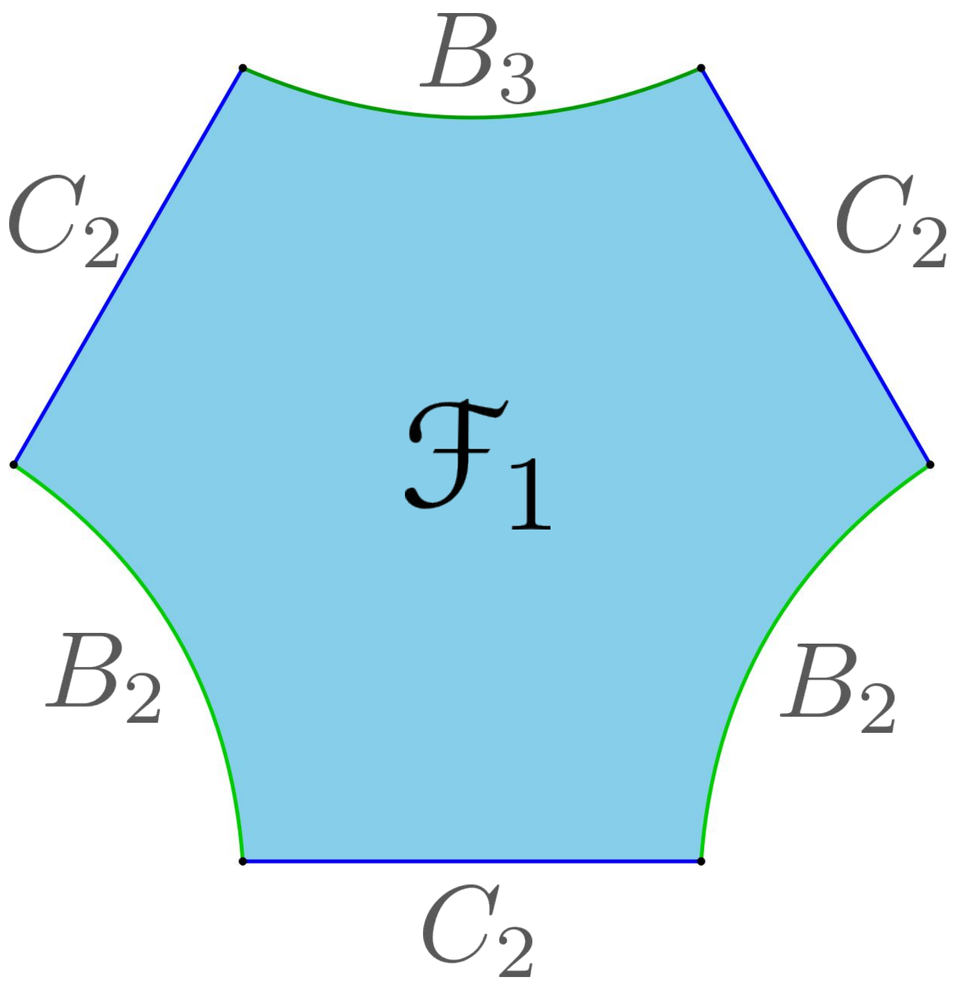
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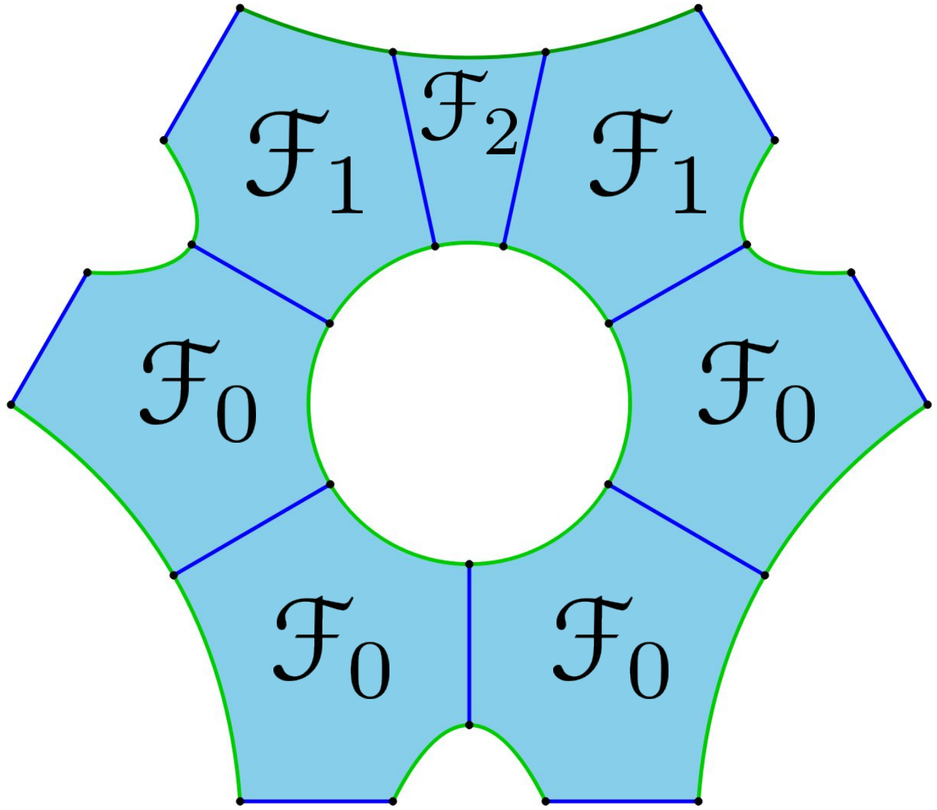




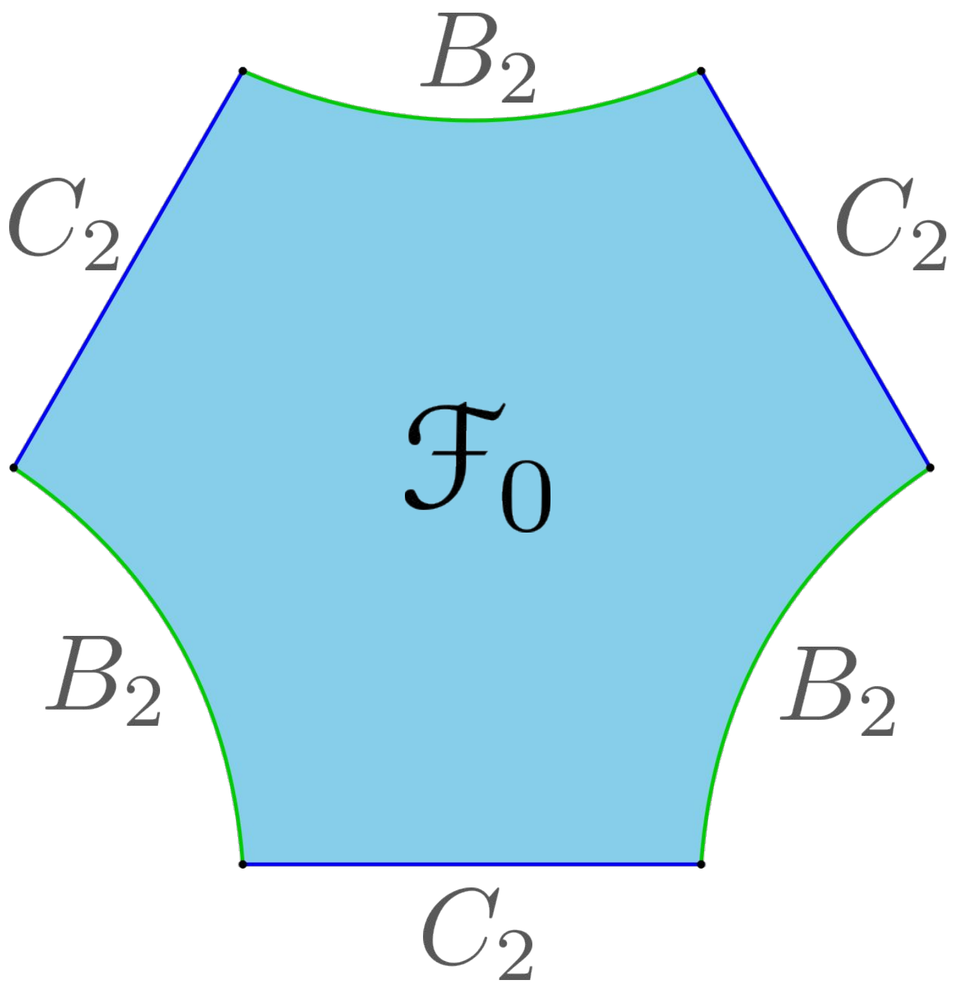
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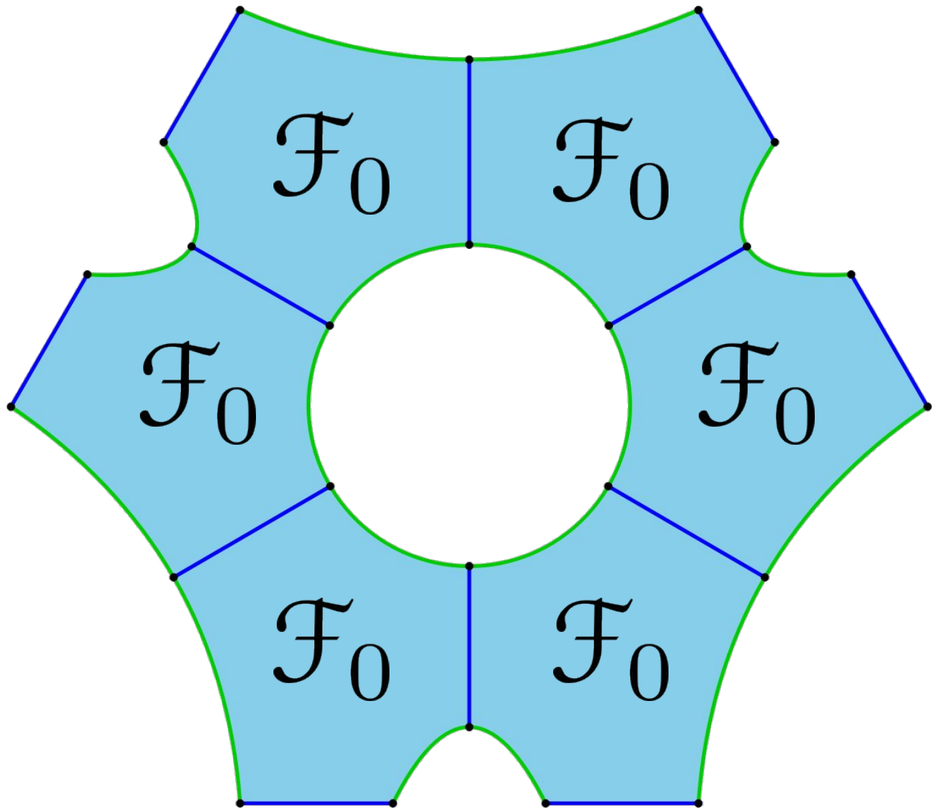
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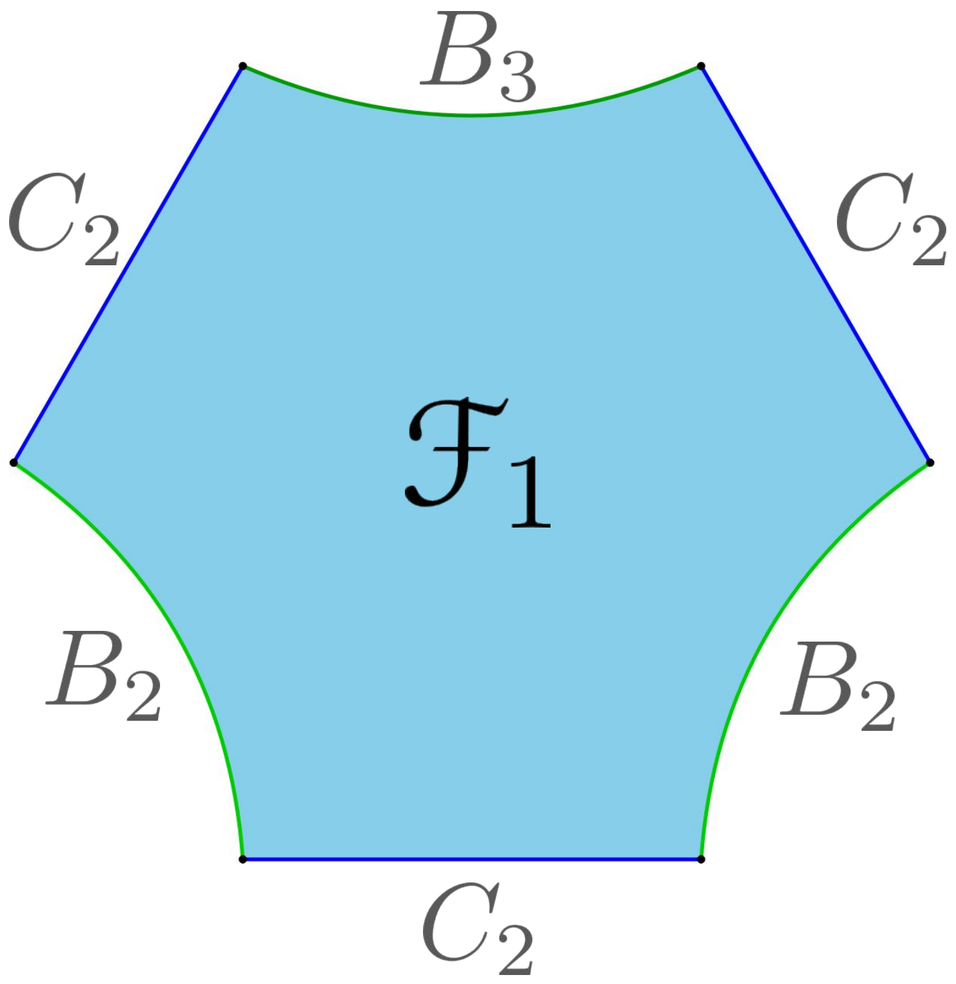


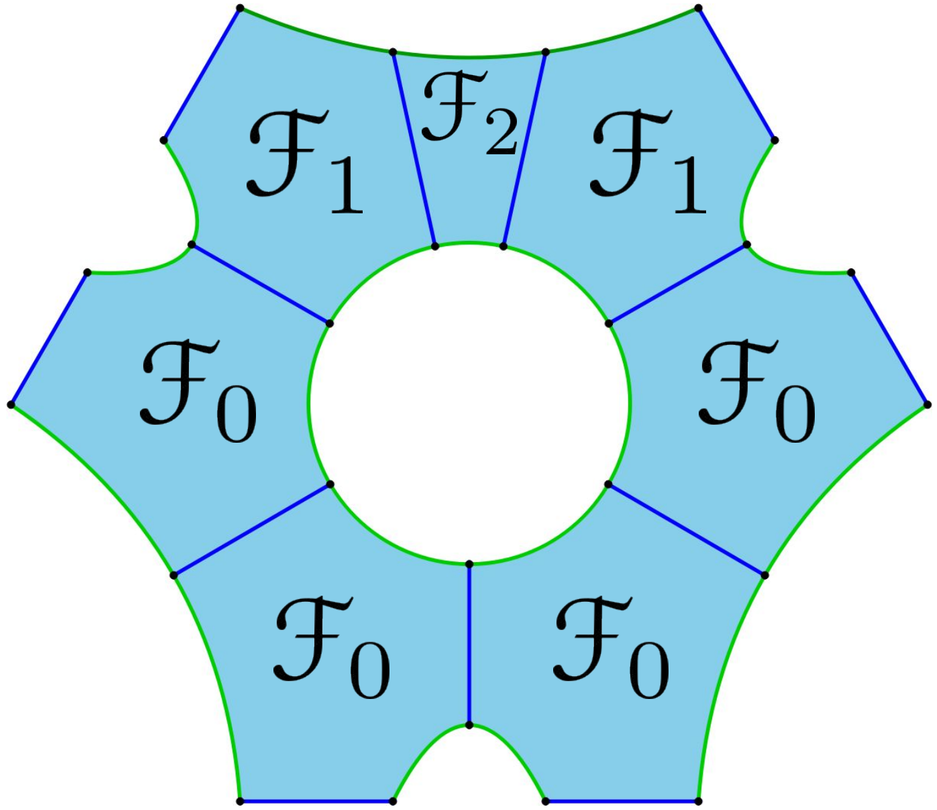
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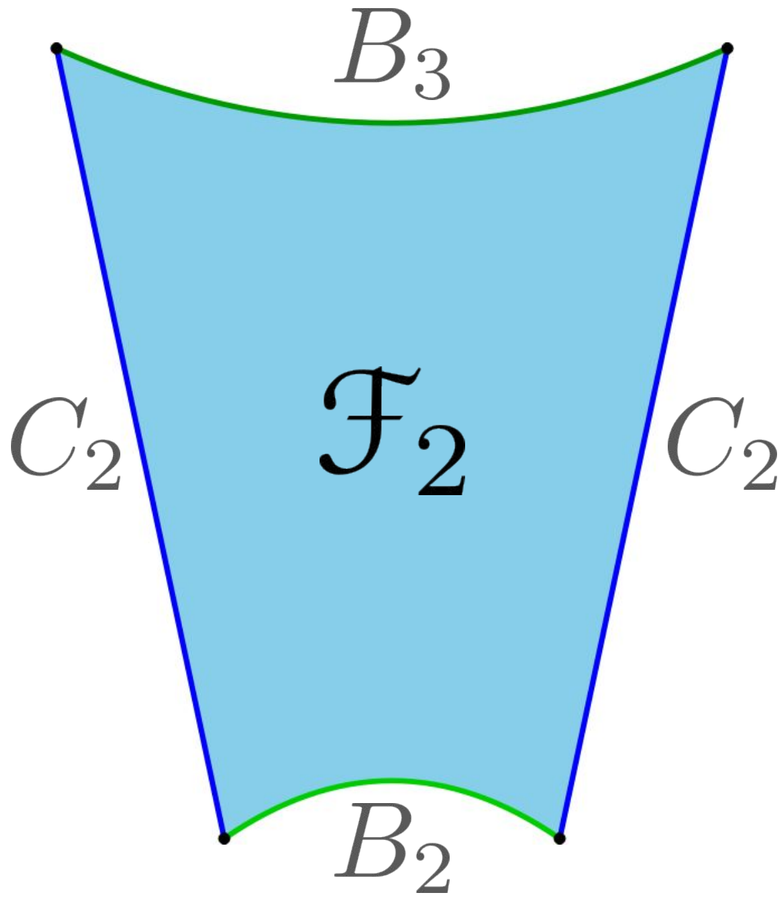


Processus de subdivision

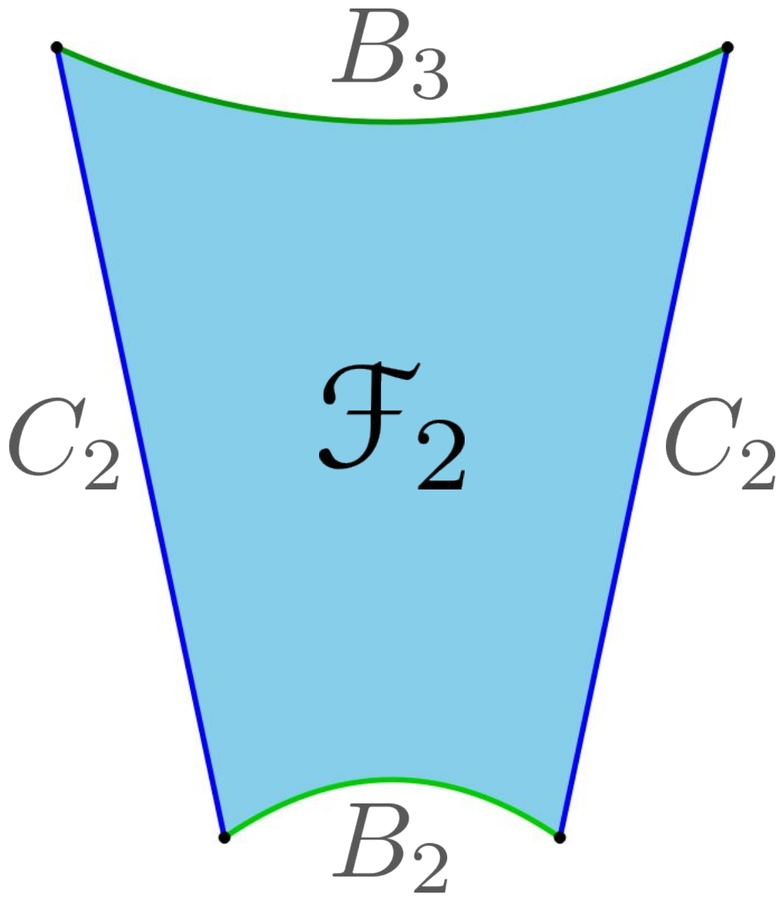


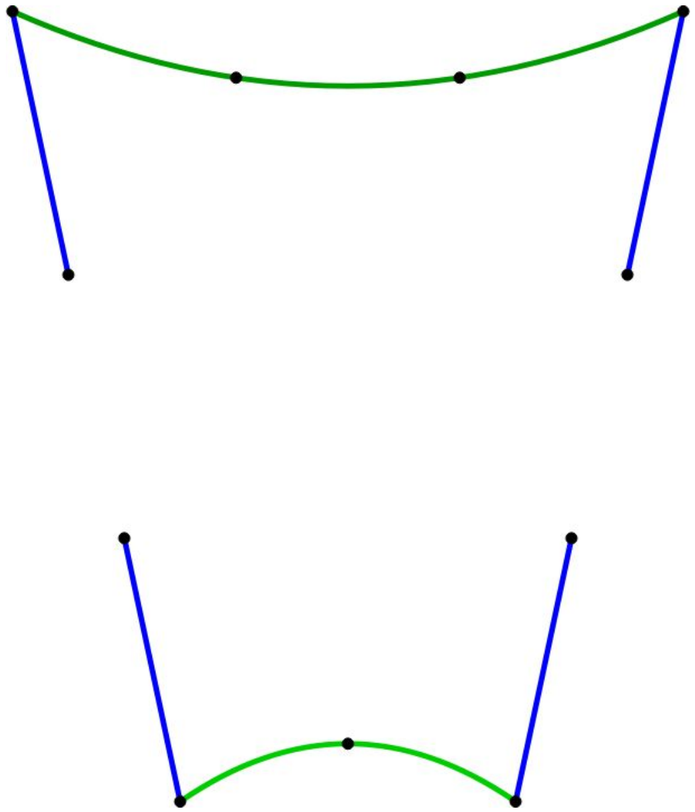


Processus de subdivision

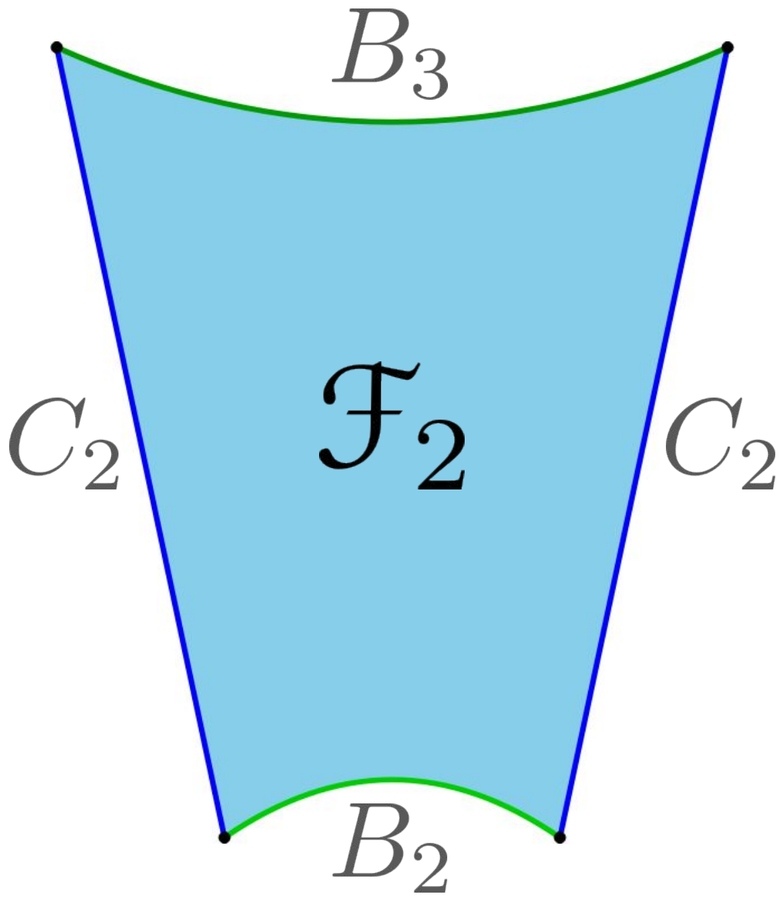


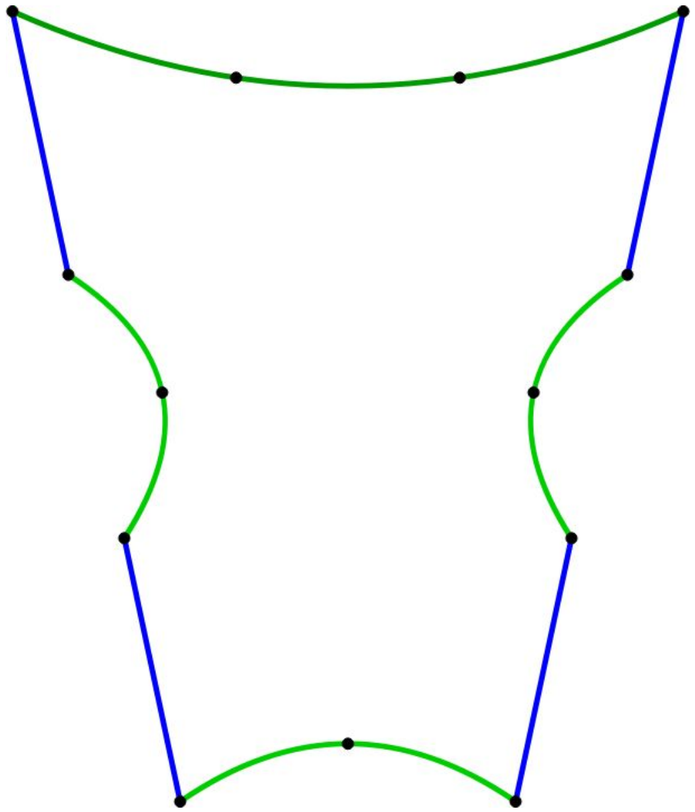
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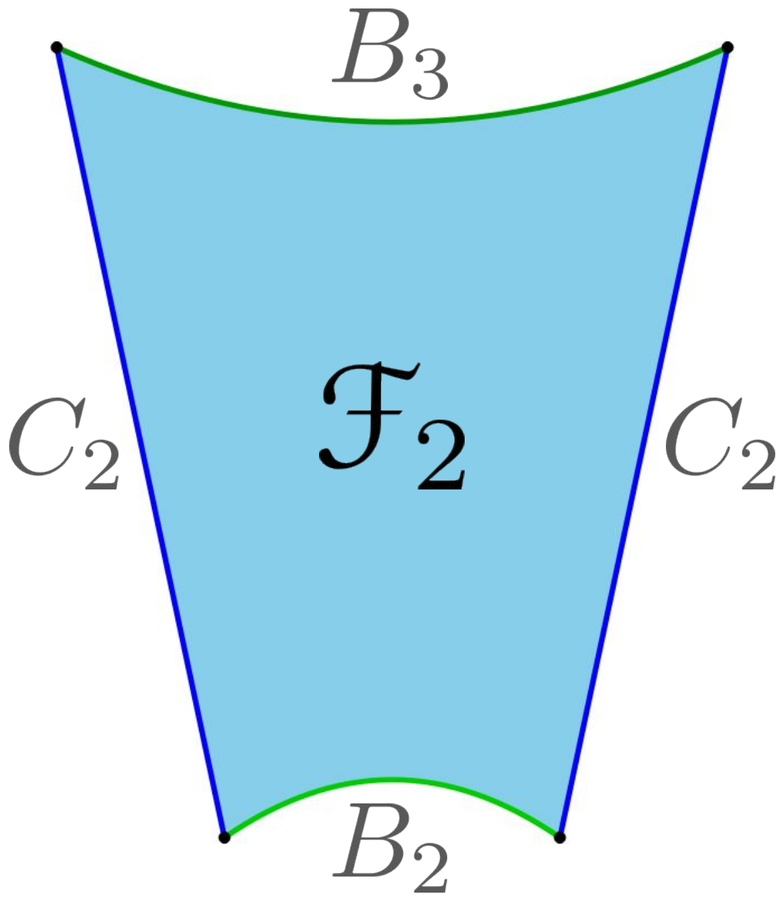


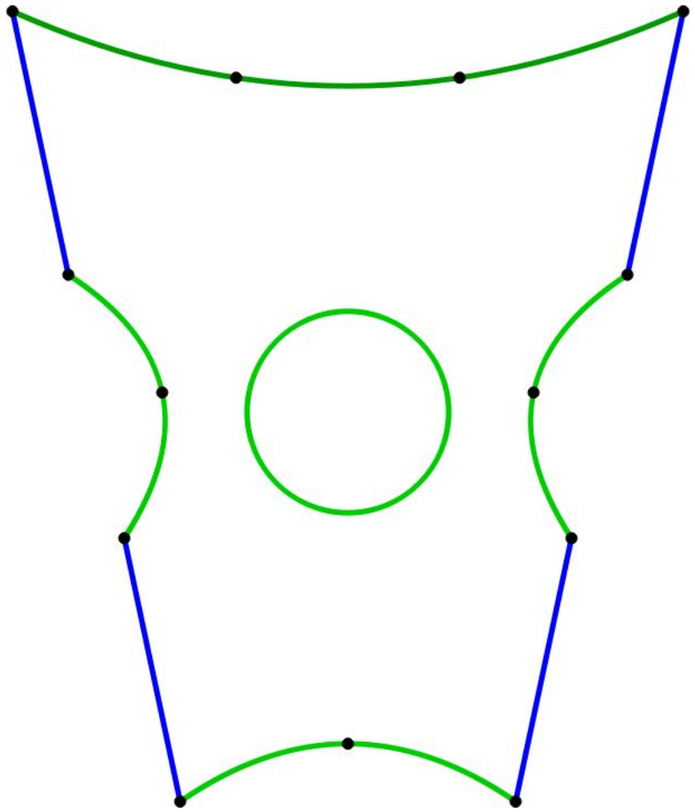
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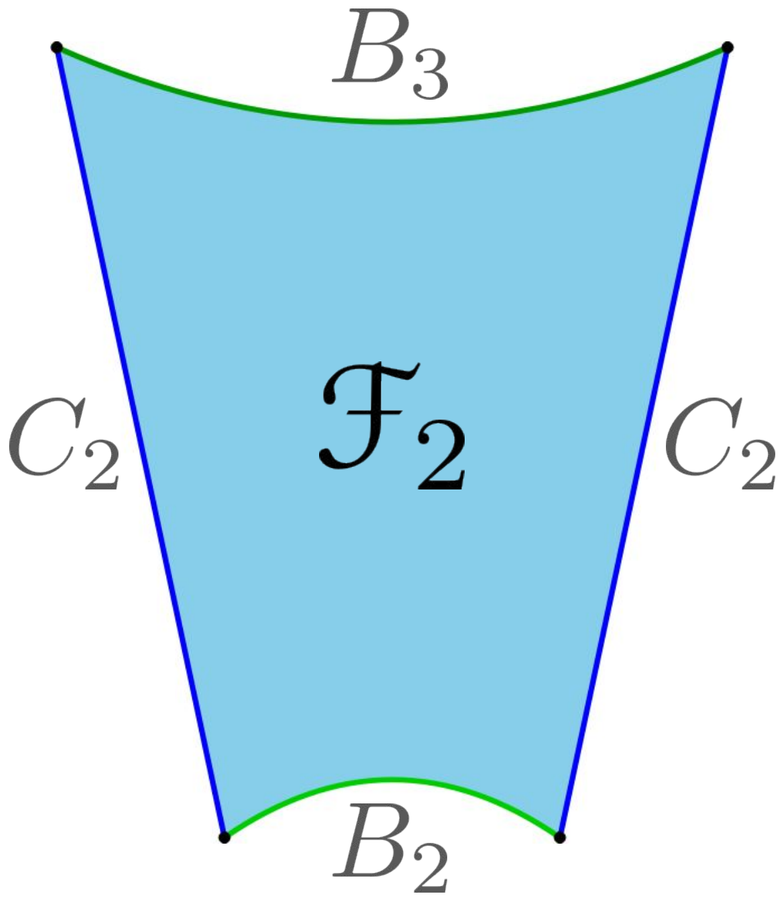


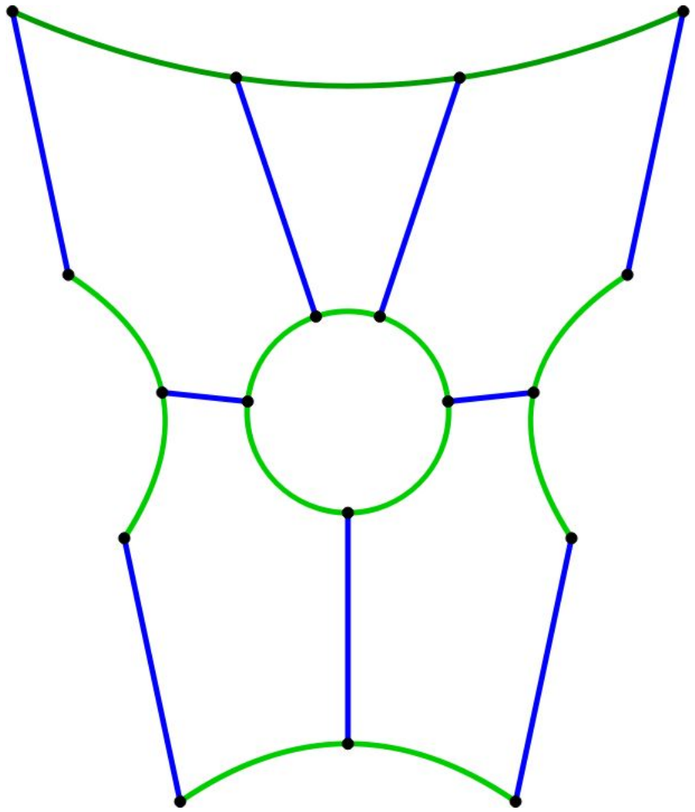
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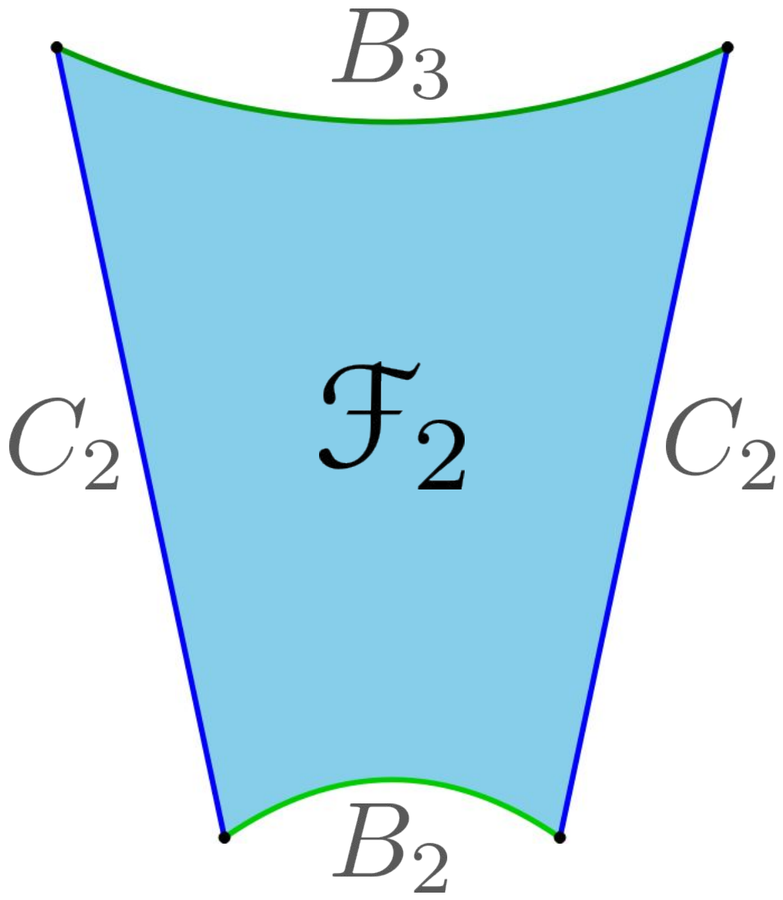


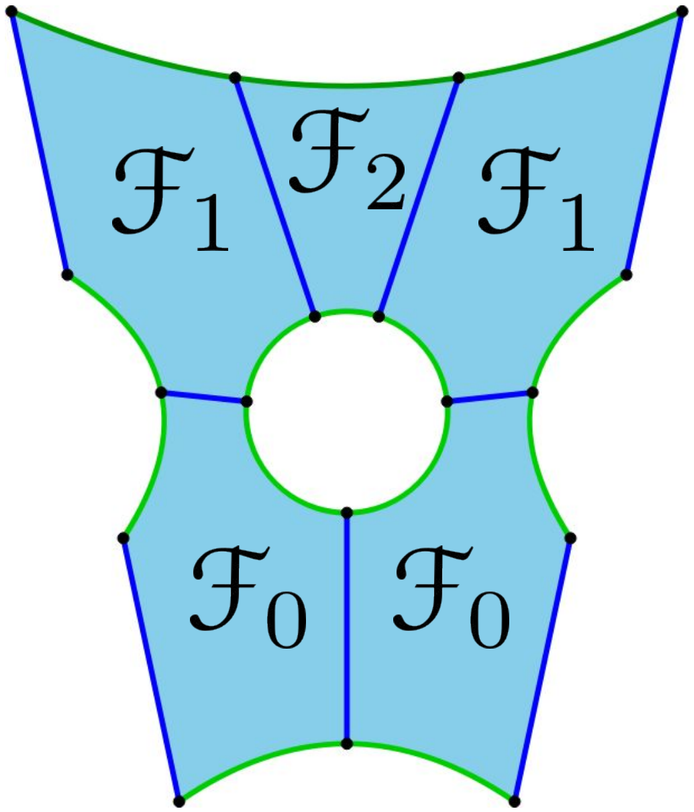
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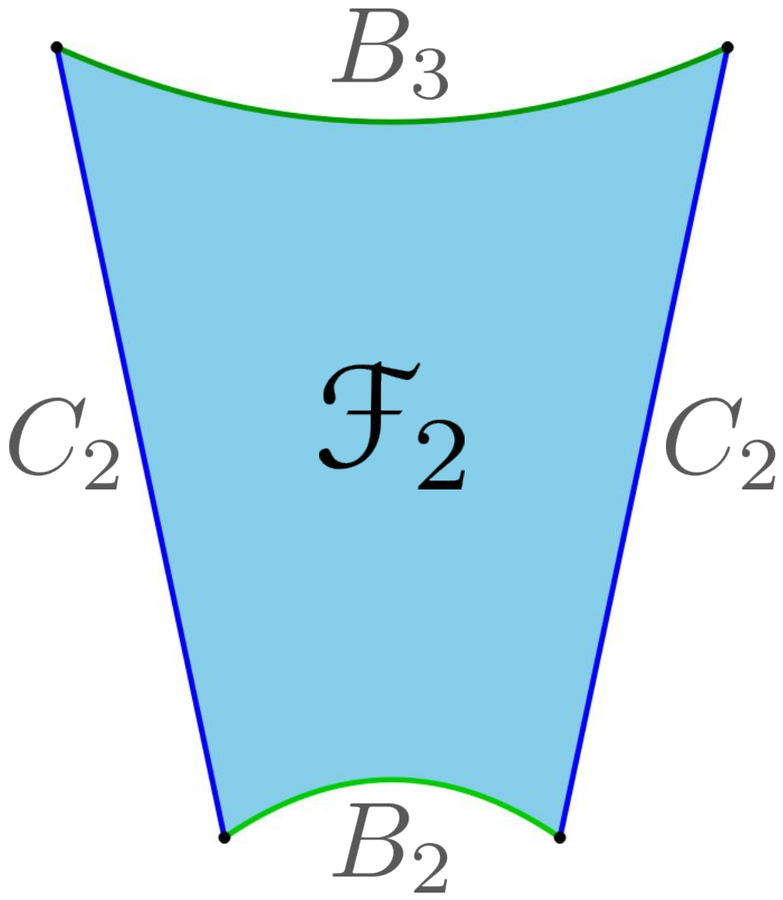


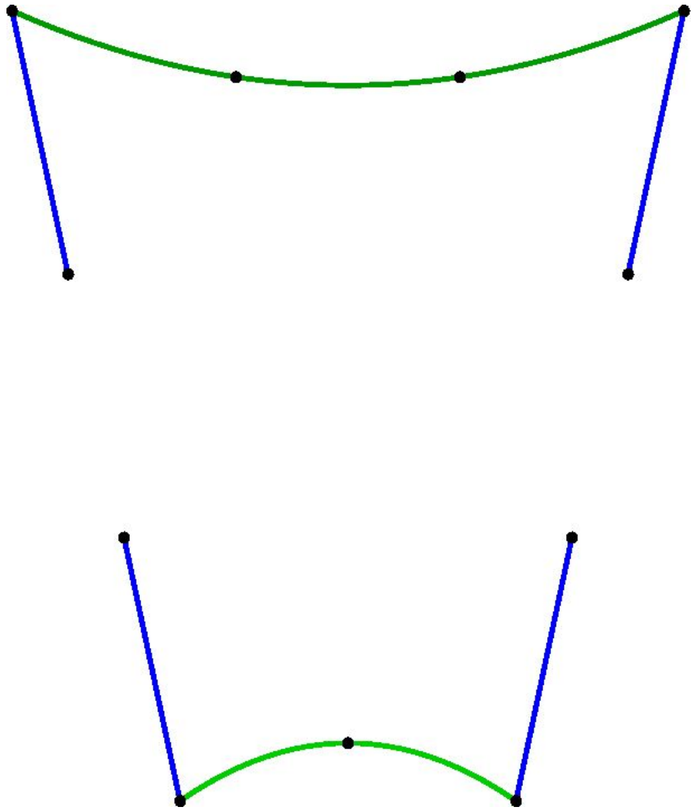
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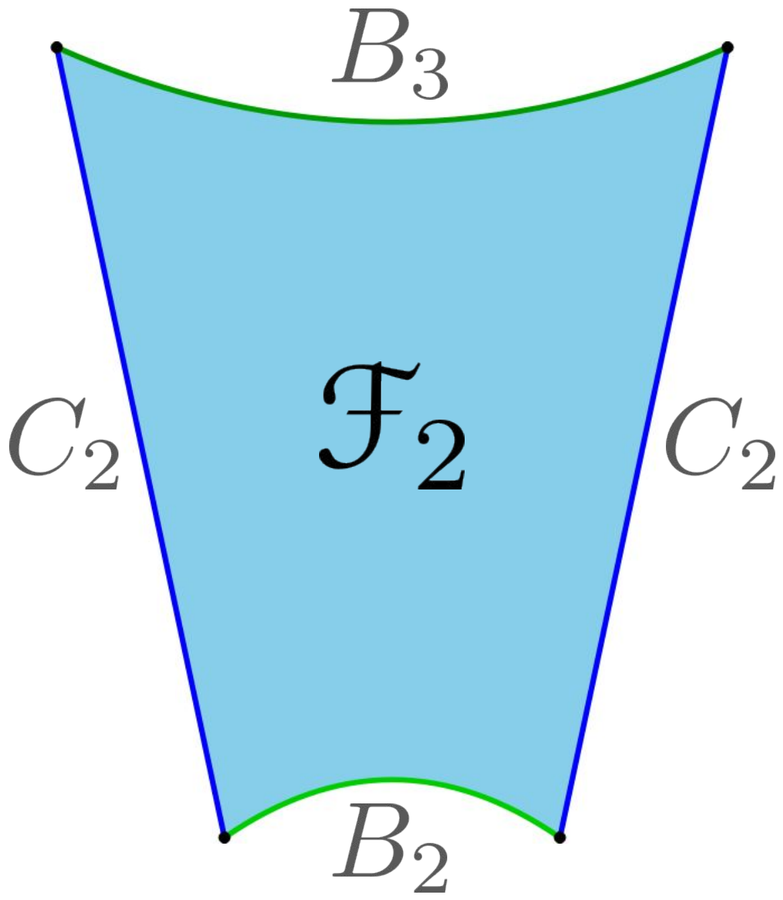


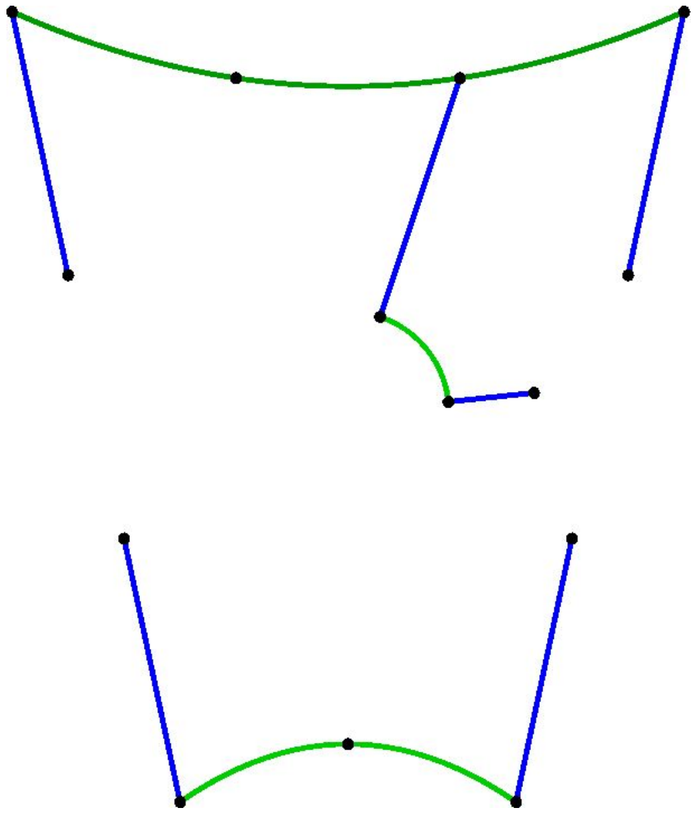
Algorithme de traduction en BC-IFS



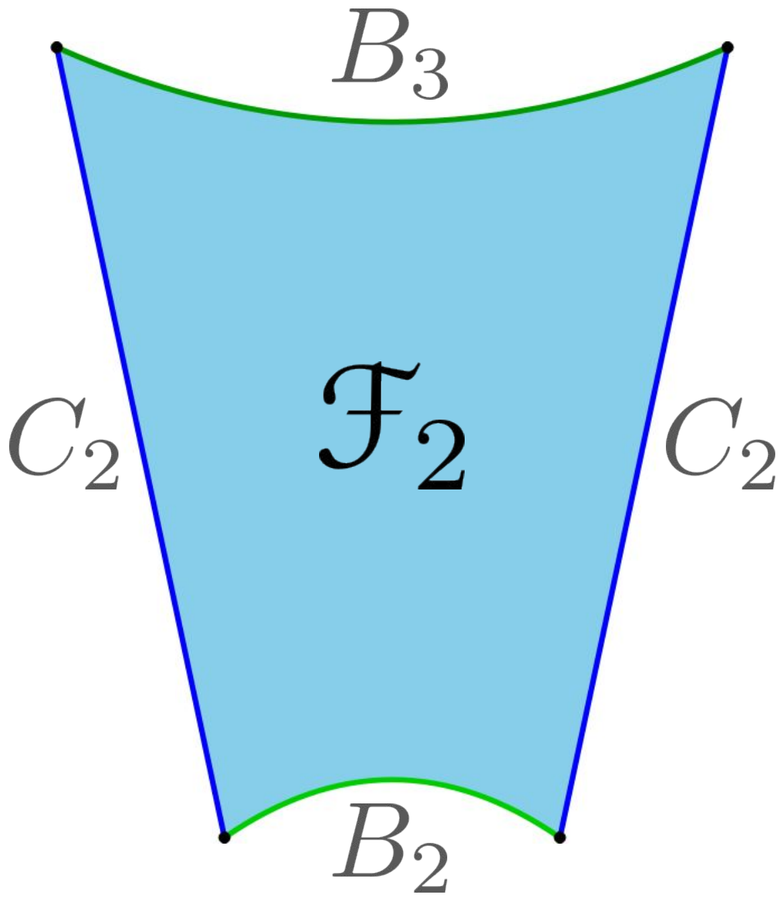


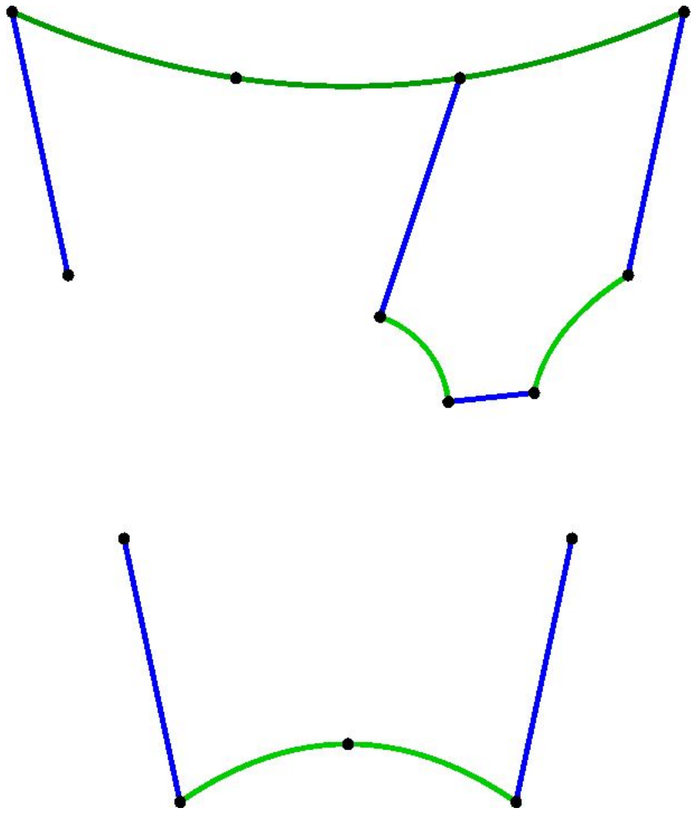
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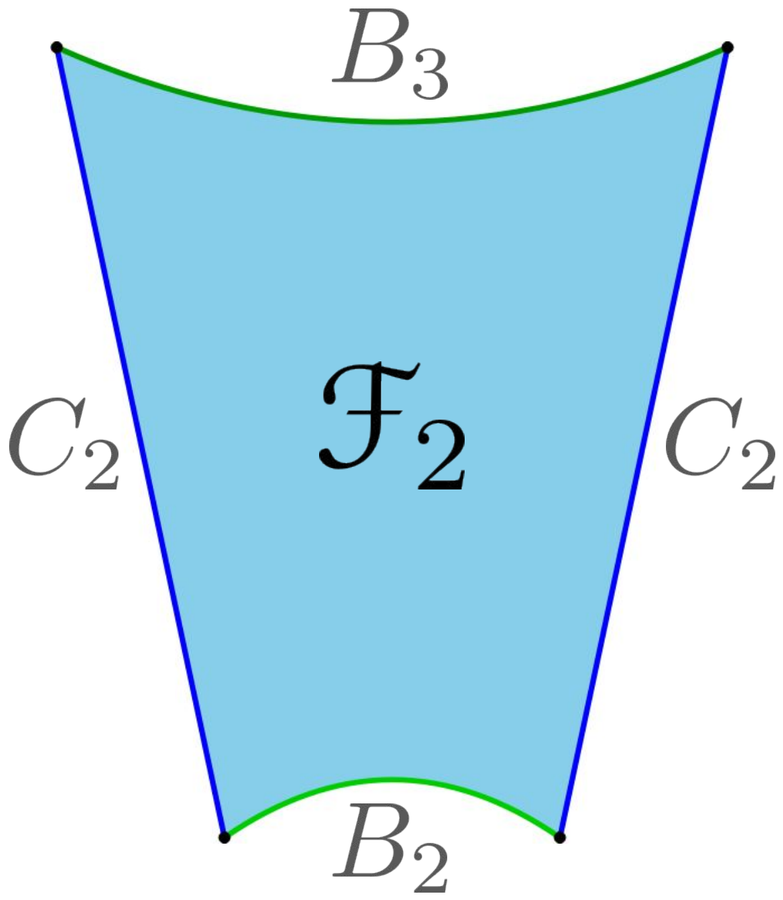


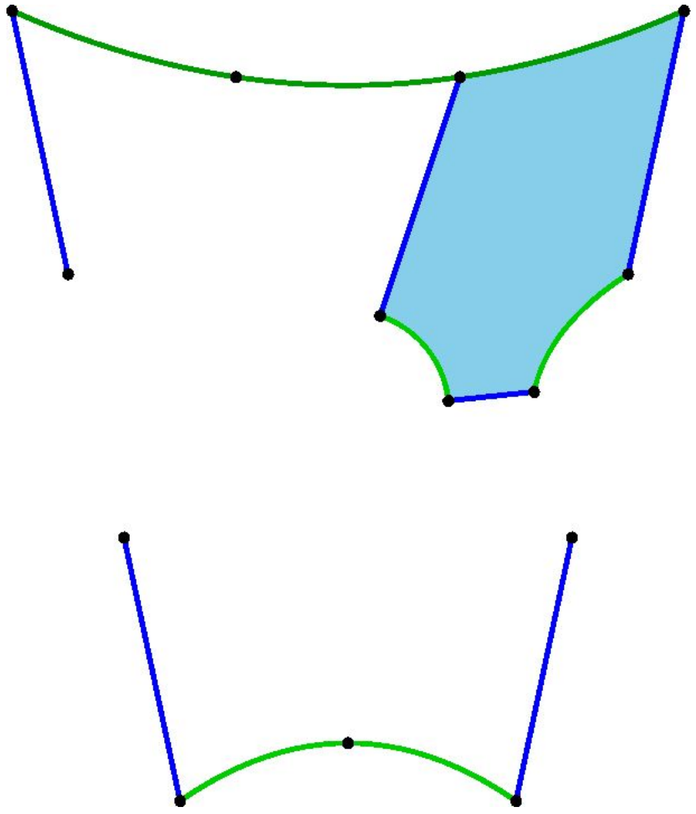
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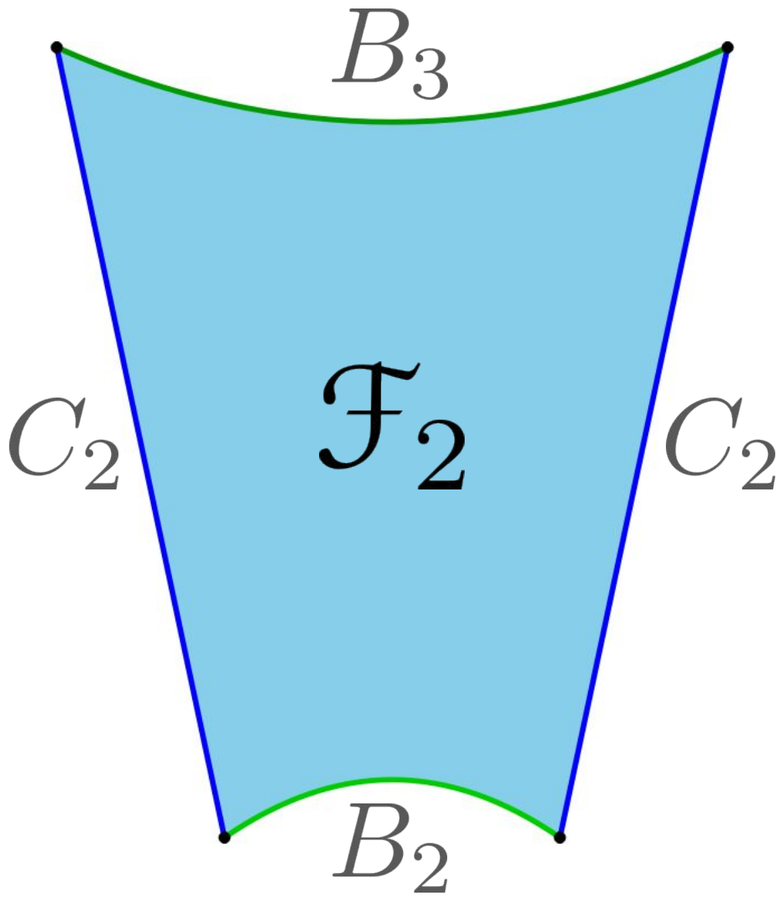


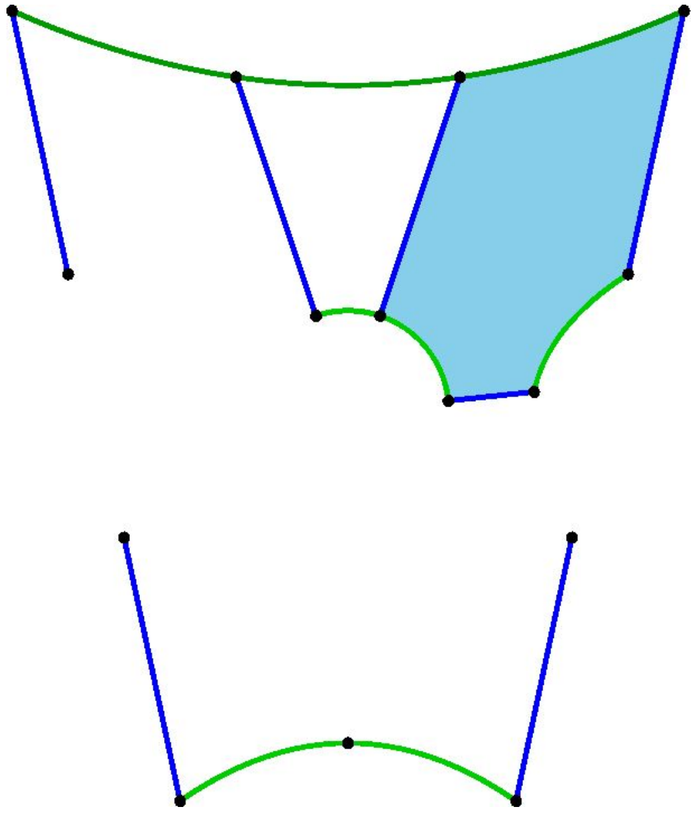
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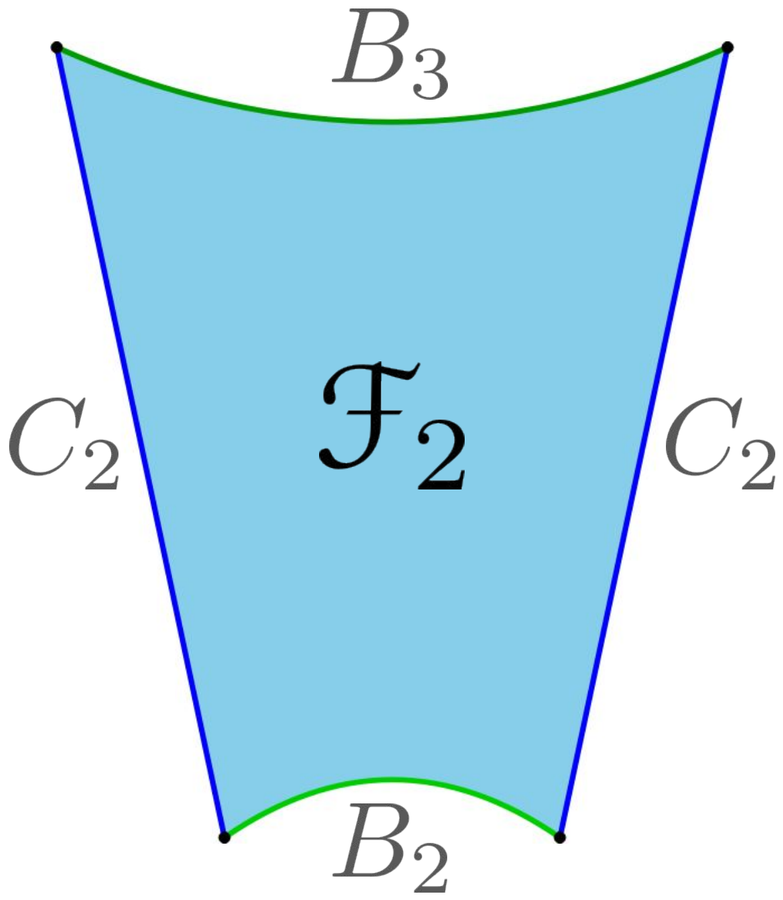


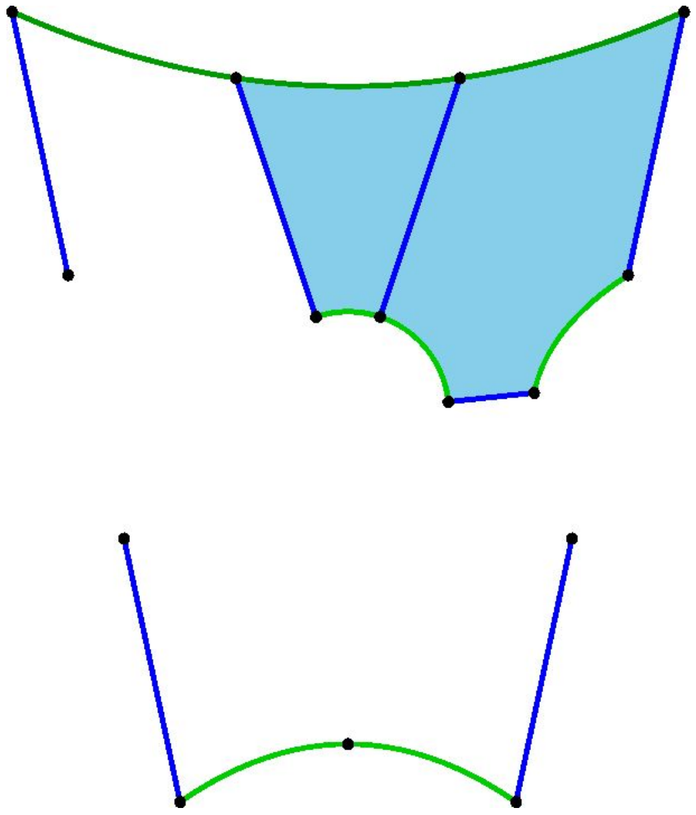
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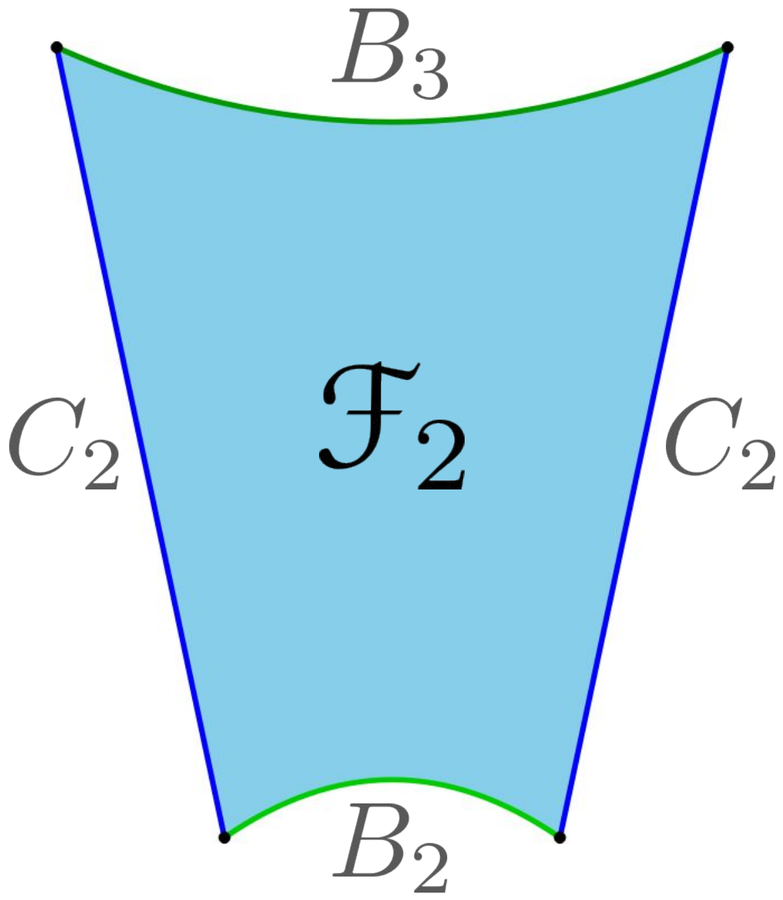


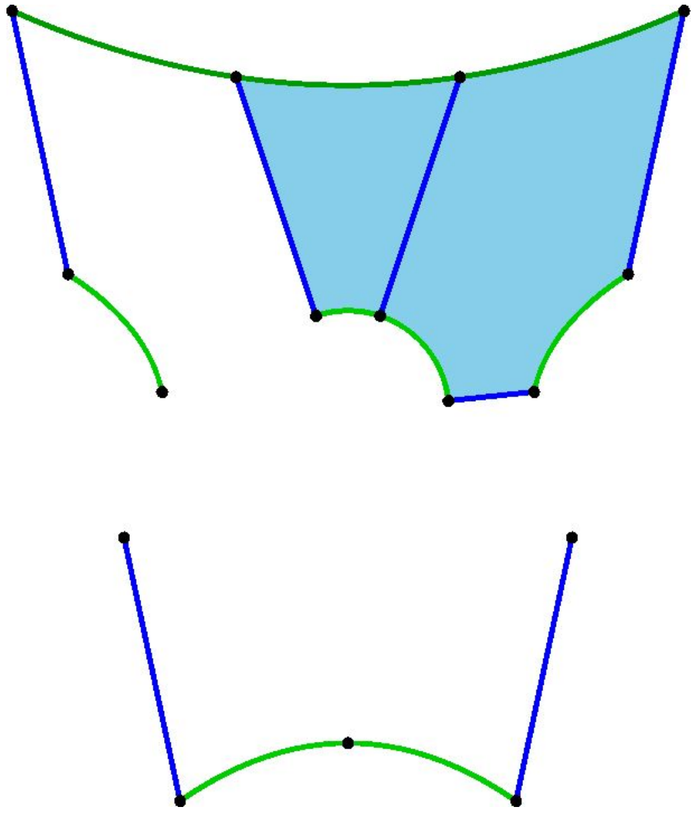
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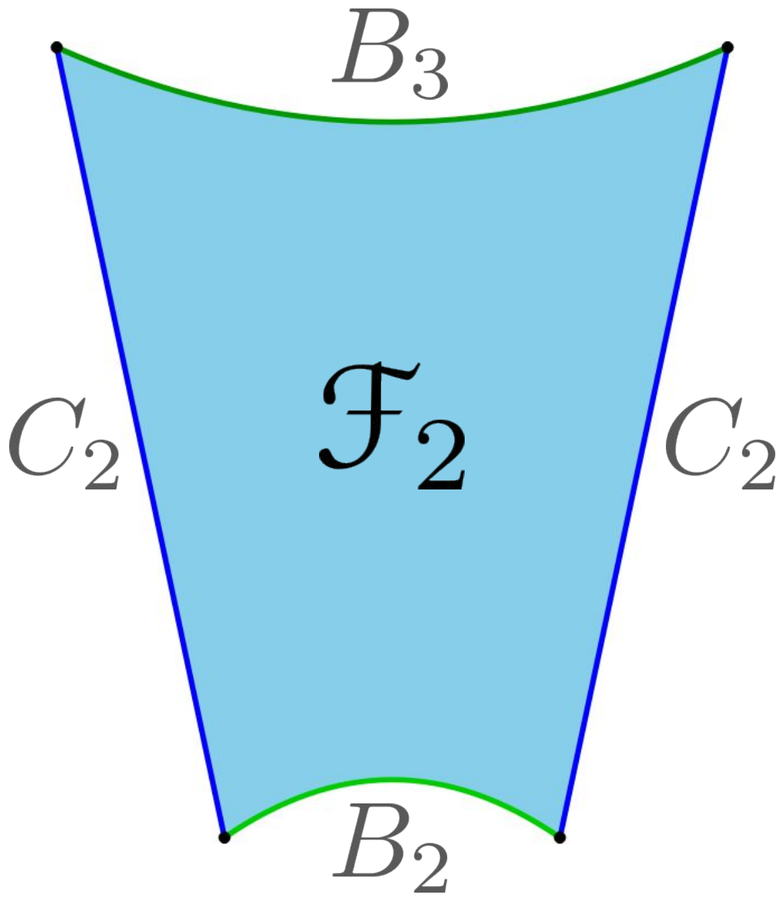


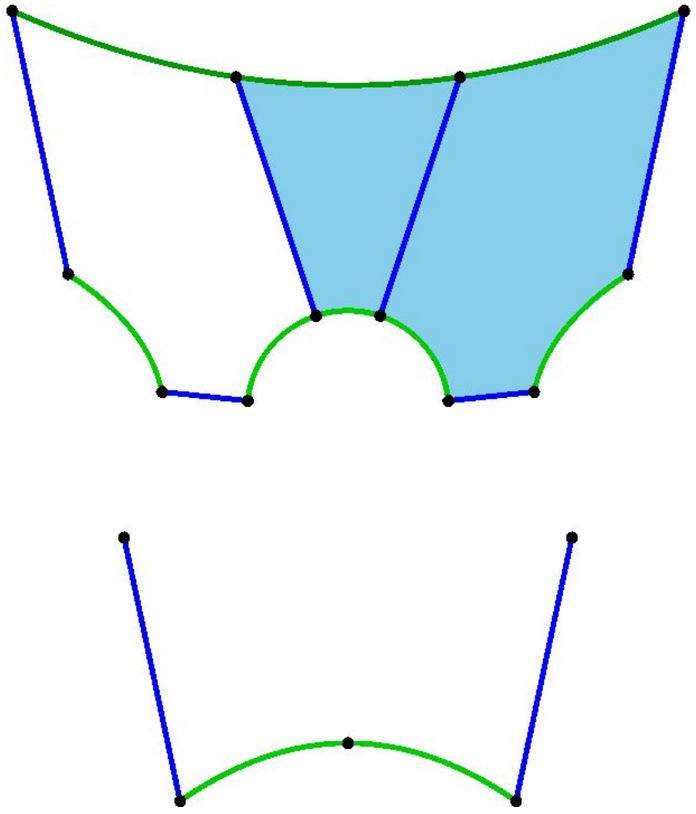
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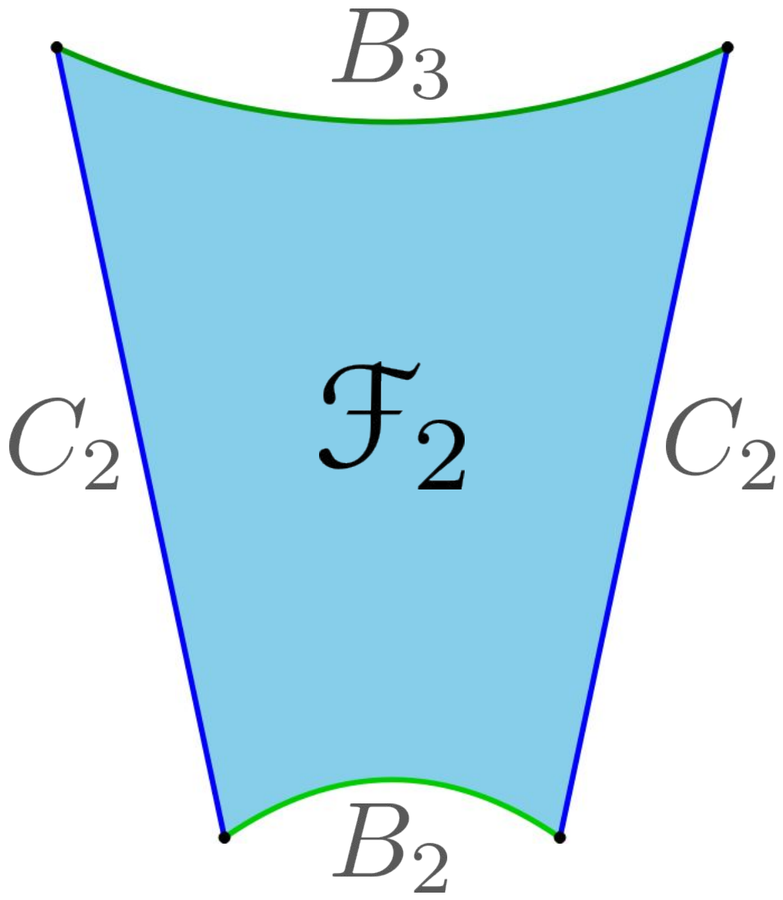


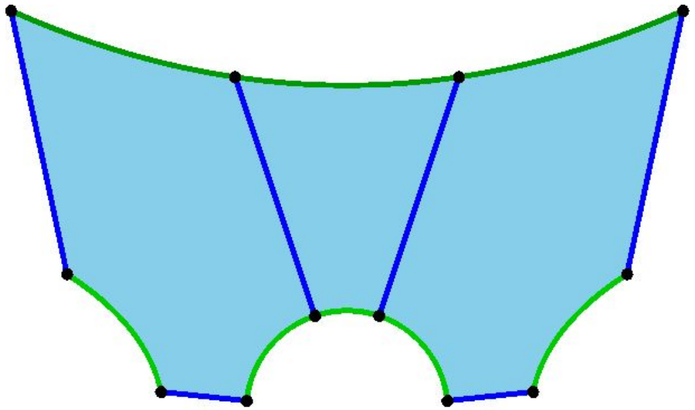
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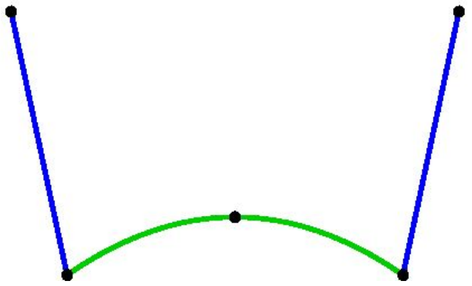




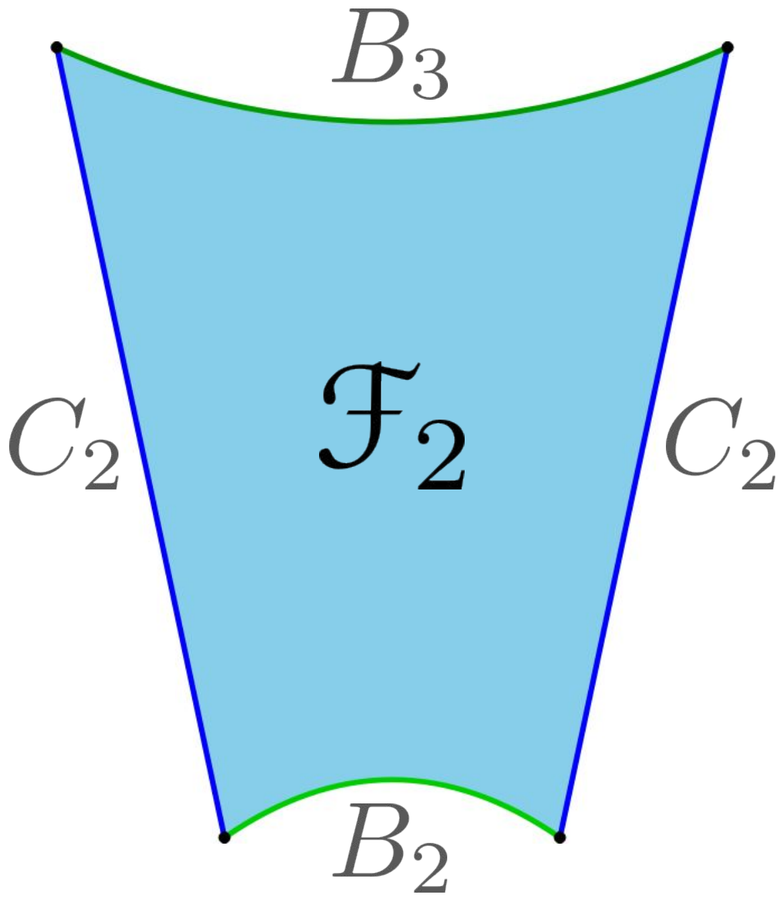
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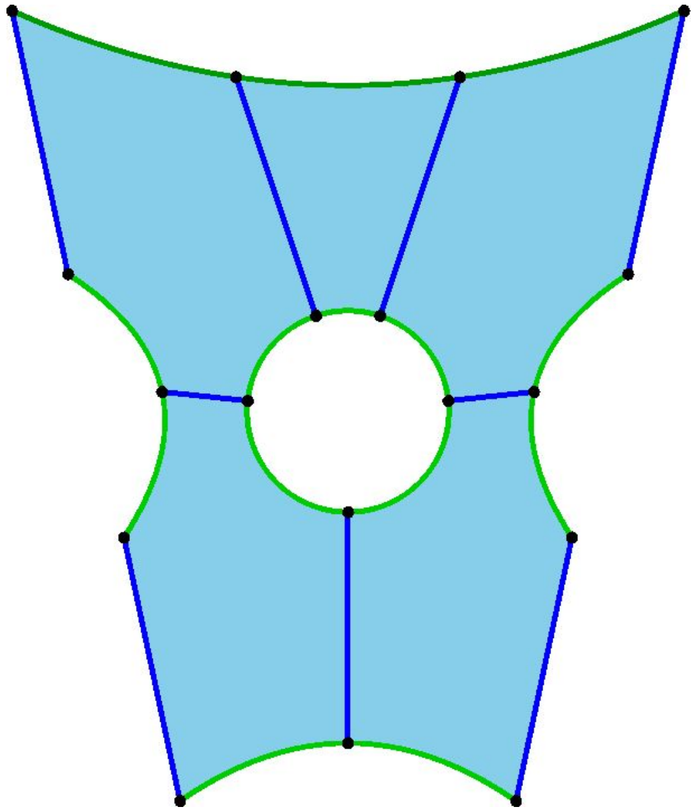




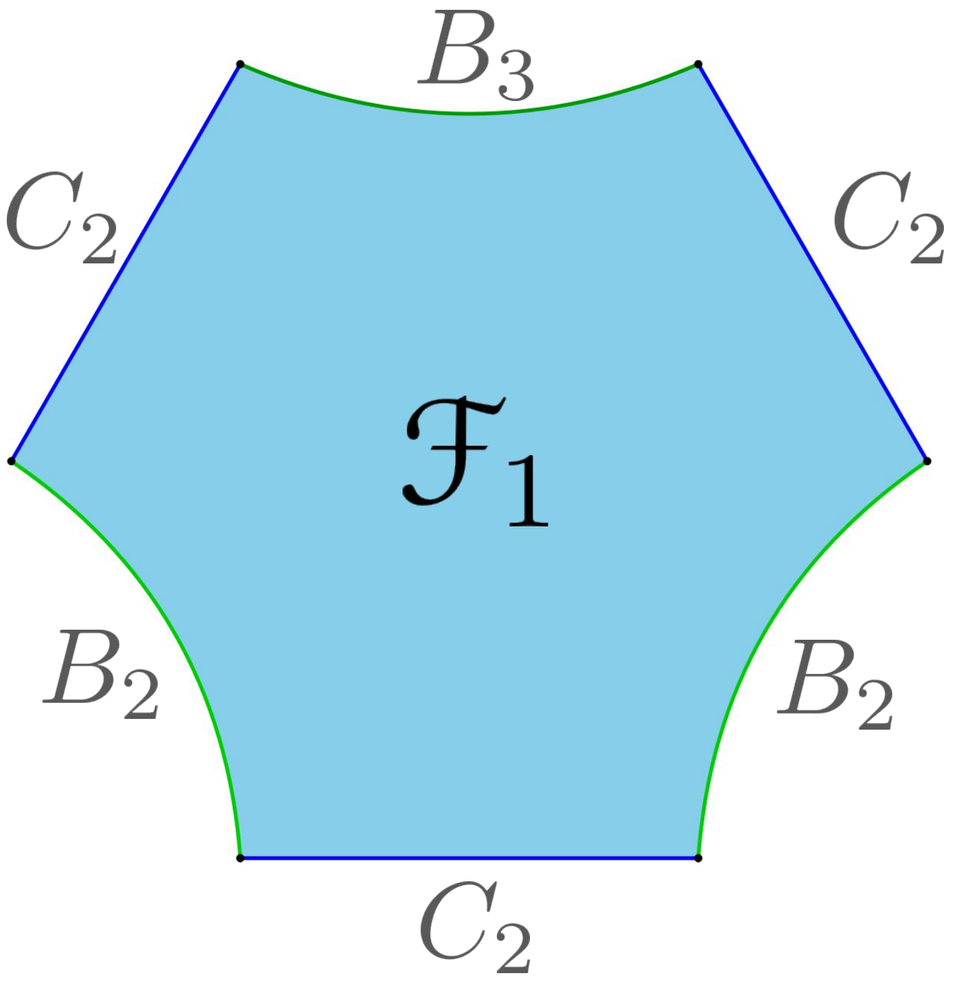


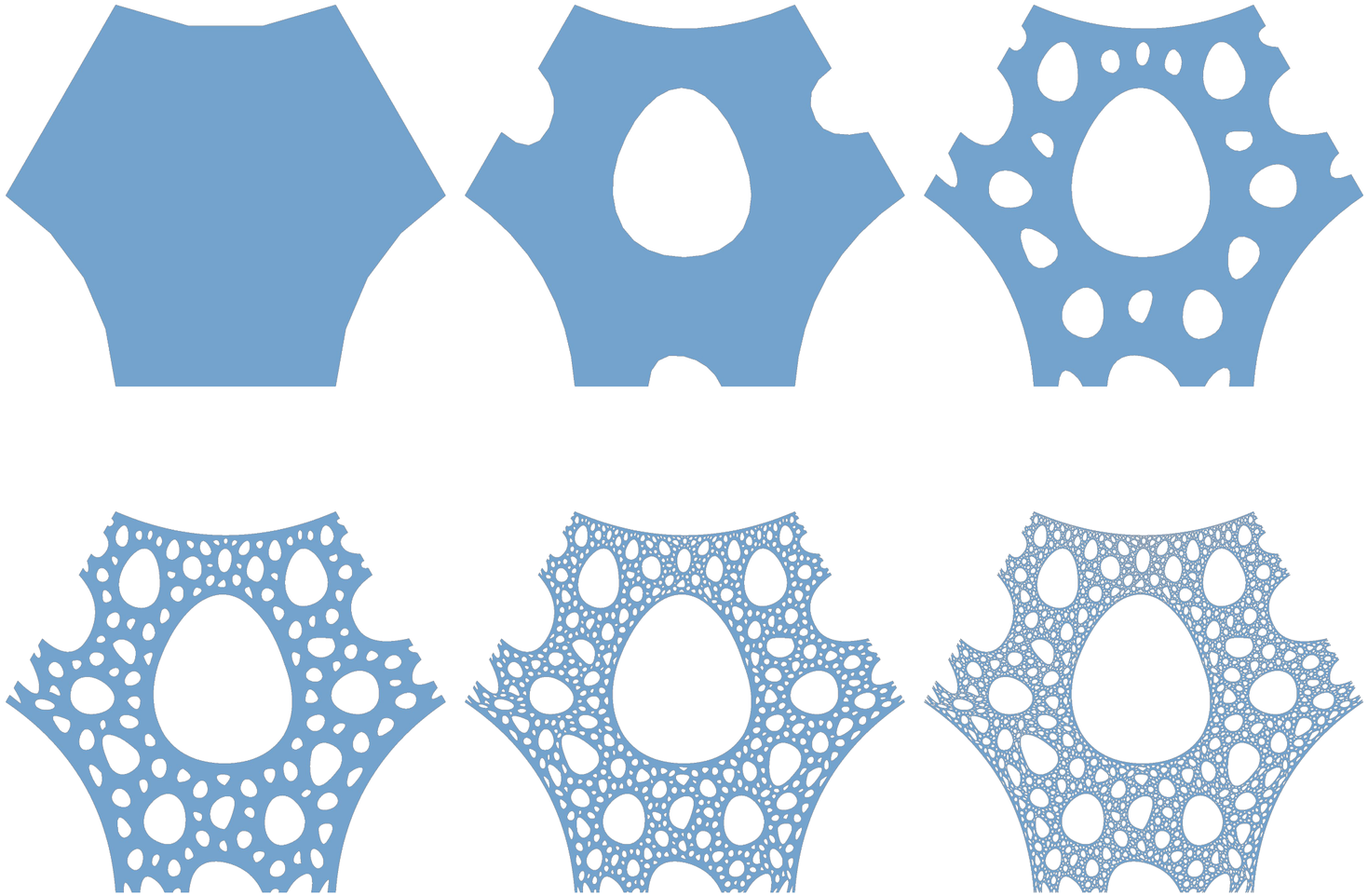
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Processus de subdivision



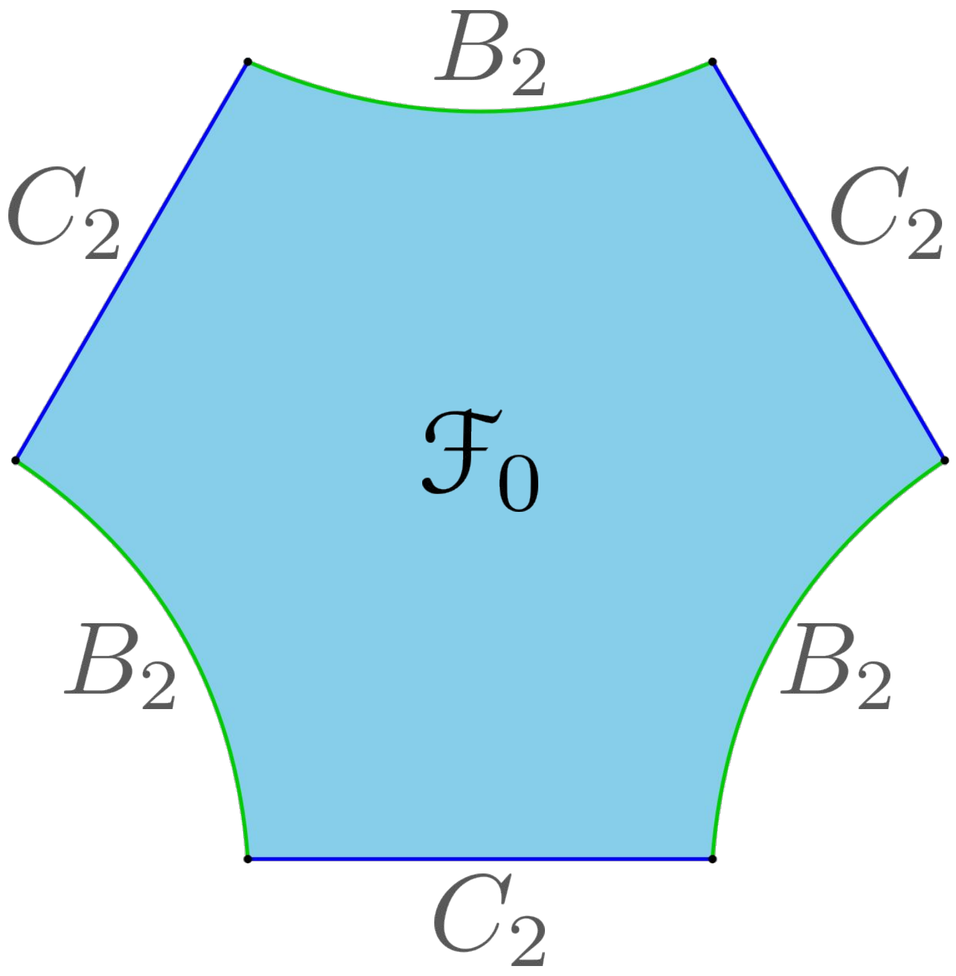


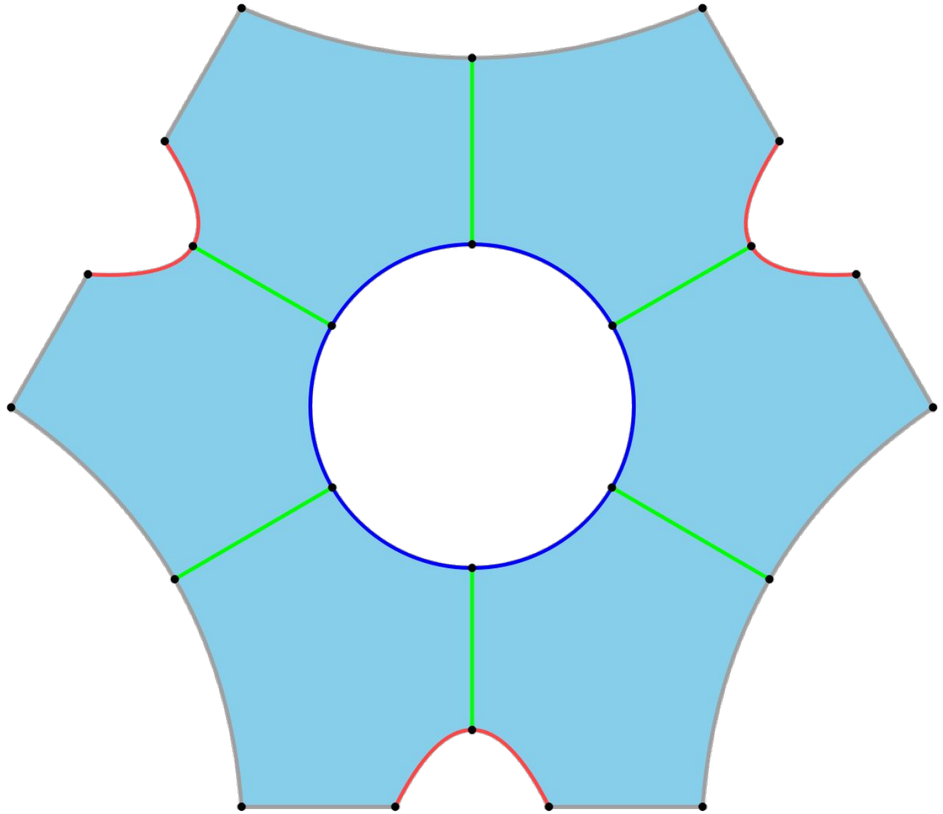
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Contrôle de la lacunarité

Paramètre $\mathcal{E}$

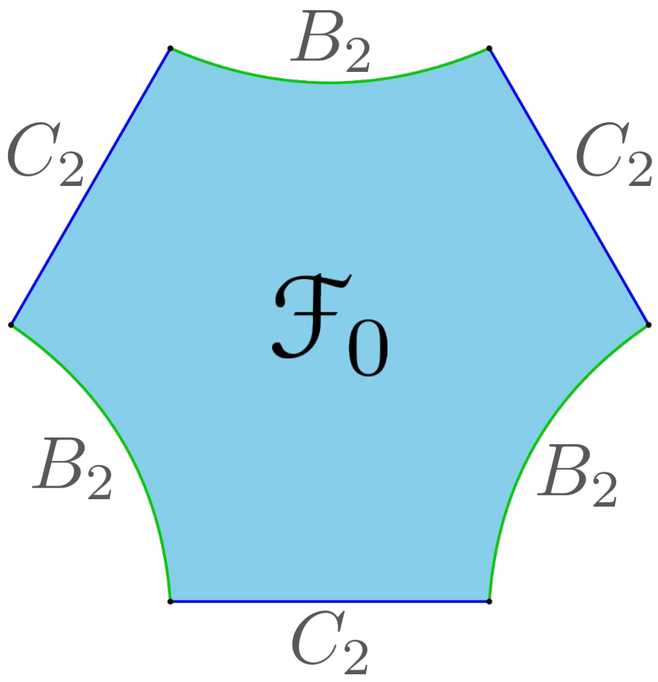
- $\mathcal{E}_a$ (vert)
- $\mathcal{E}_l$ (bleu)
- $\mathcal{E}_c$ (rouge)

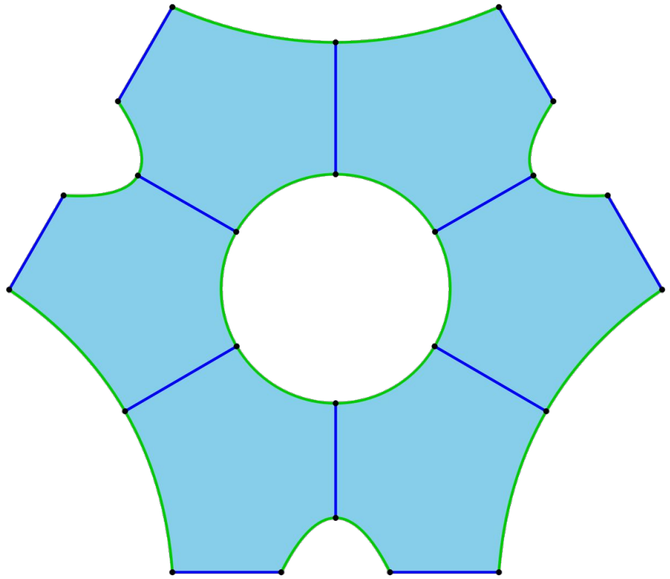


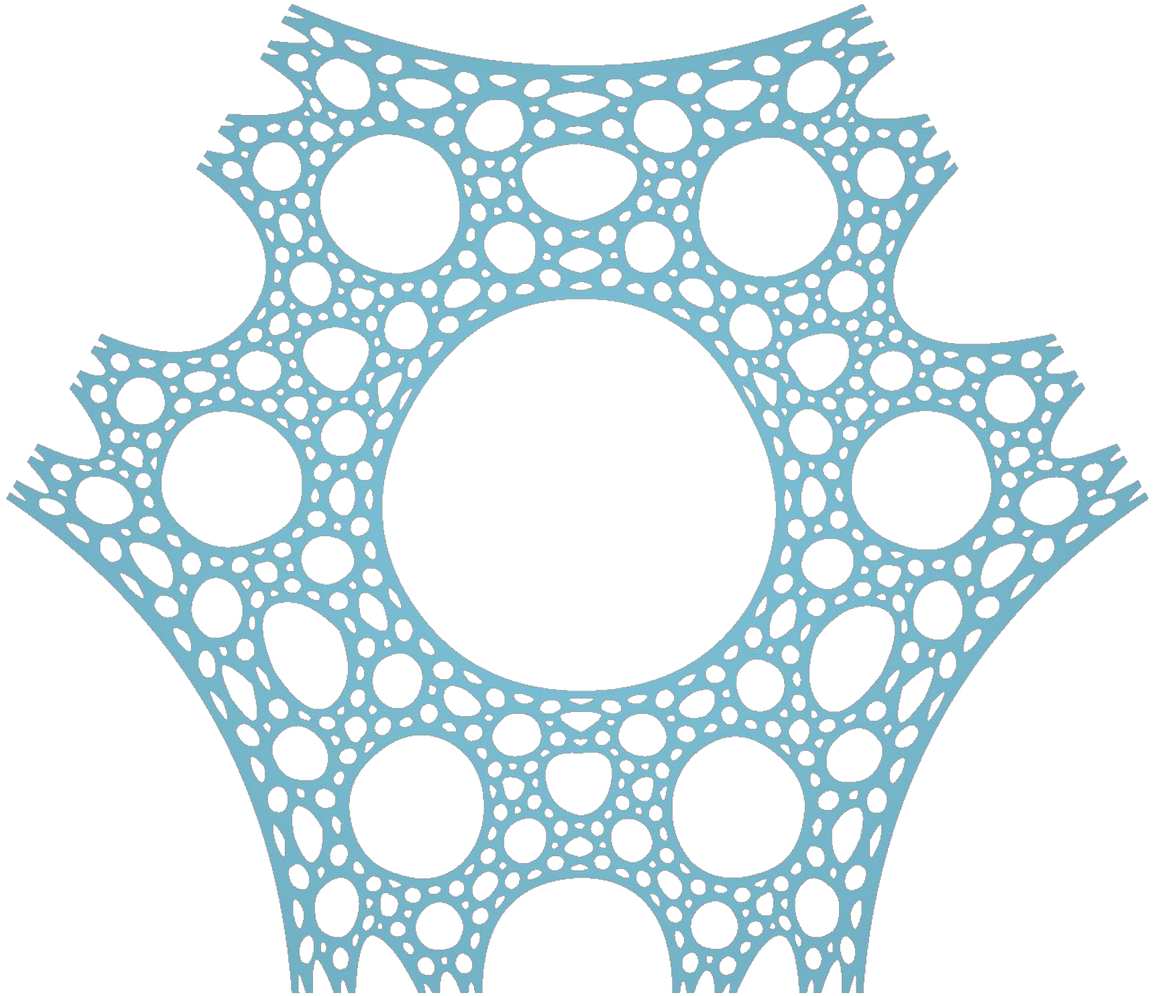


Paramètre $\mathcal{E}$

- $\mathcal{E}_a = C_2$
- $\mathcal{E}_l = B_2$
- $\mathcal{E}_c = B_2$

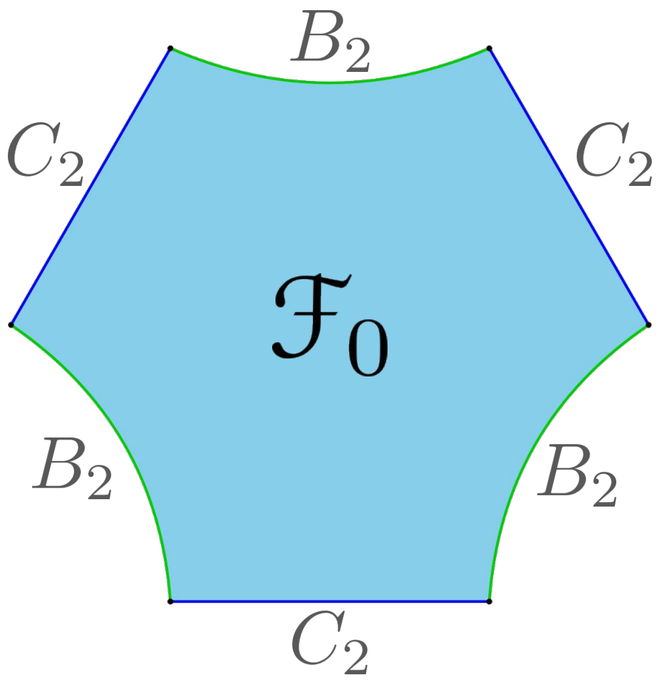


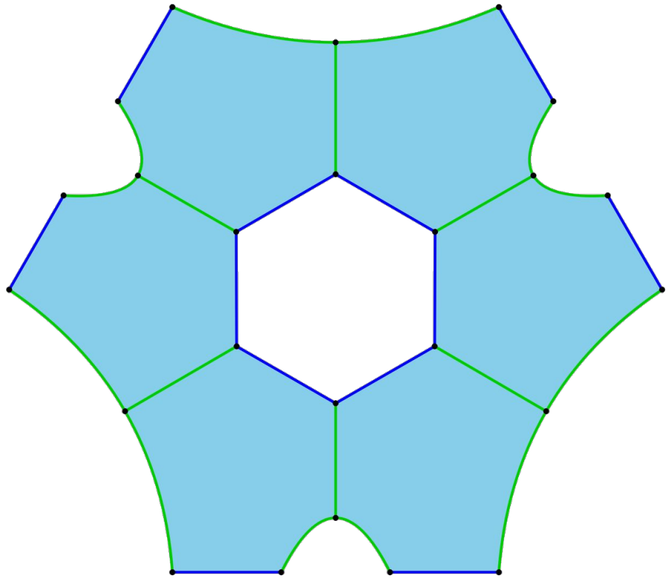


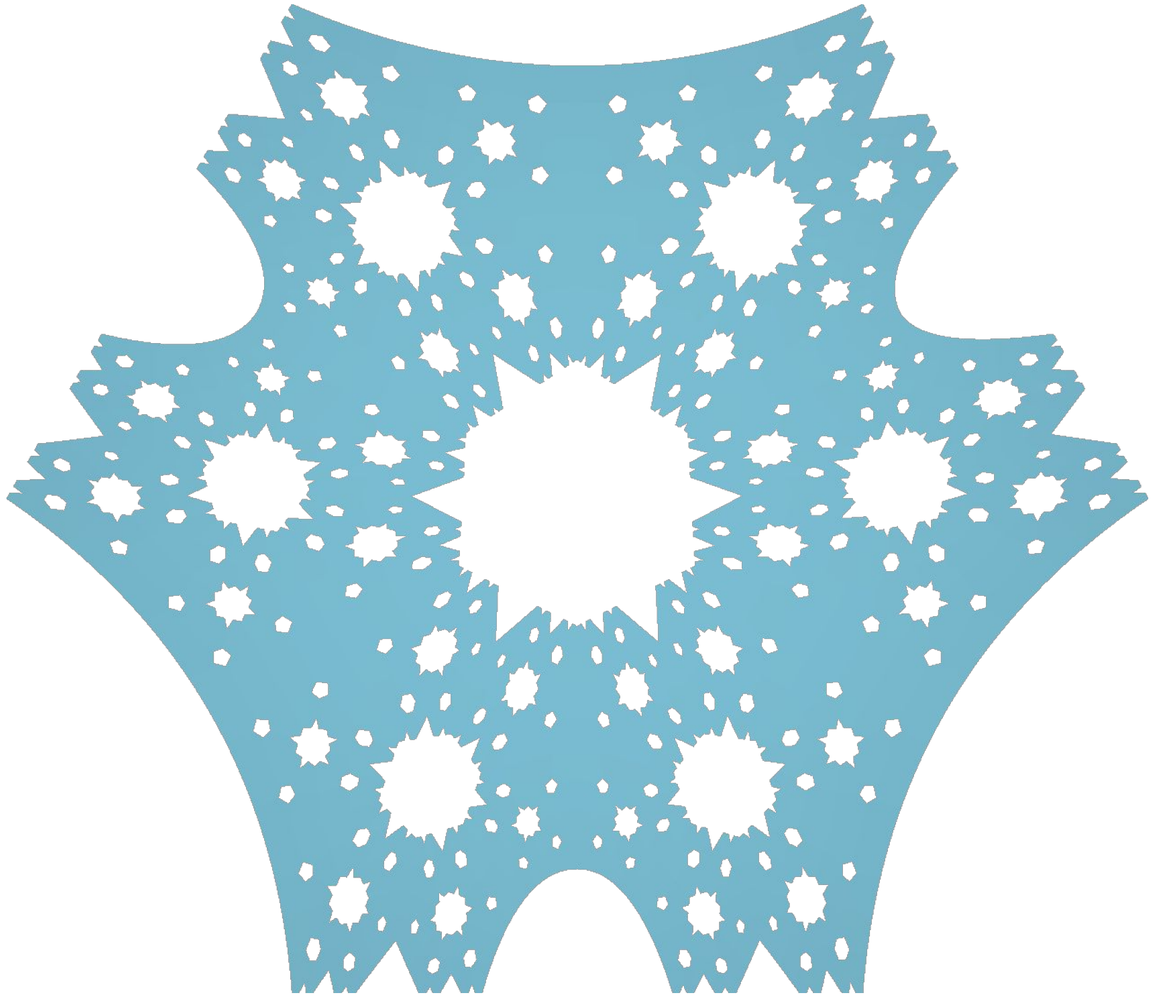


Paramètre $\mathcal{E}$

- $\mathcal{E}_a = B_2$
- $\mathcal{E}_l = C_2$
- $\mathcal{E}_c = B_2$

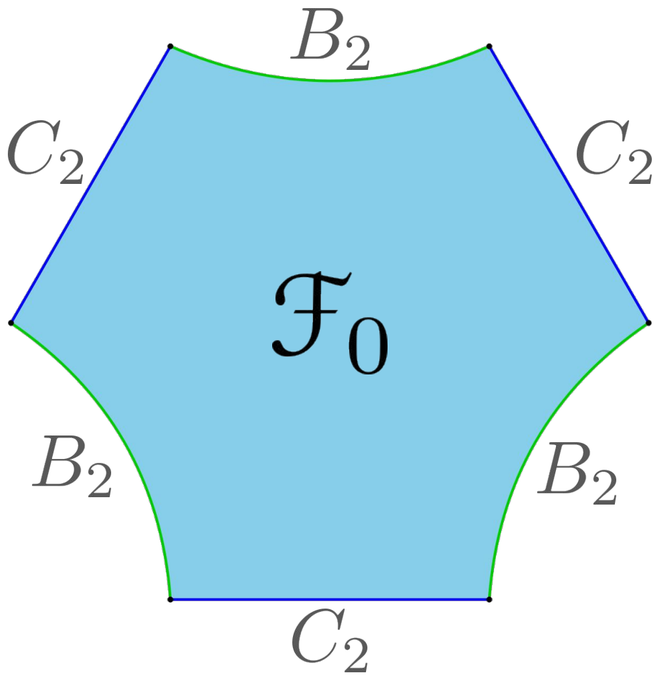


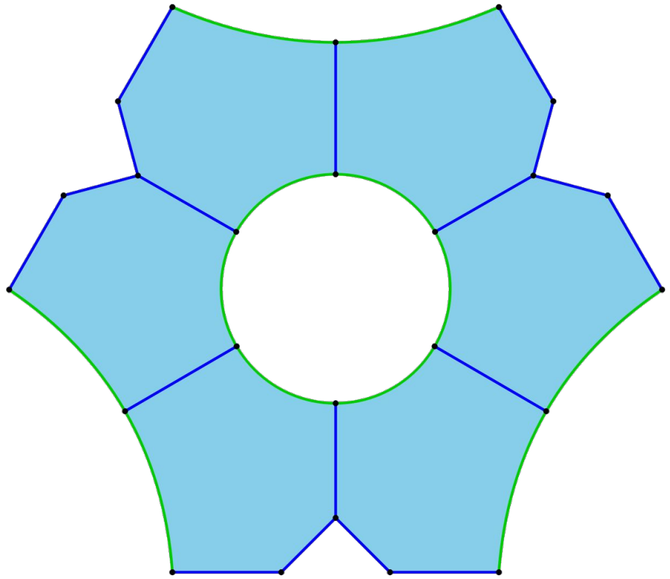


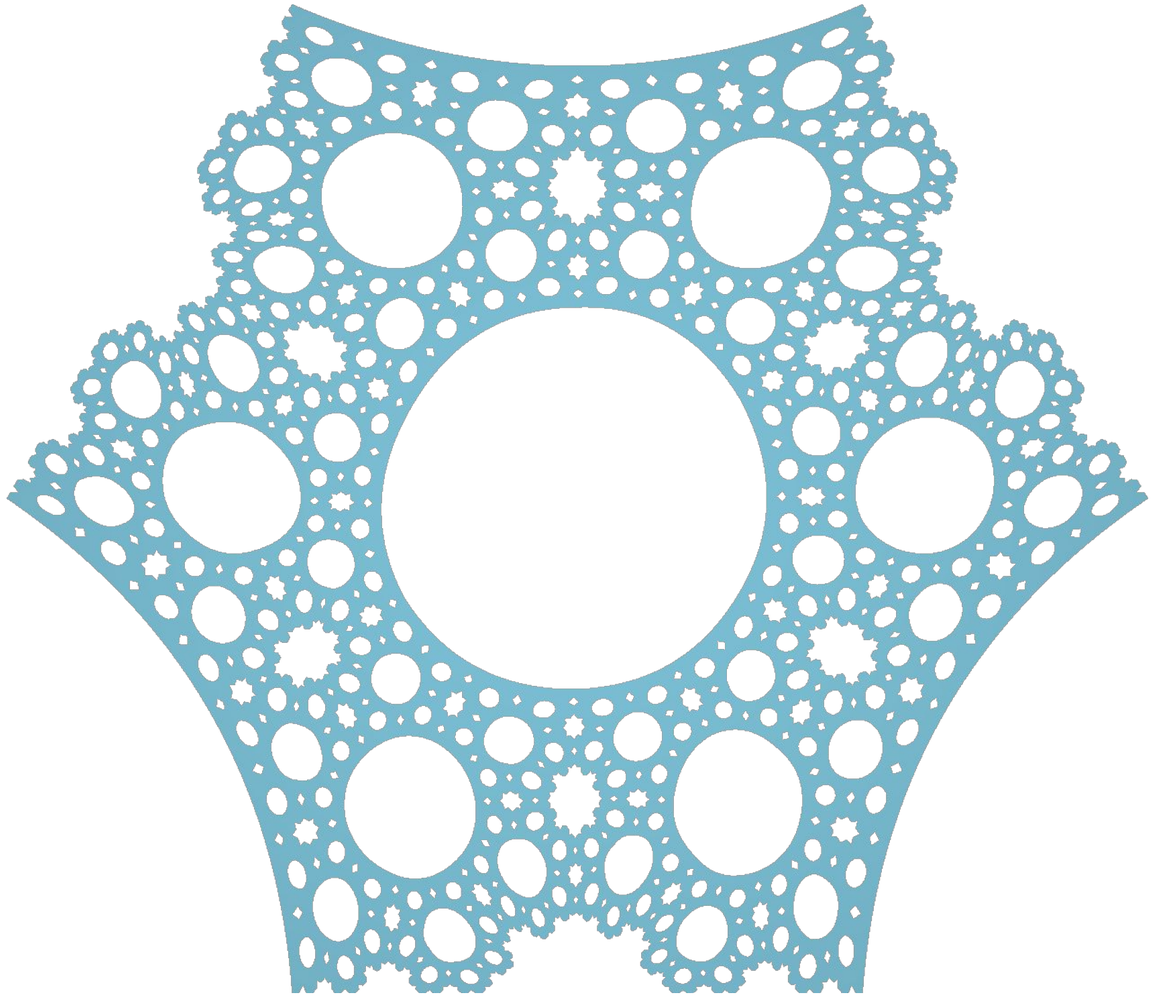


Paramètre $\mathcal{E}$

- $\mathcal{E}_a = C_2$
- $\mathcal{E}_l = B_2$
- $\mathcal{E}_c = C_2$

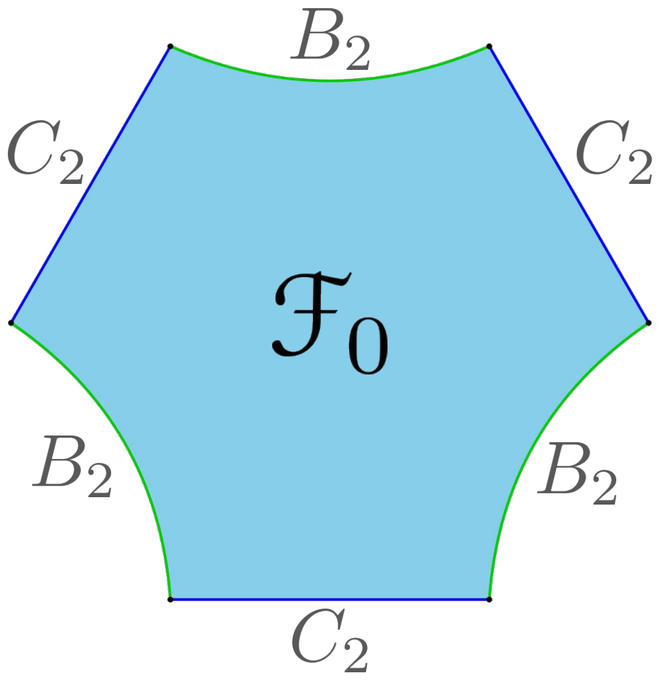


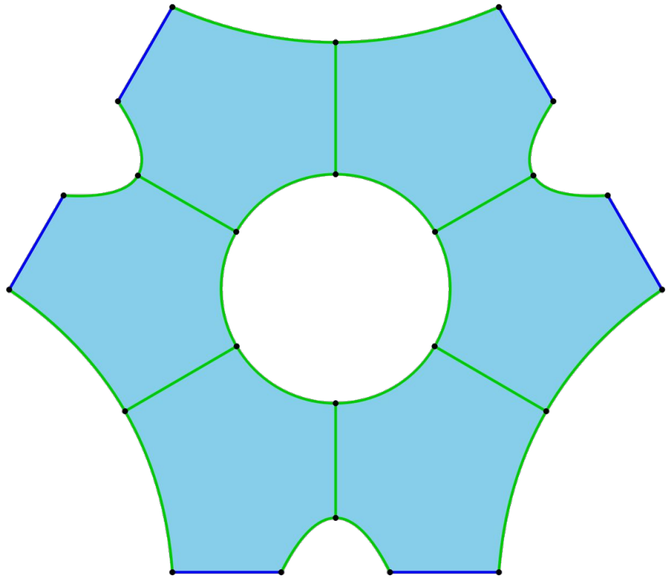


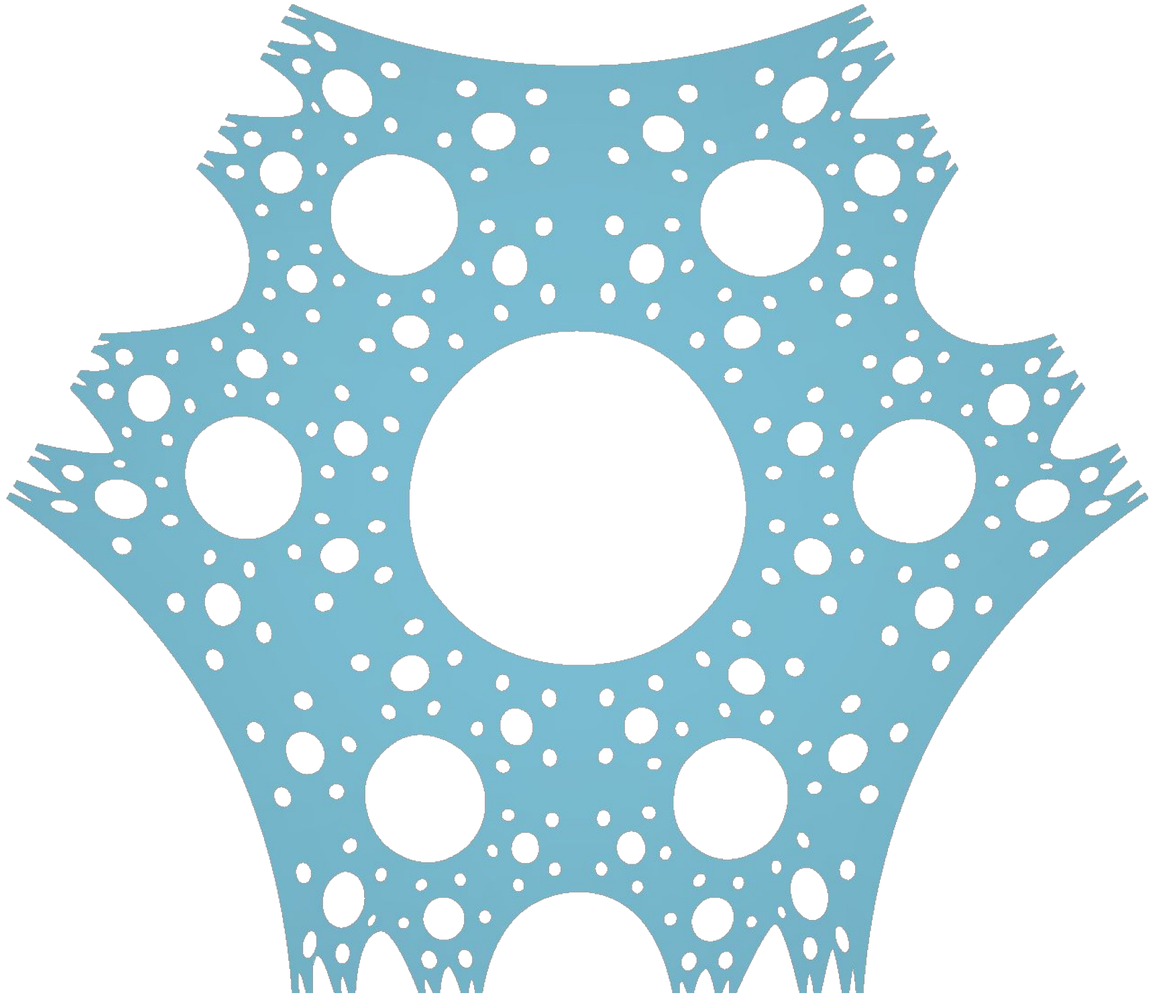


Paramètre $\mathcal{E}$

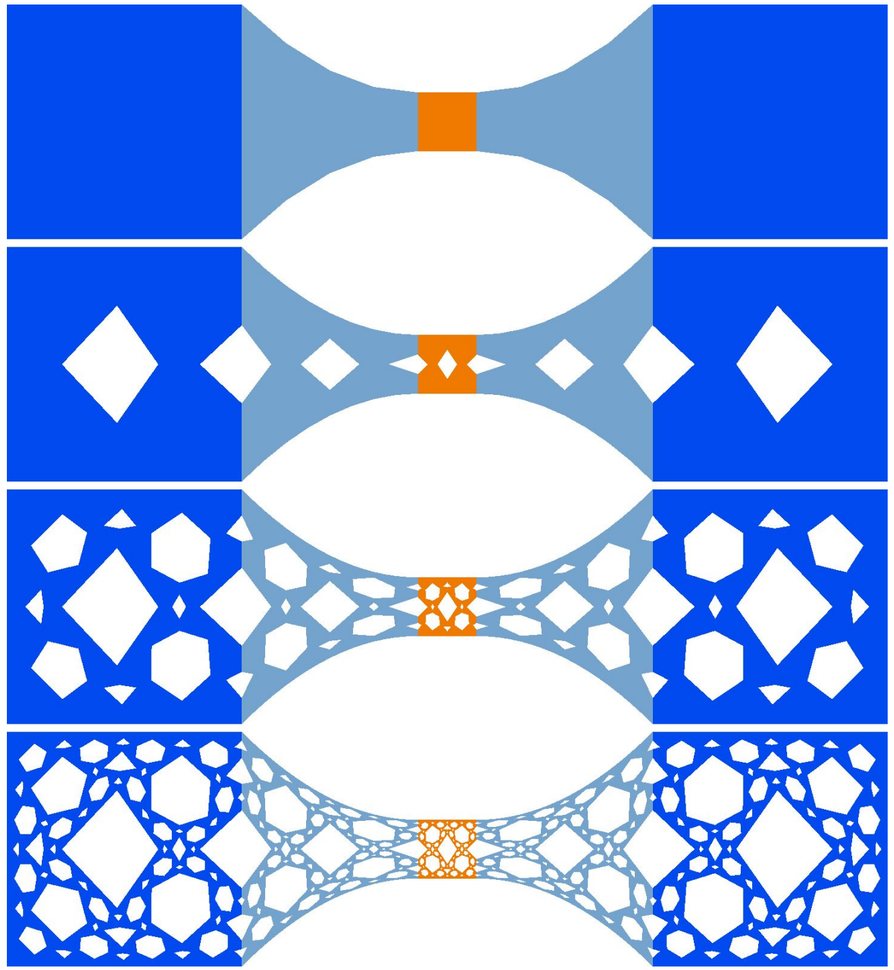
- $\mathcal{E}_a = B_2$
- $\mathcal{E}_l = B_2$
- $\mathcal{E}_c = B_2$

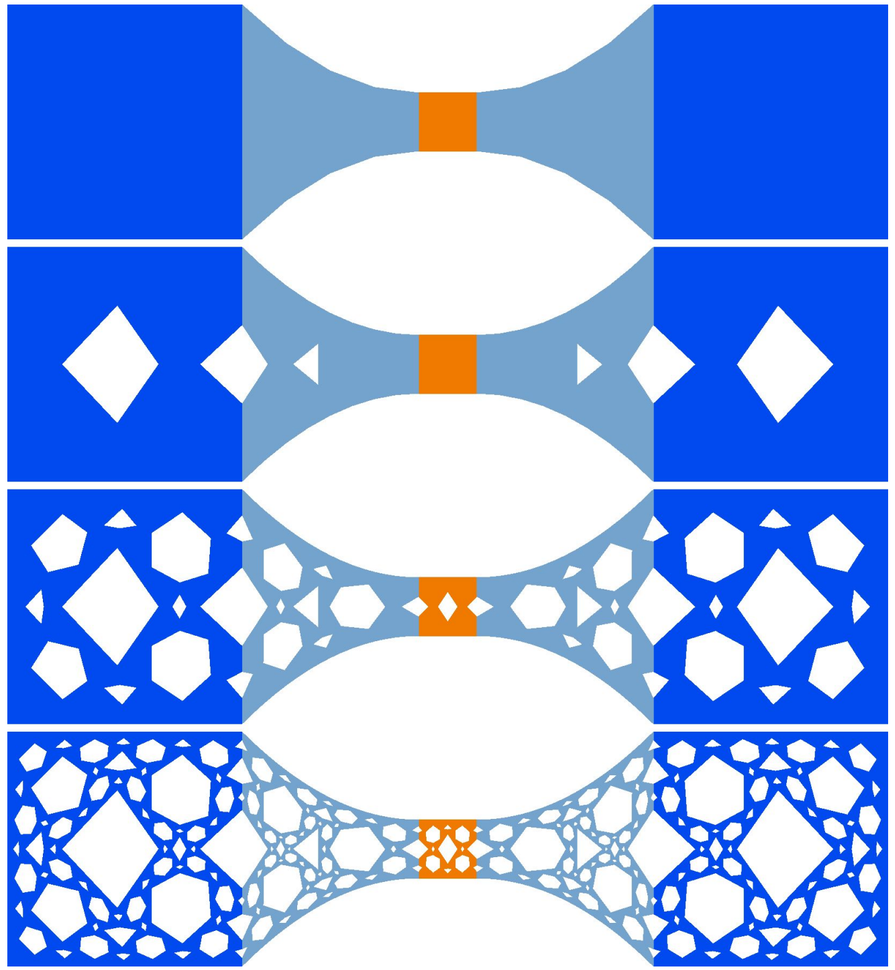






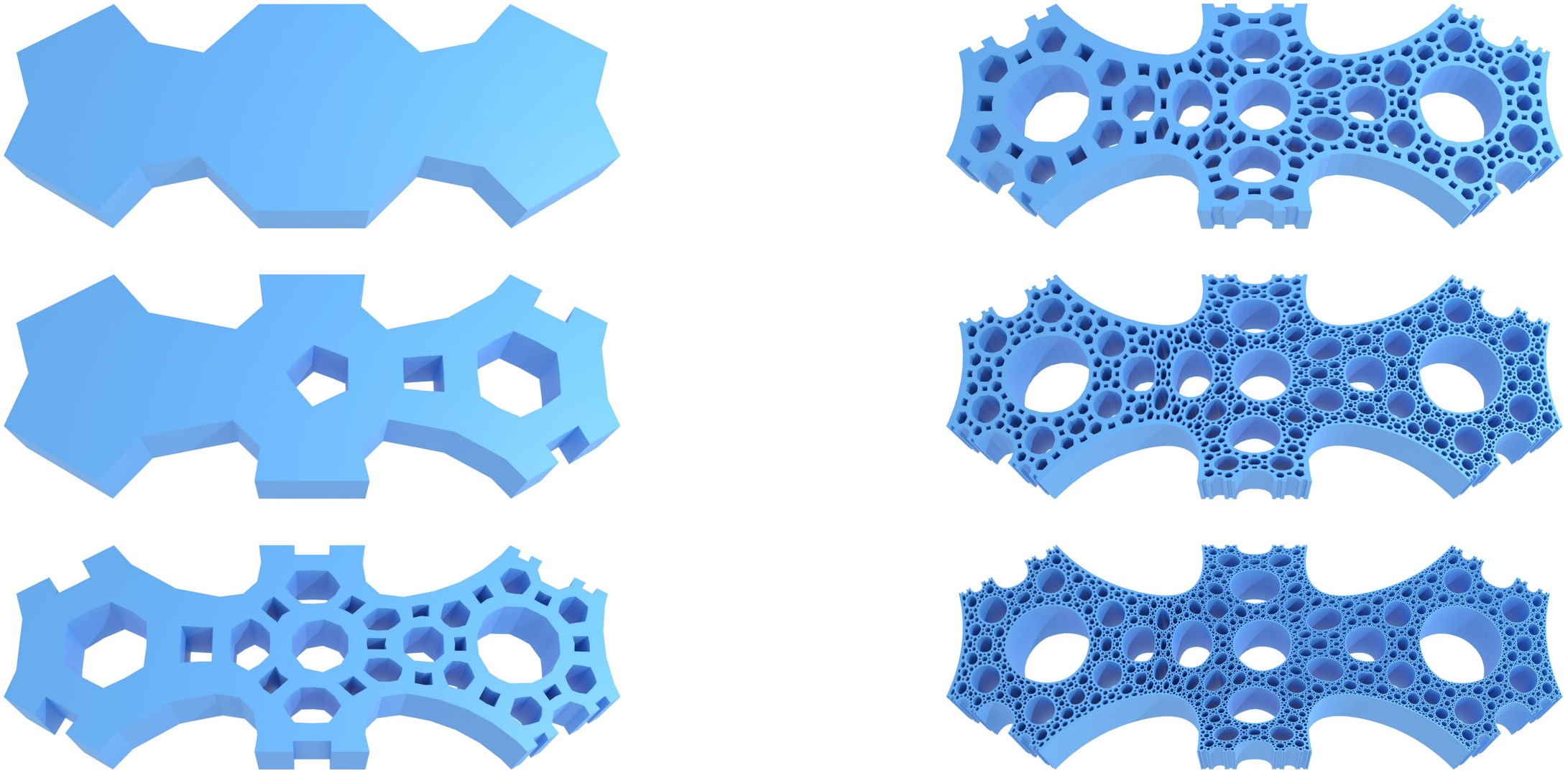
Délai dans la subdivision





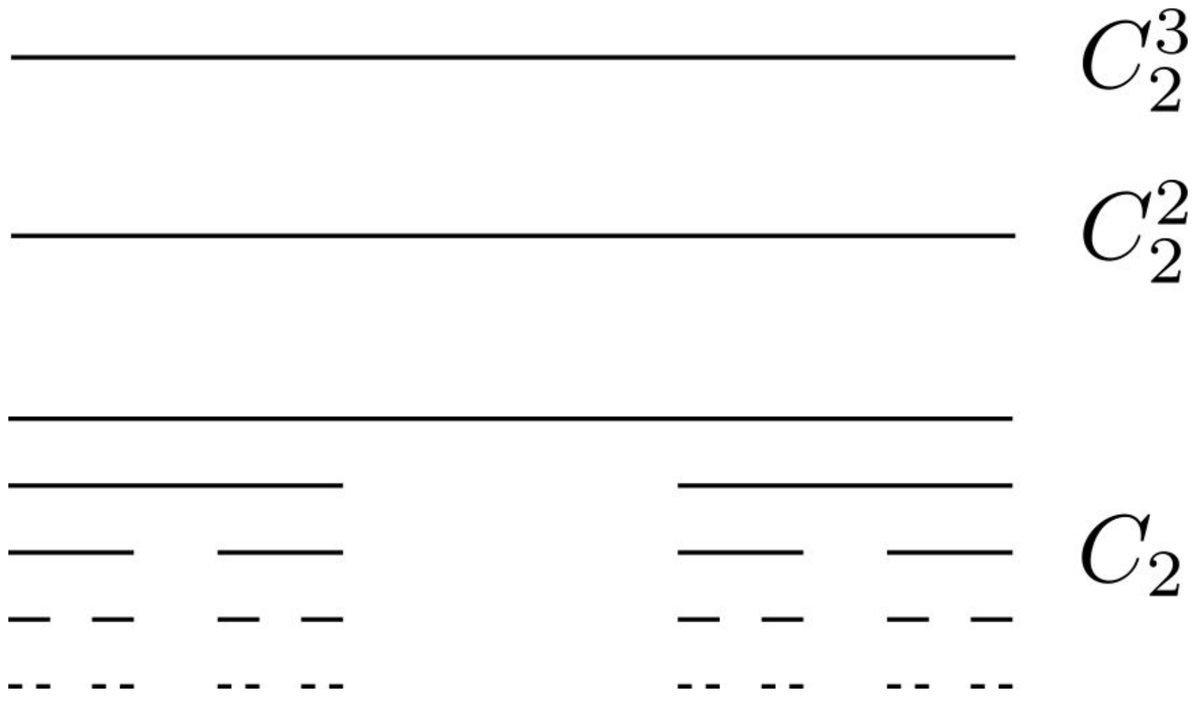
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Délai dans la subdivision



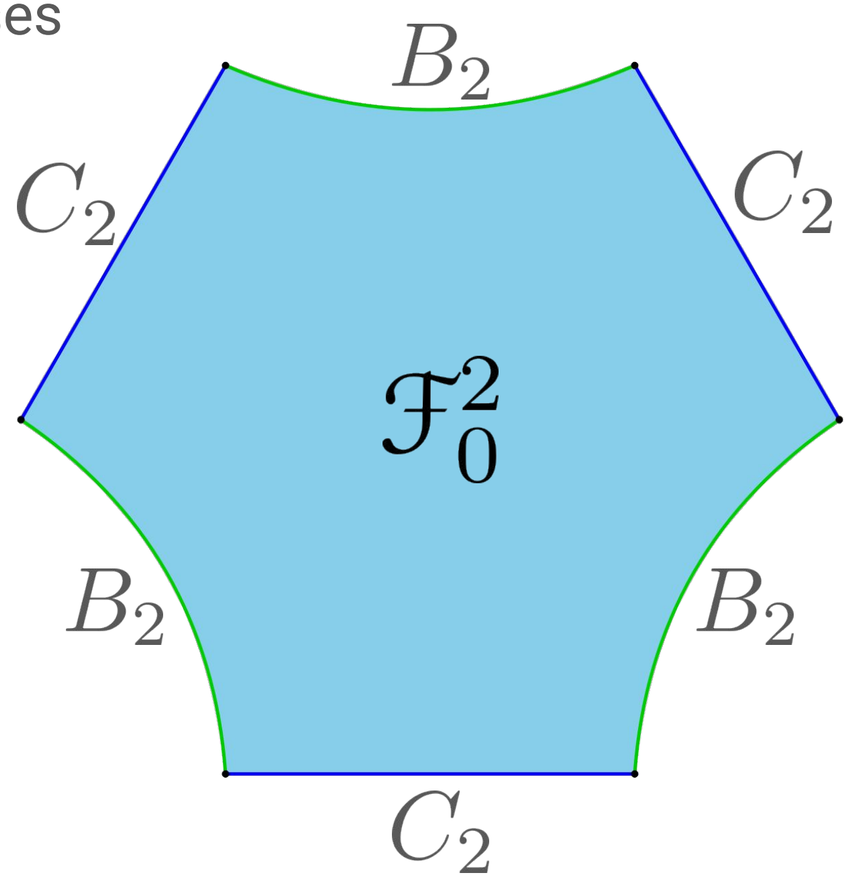
Délai dans la subdivision

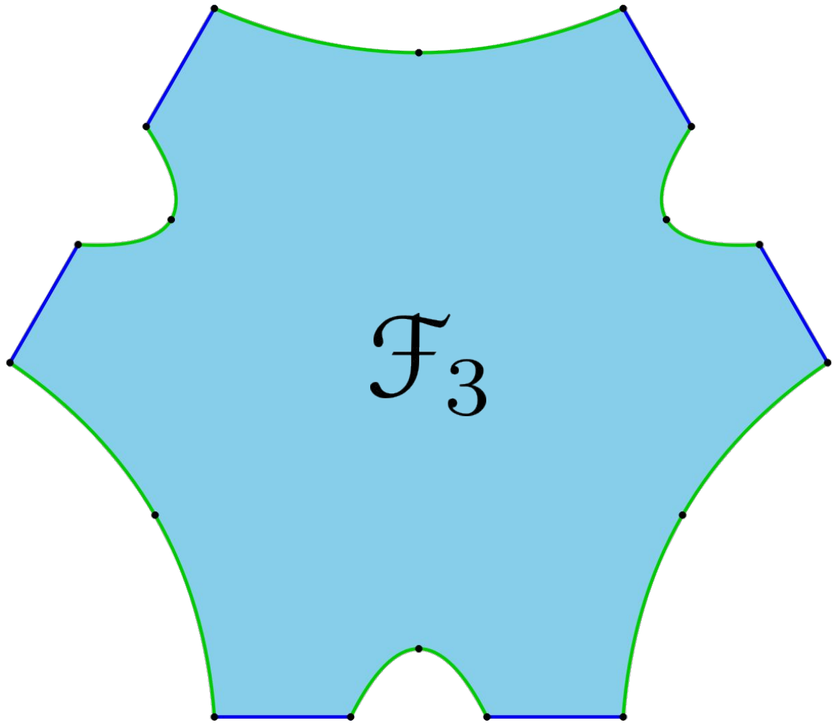
- Arêtes
  - Le délai diminue



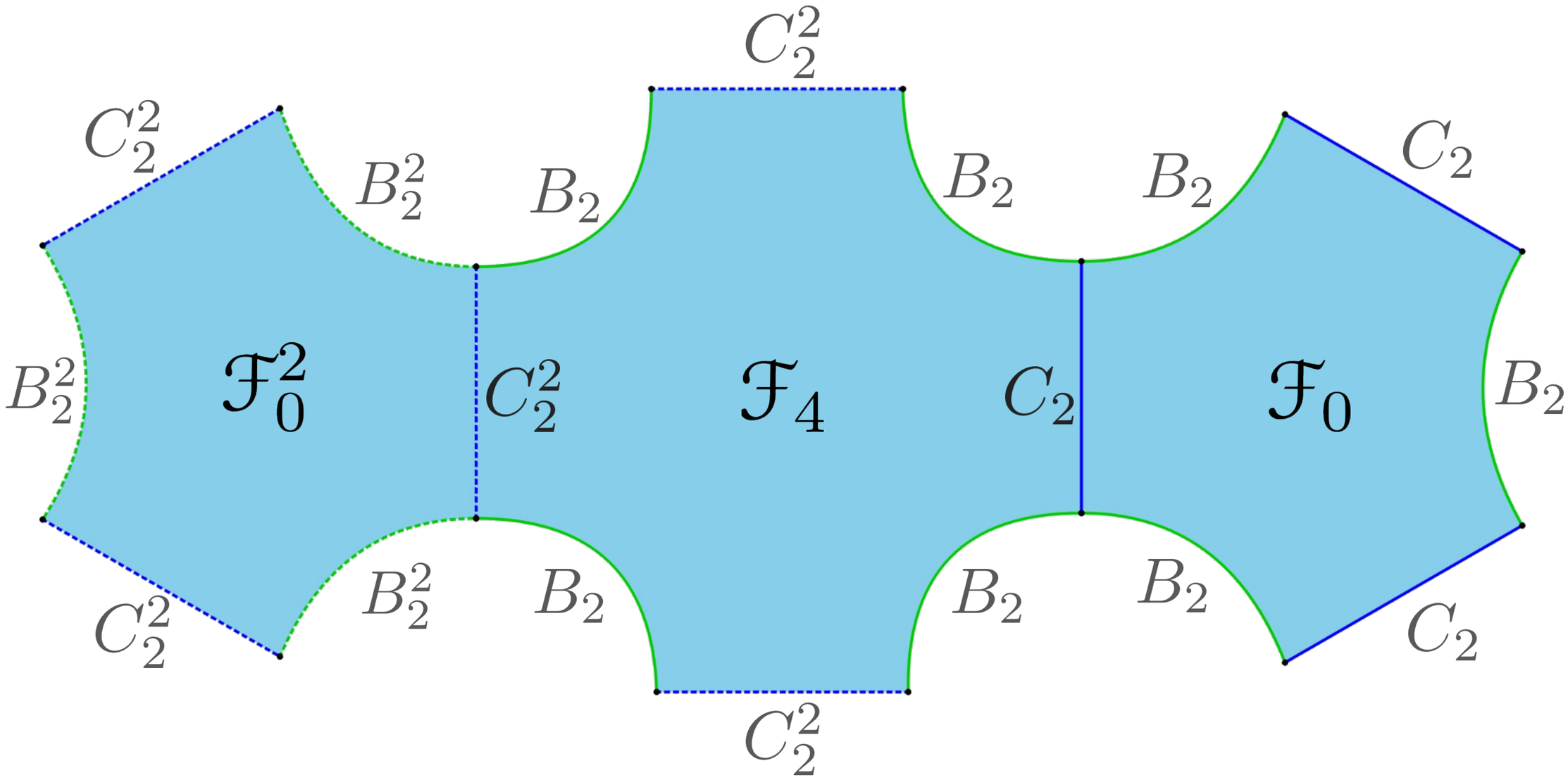
Délai dans la subdivision

- Faces
  - Pas de lacune centrale
  - Les bords sont subdivisés



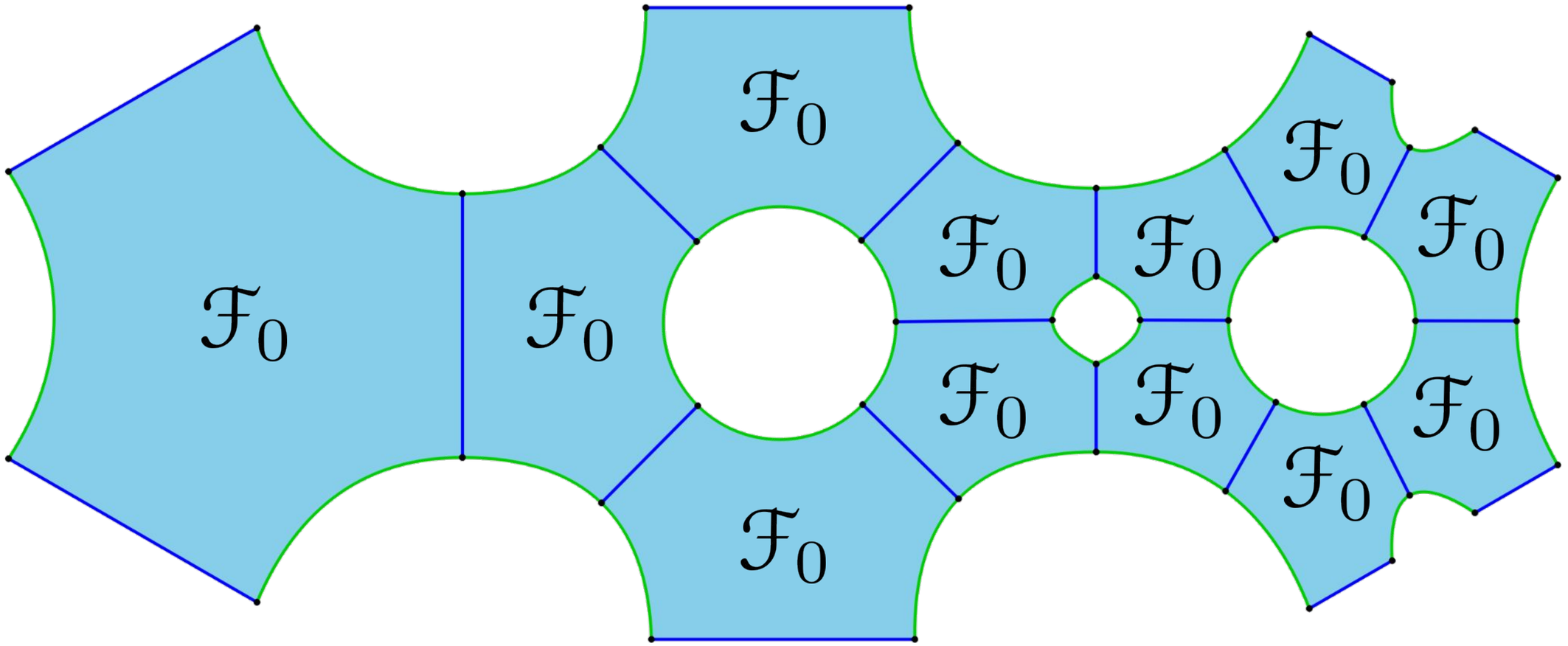


Délai dans la subdivision

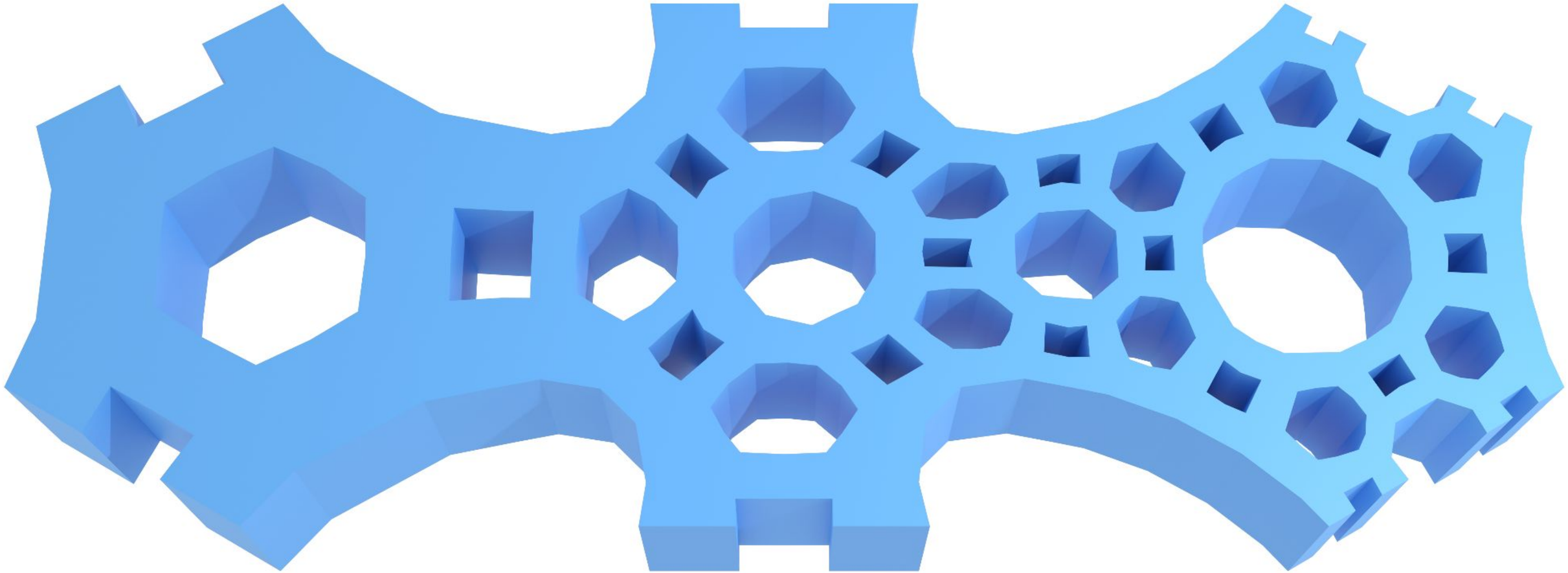


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Délai dans la subdivision

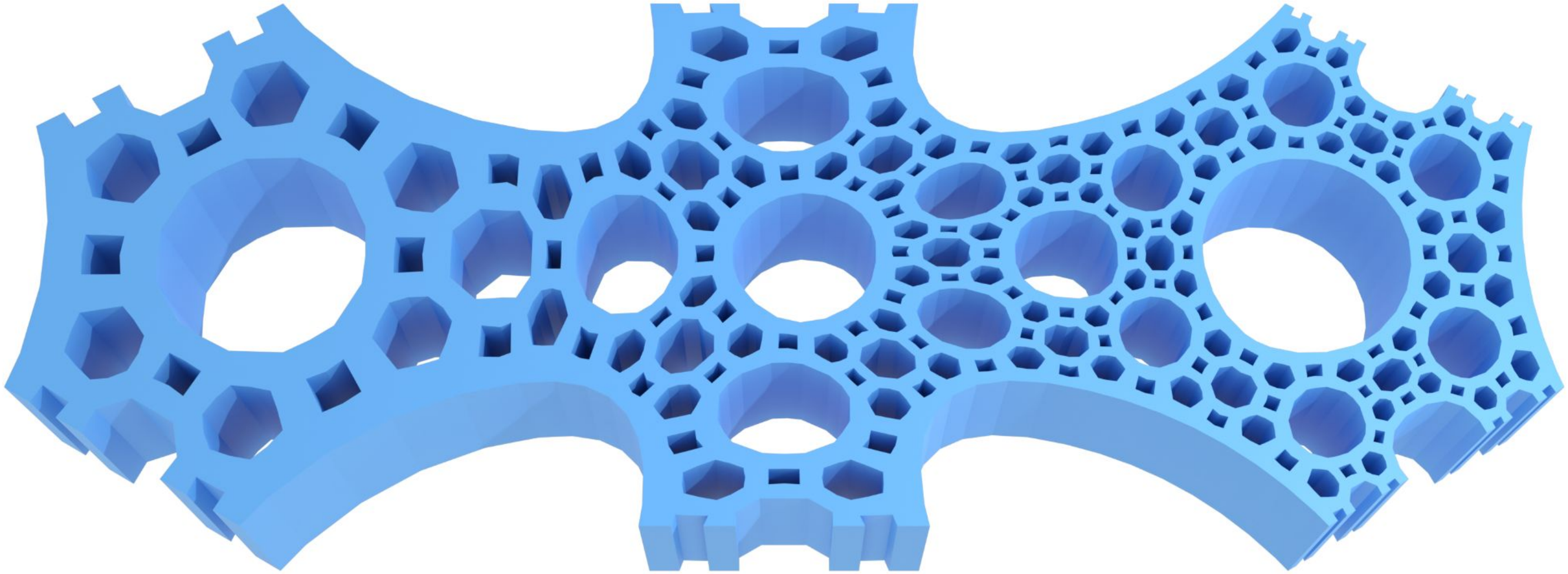


Délai dans la subdivision

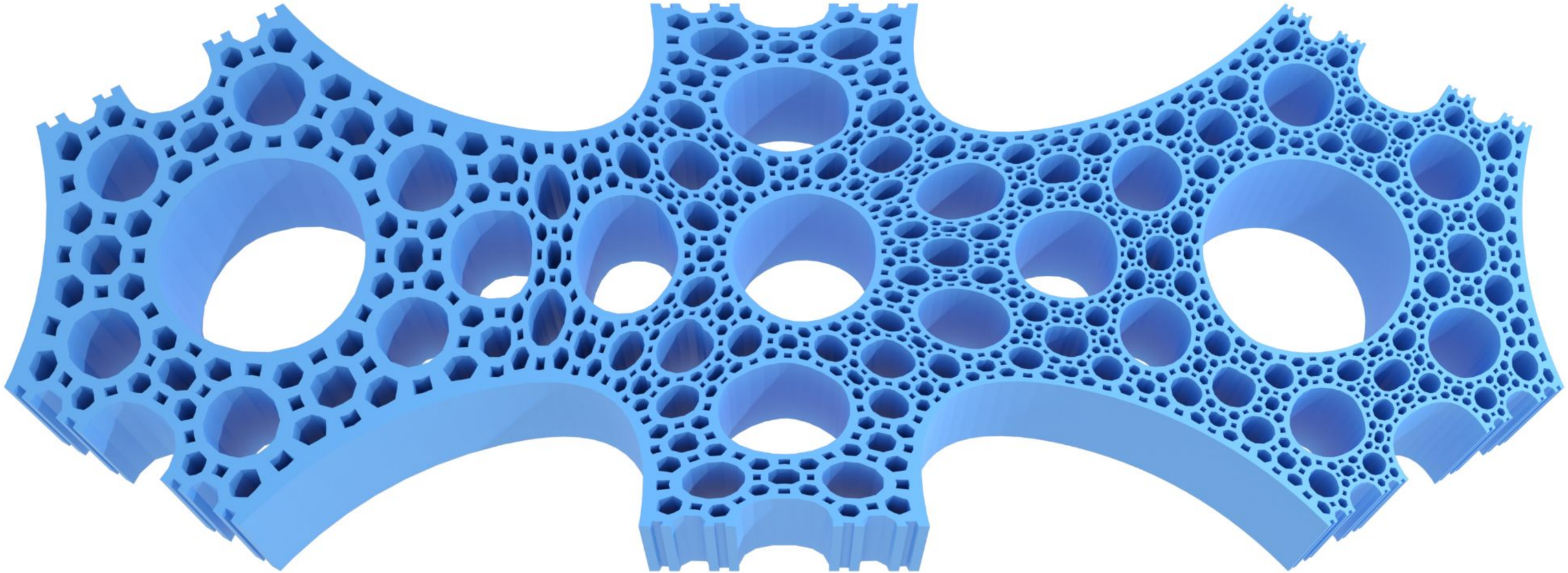


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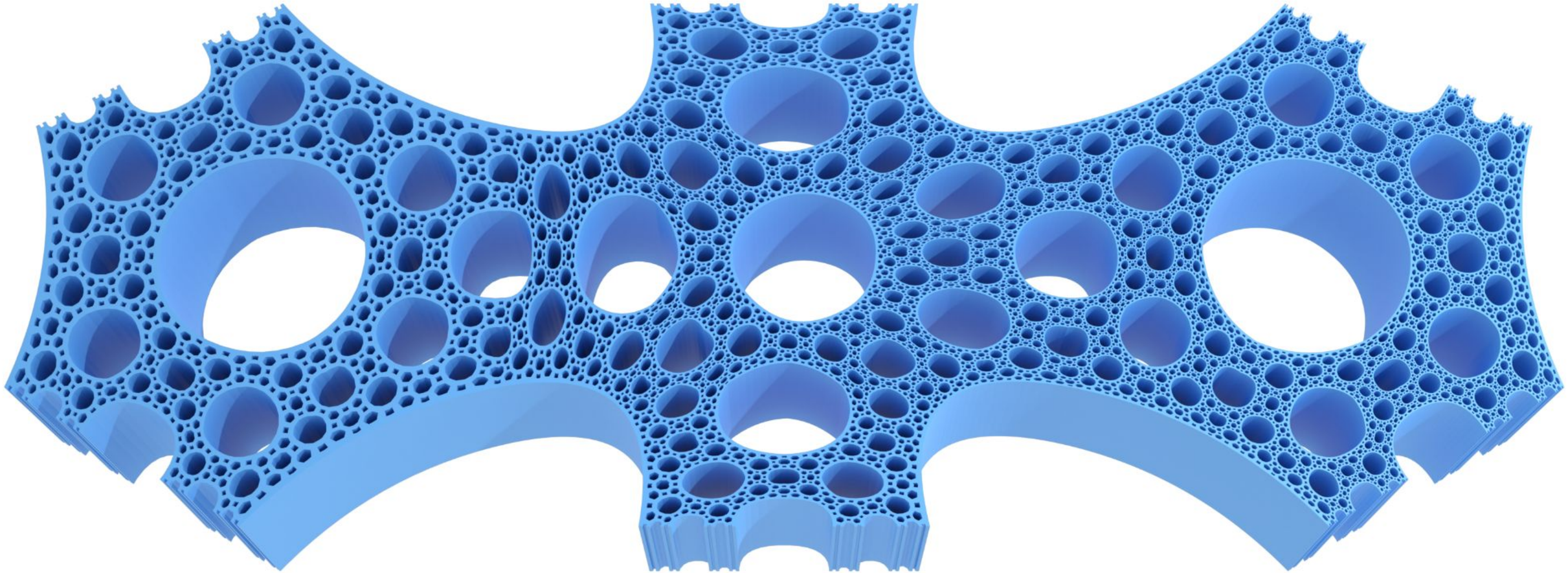
Délai dans la subdivision



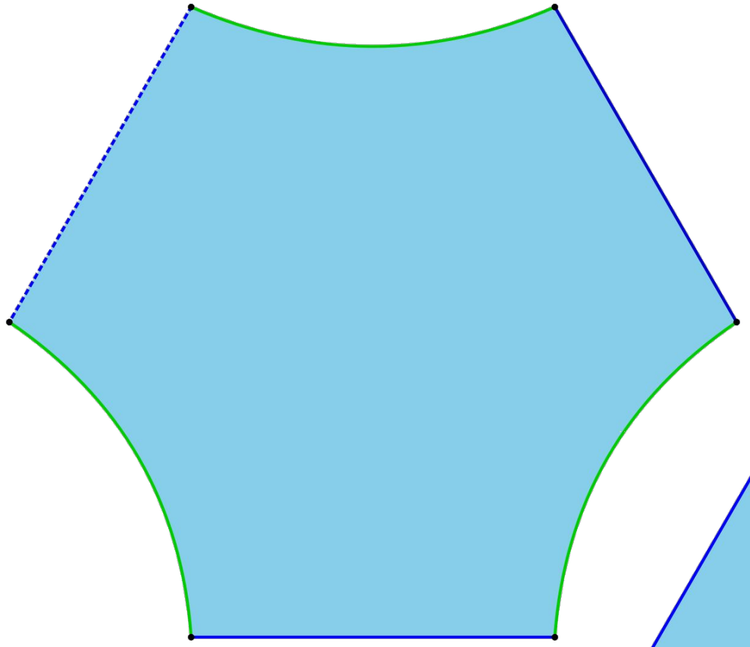
Délai dans la subdivision

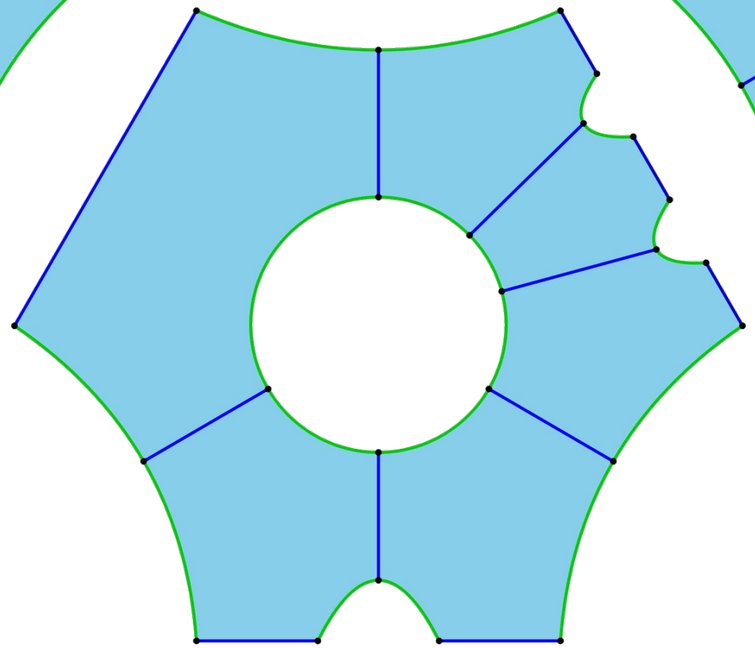


Délai dans la subdivision

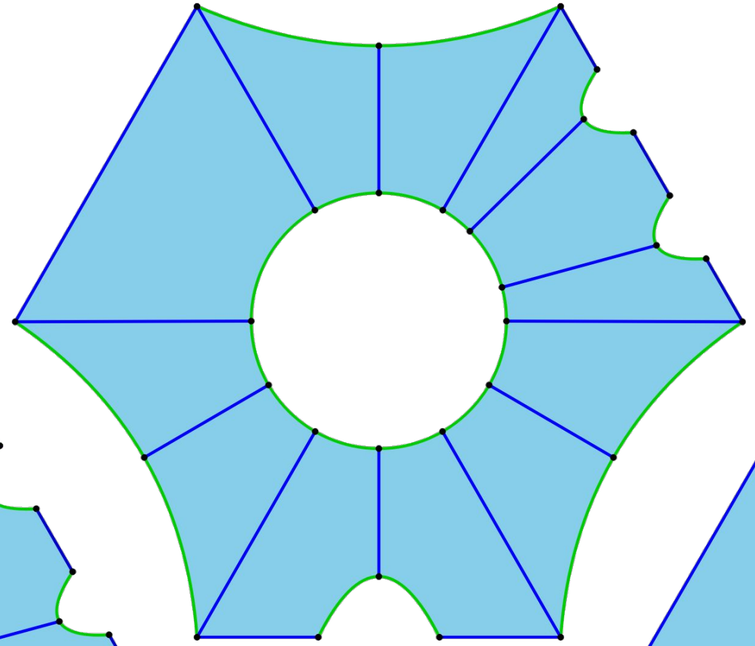


Autres processus de subdivision

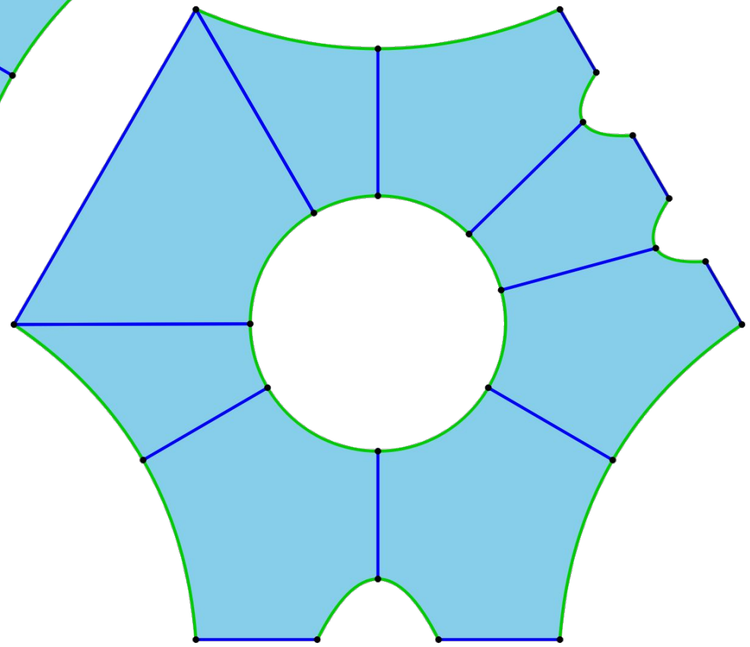




Processus 1



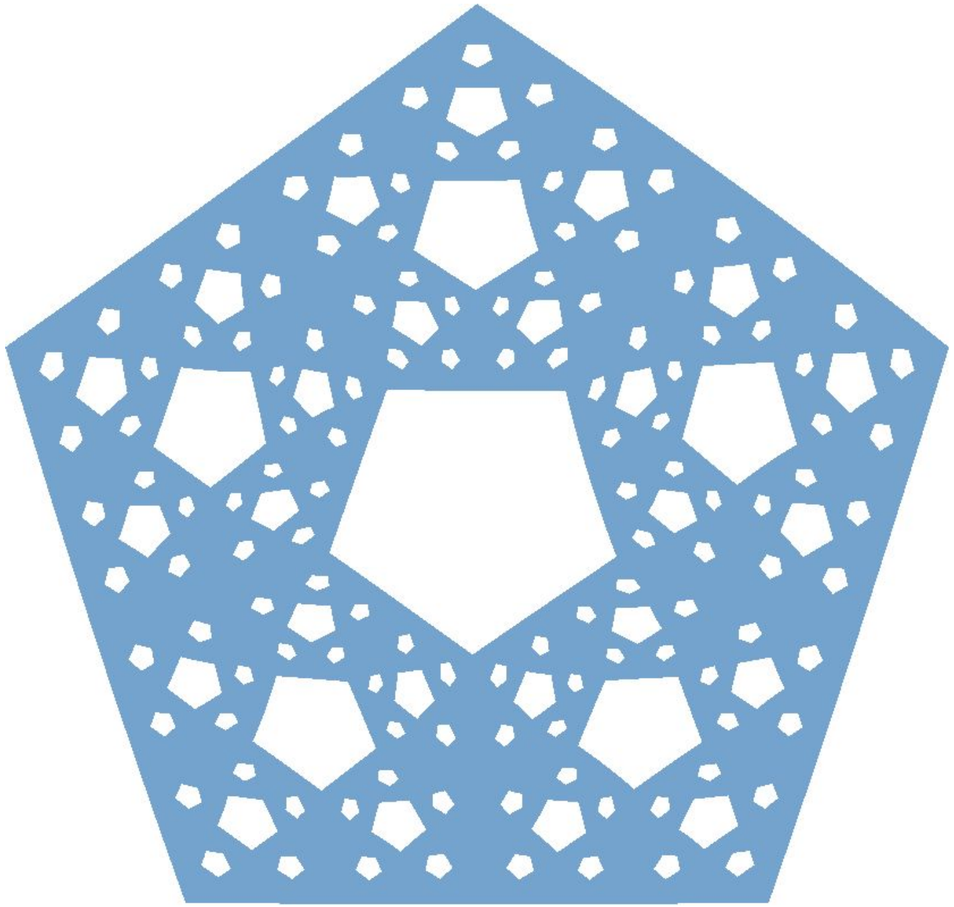
Processus 2

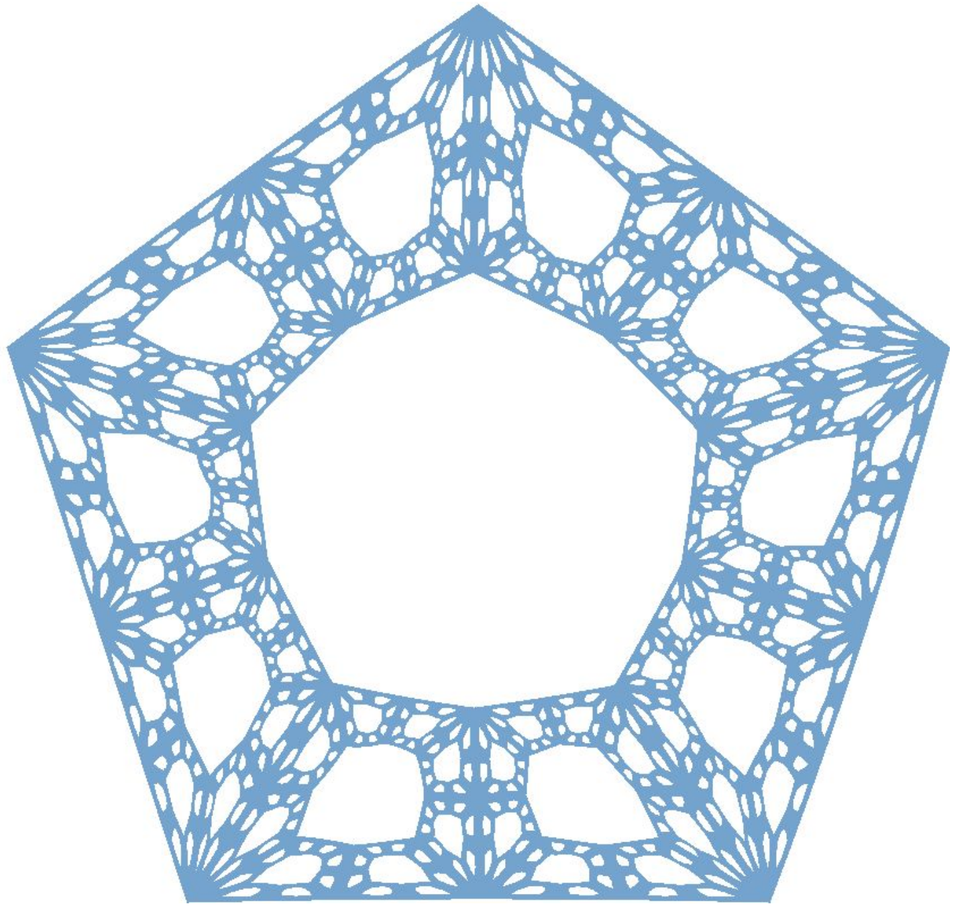


Processus 3

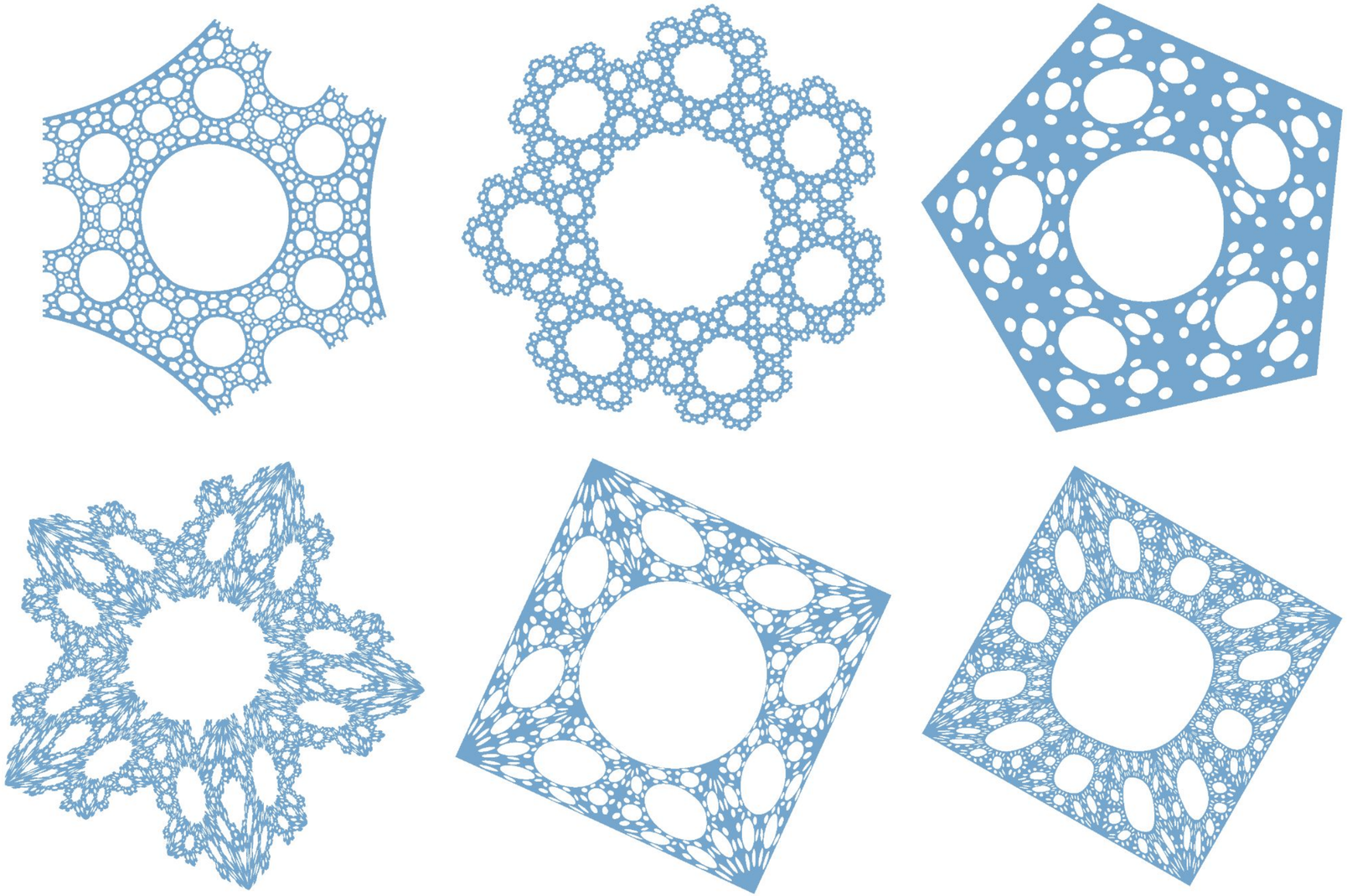
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Autres processus de subdivision

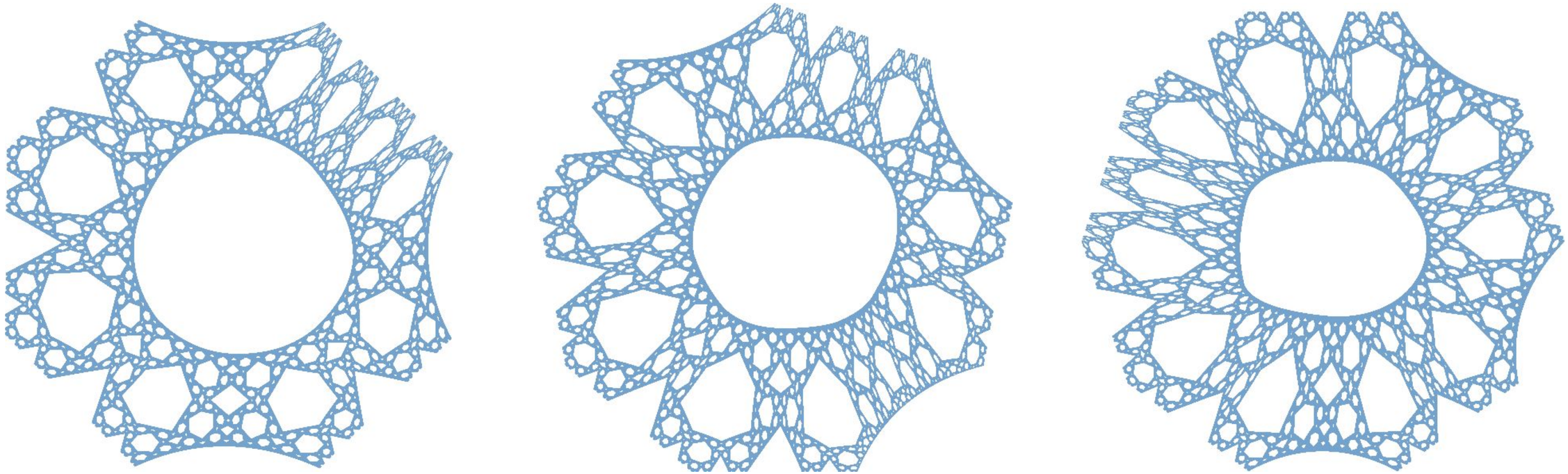




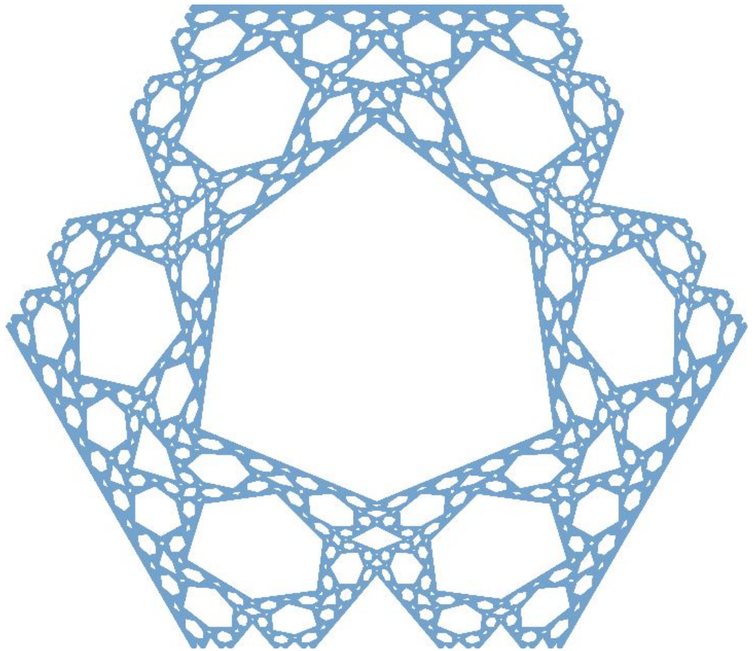
Exemples de formes

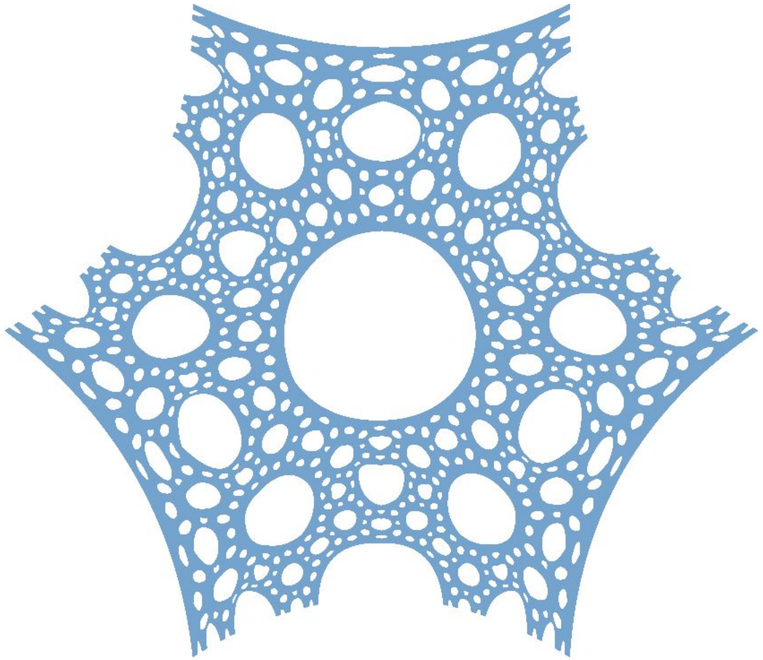


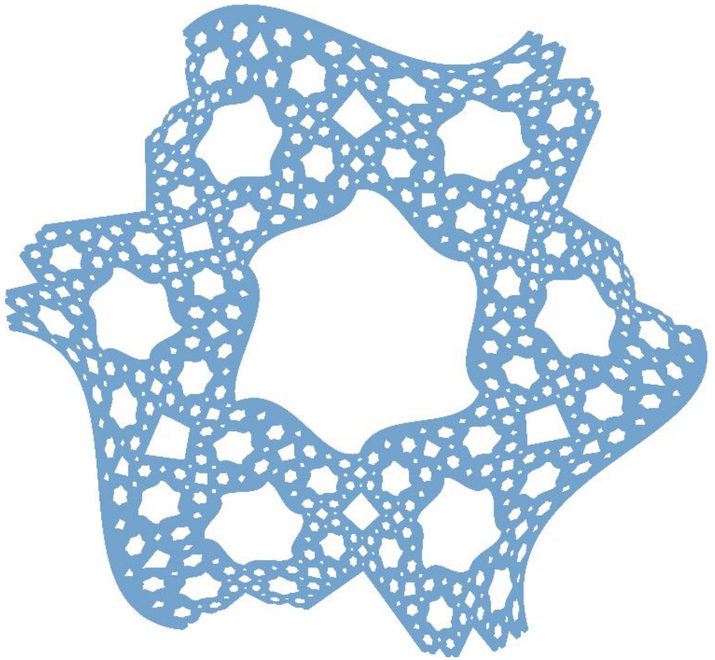
Exemples de formes



Exemples de formes





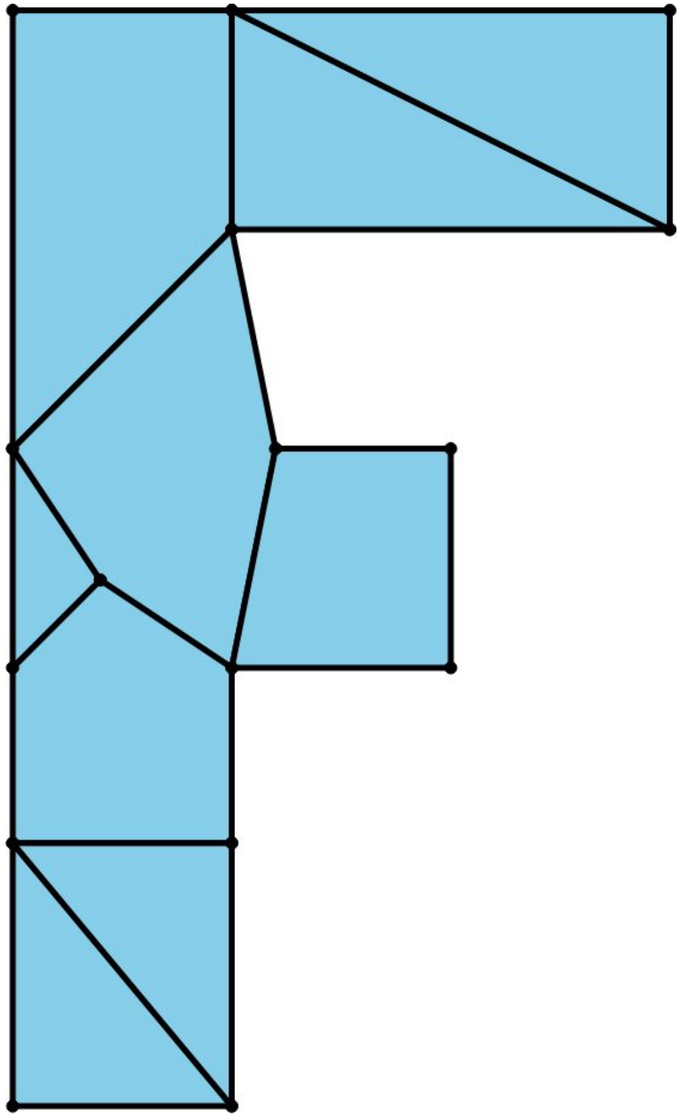


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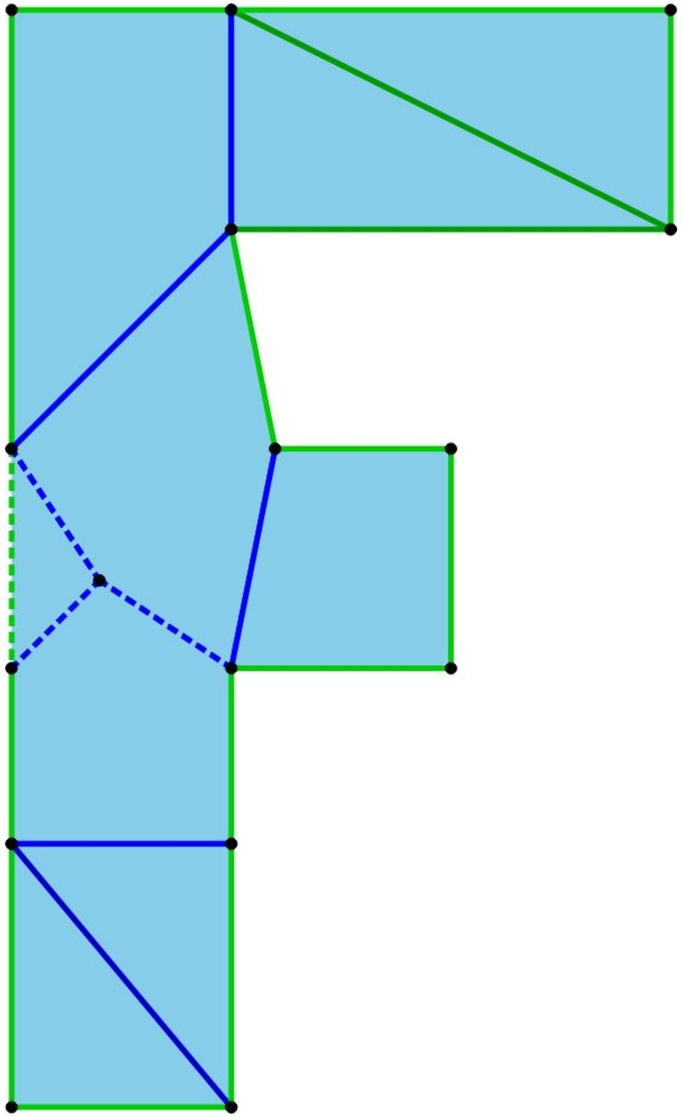
Assemblage



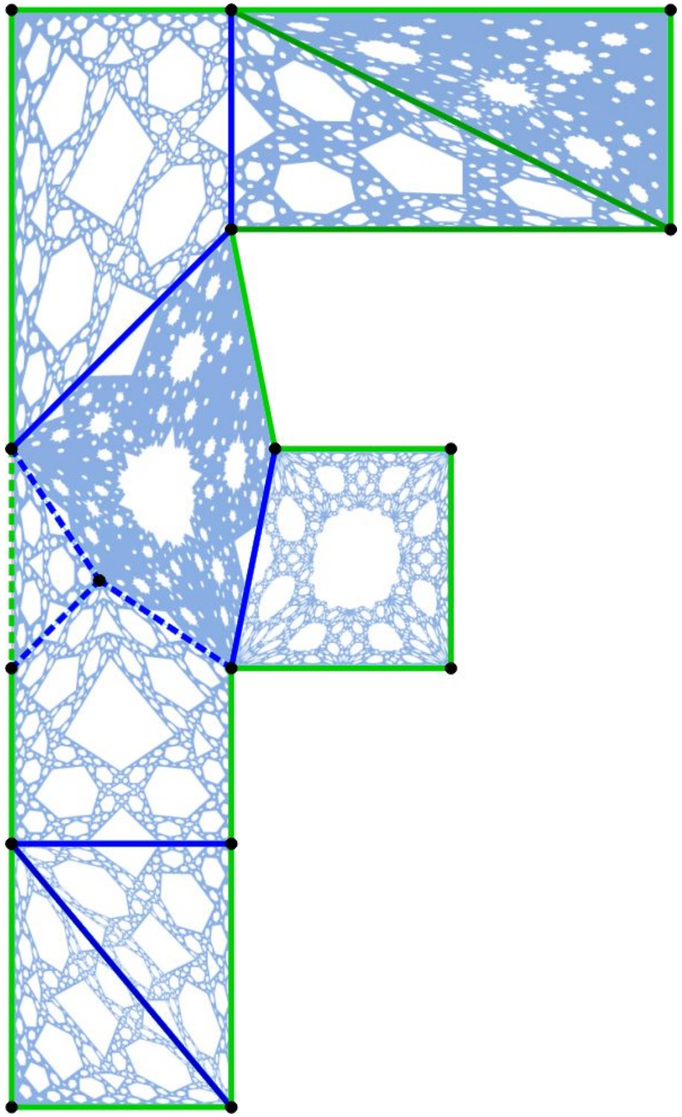
Assemblage



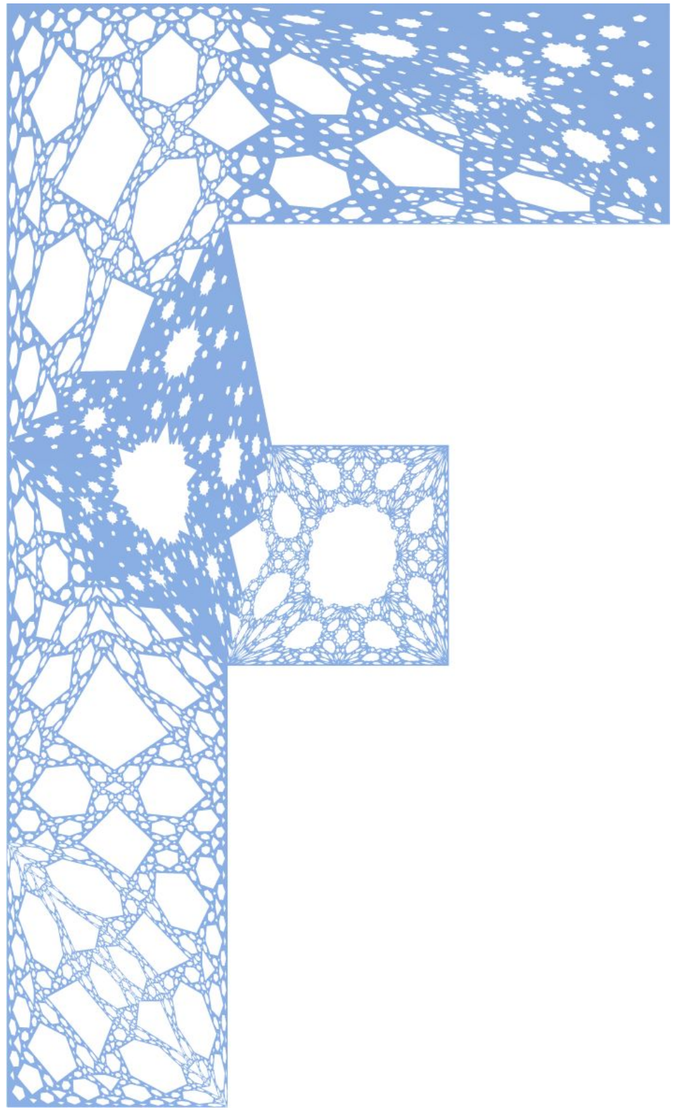
Assemblage



Assemblage

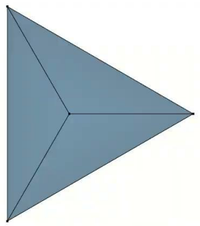


Assemblage

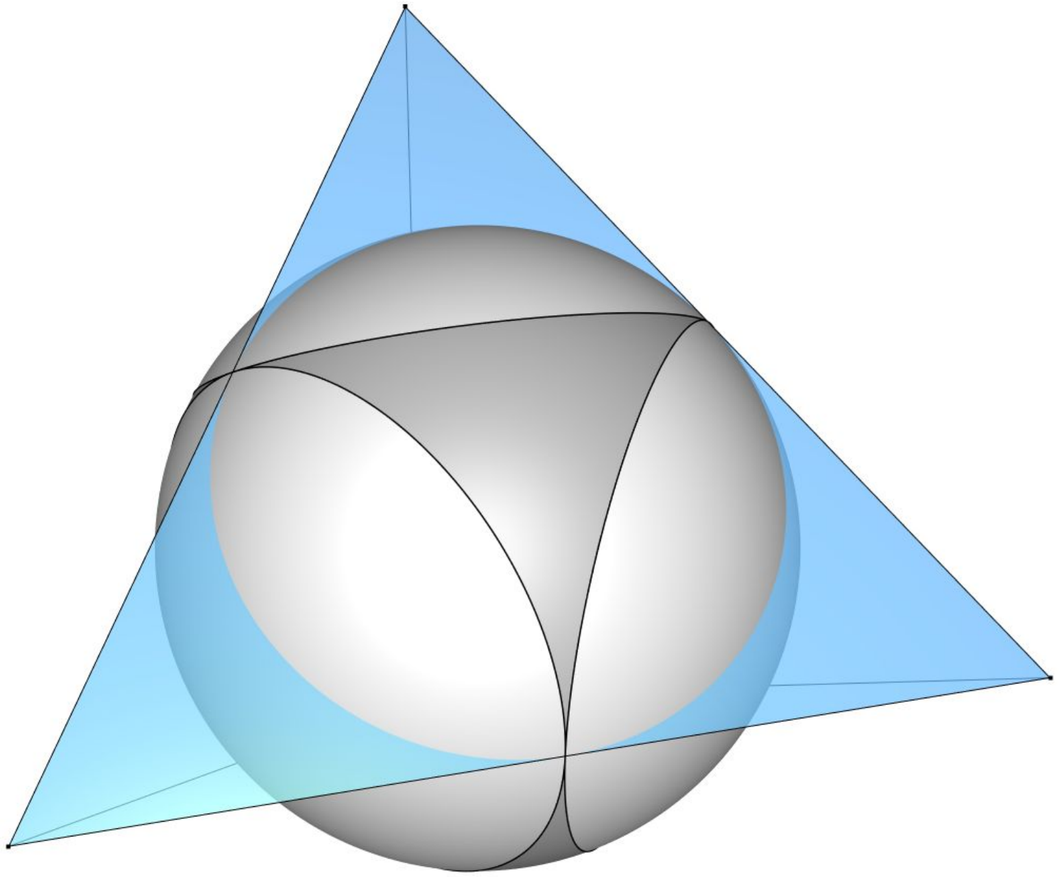


Conception automatique 2D à partir d’un polyèdre

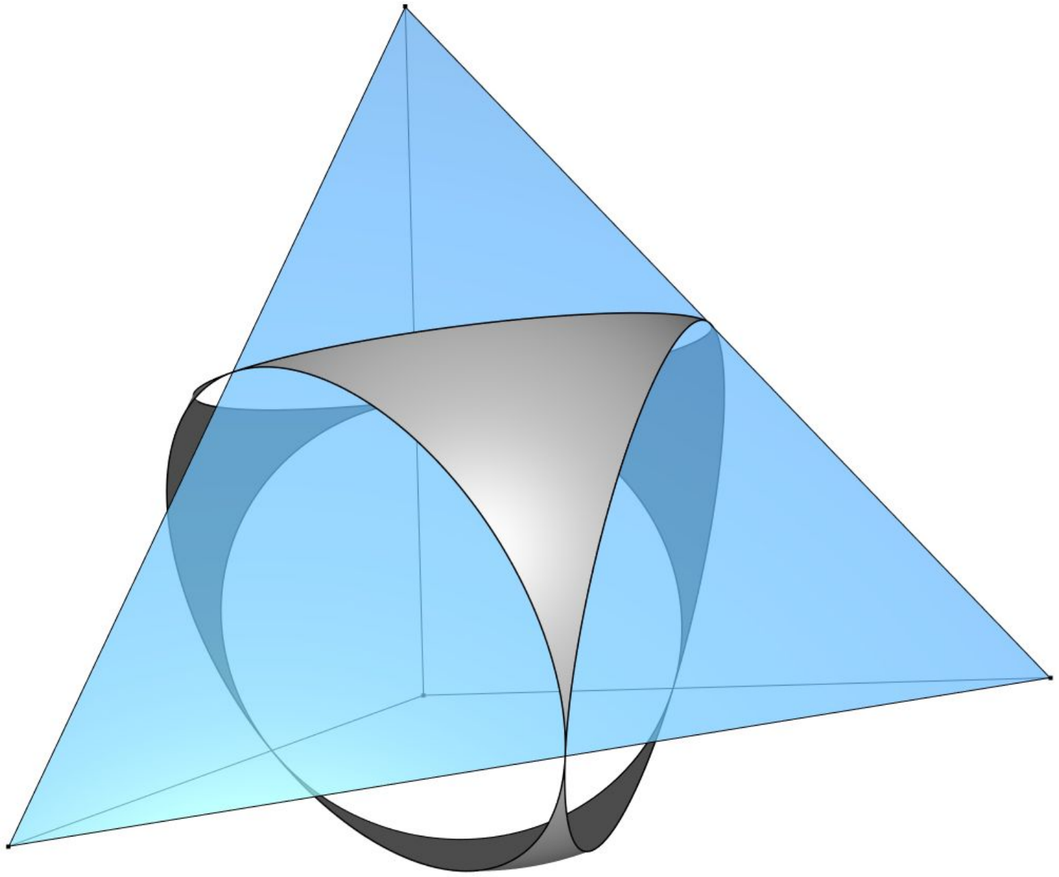
Empilements polyédriques de cercles



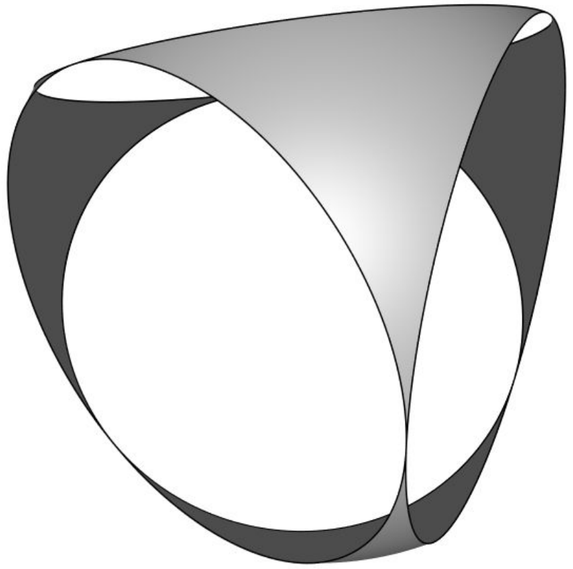
Tétraèdre



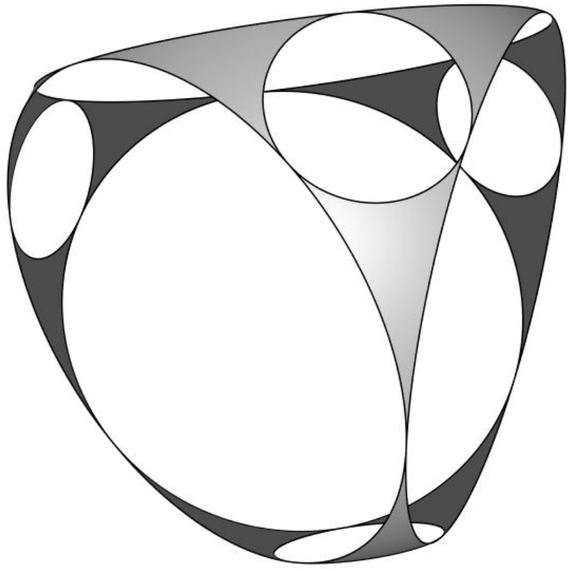
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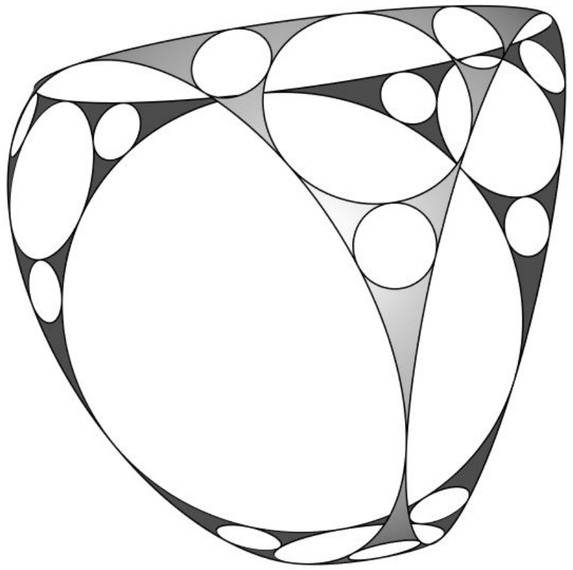
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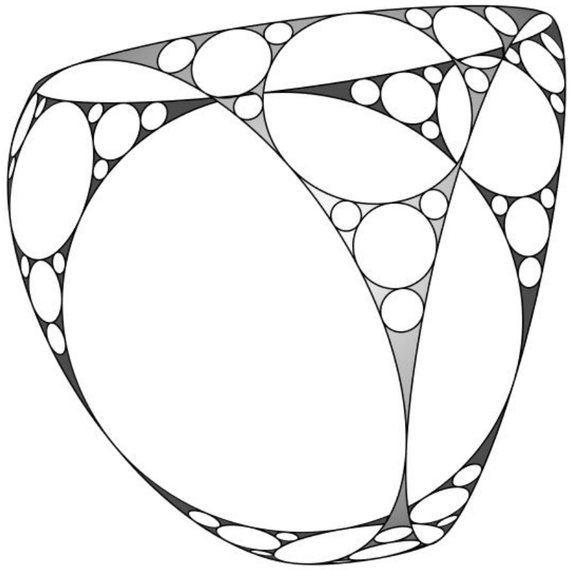
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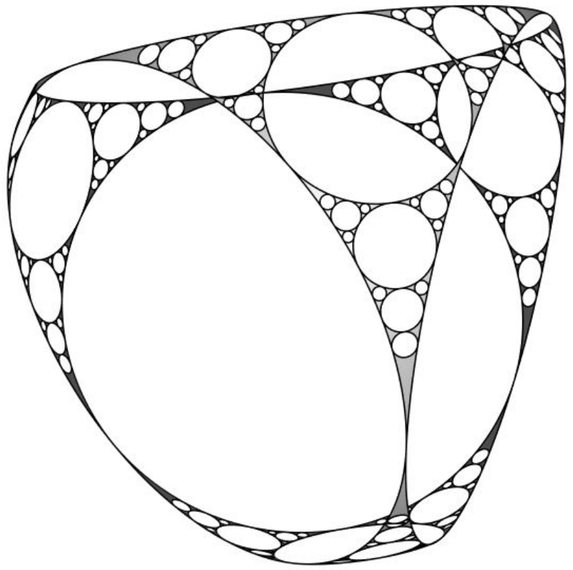
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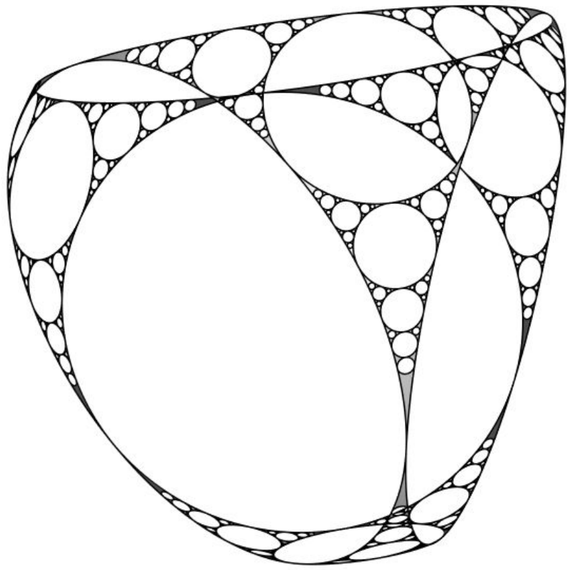
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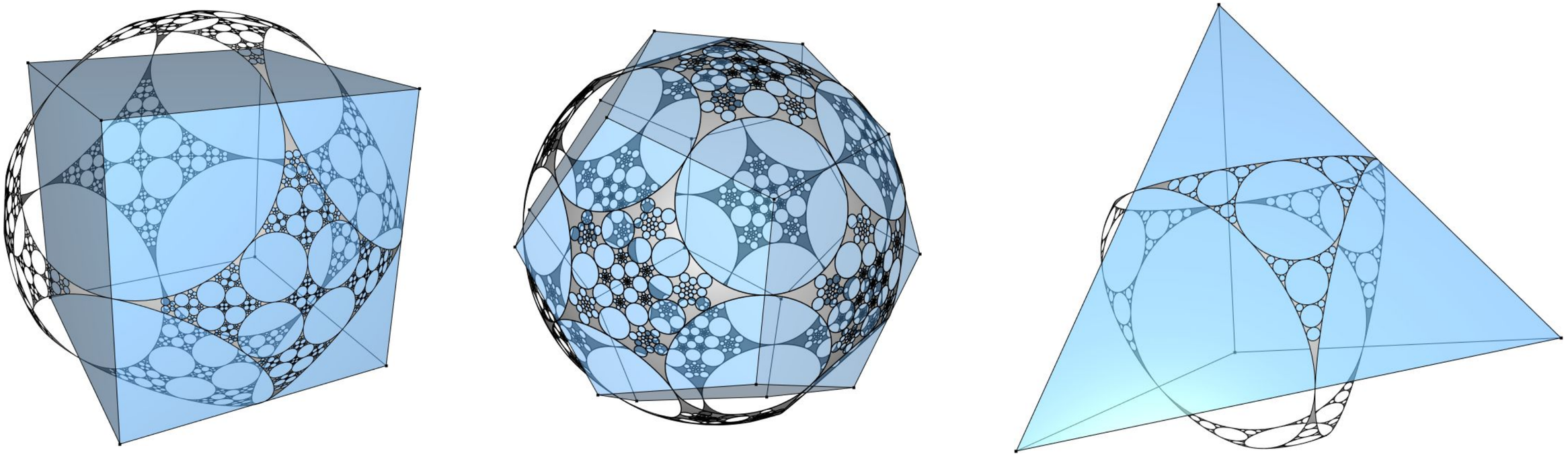
Tétraèdre



Tétraèdre



Processus de subdivision

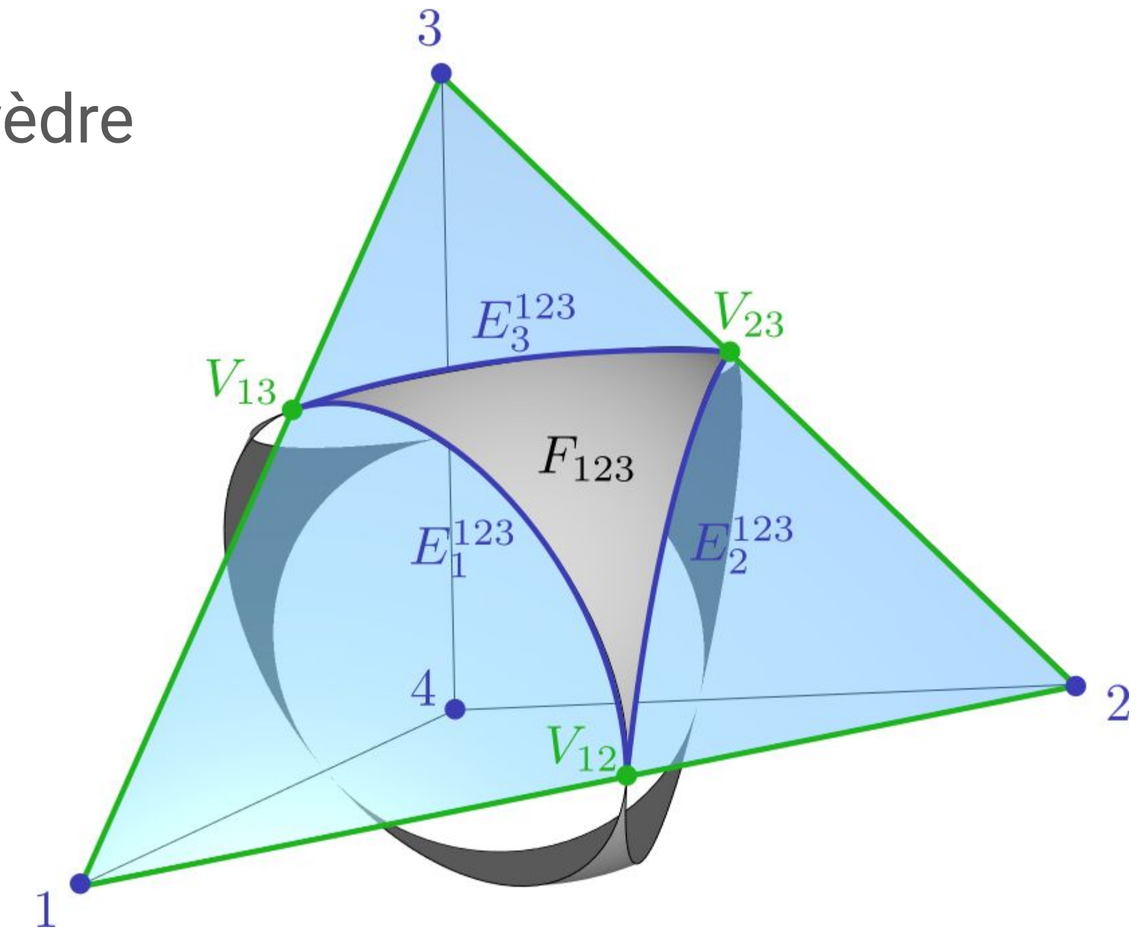


Encodage de la topologie avec un BC-IFS

- Automate
  - États
  - Transitions
    - Opérateurs de bords
    - Opérateurs de subdivision
- Contraintes d’incidence
- Contraintes d’adjacence

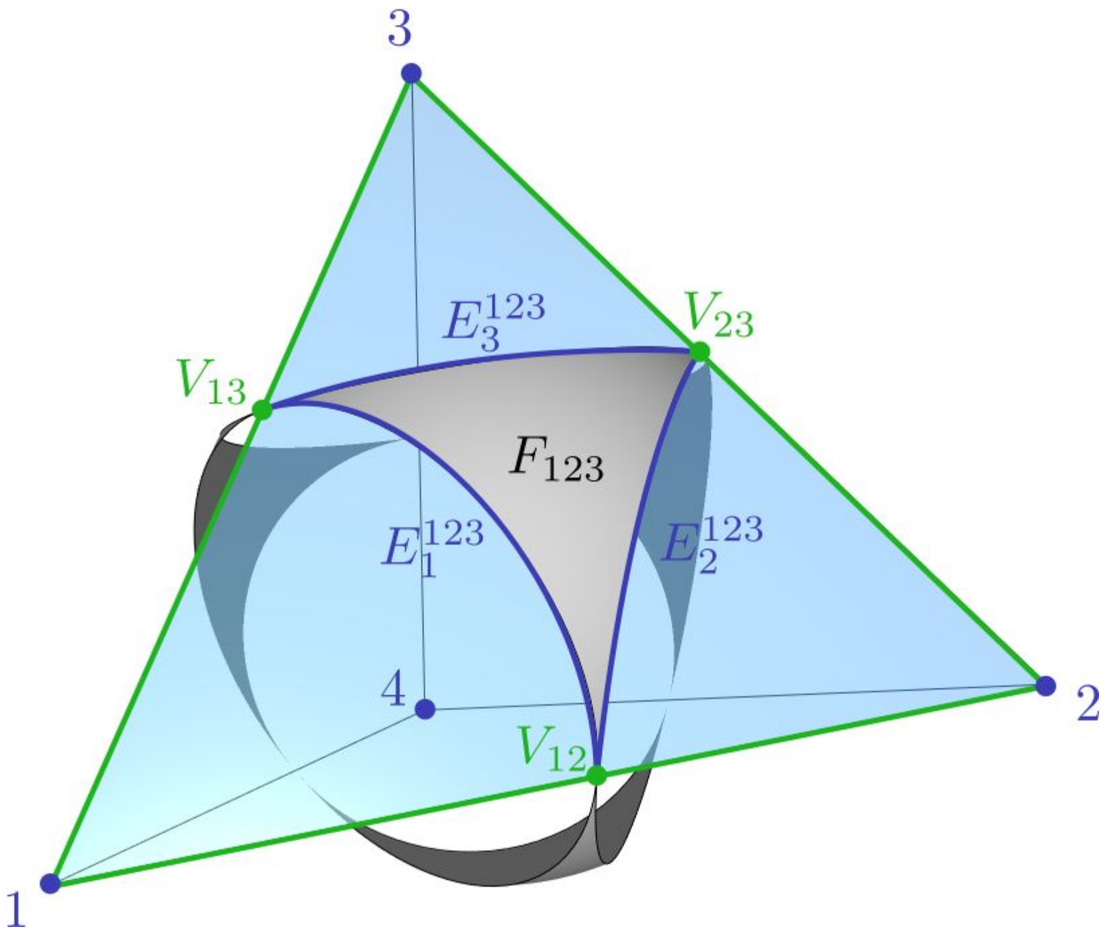
Bijections

- Sommet fractal = arête du polyèdre
- Arête fractale = sommet d'une face du polyèdre
- Face fractale = face du polyèdre



Automate

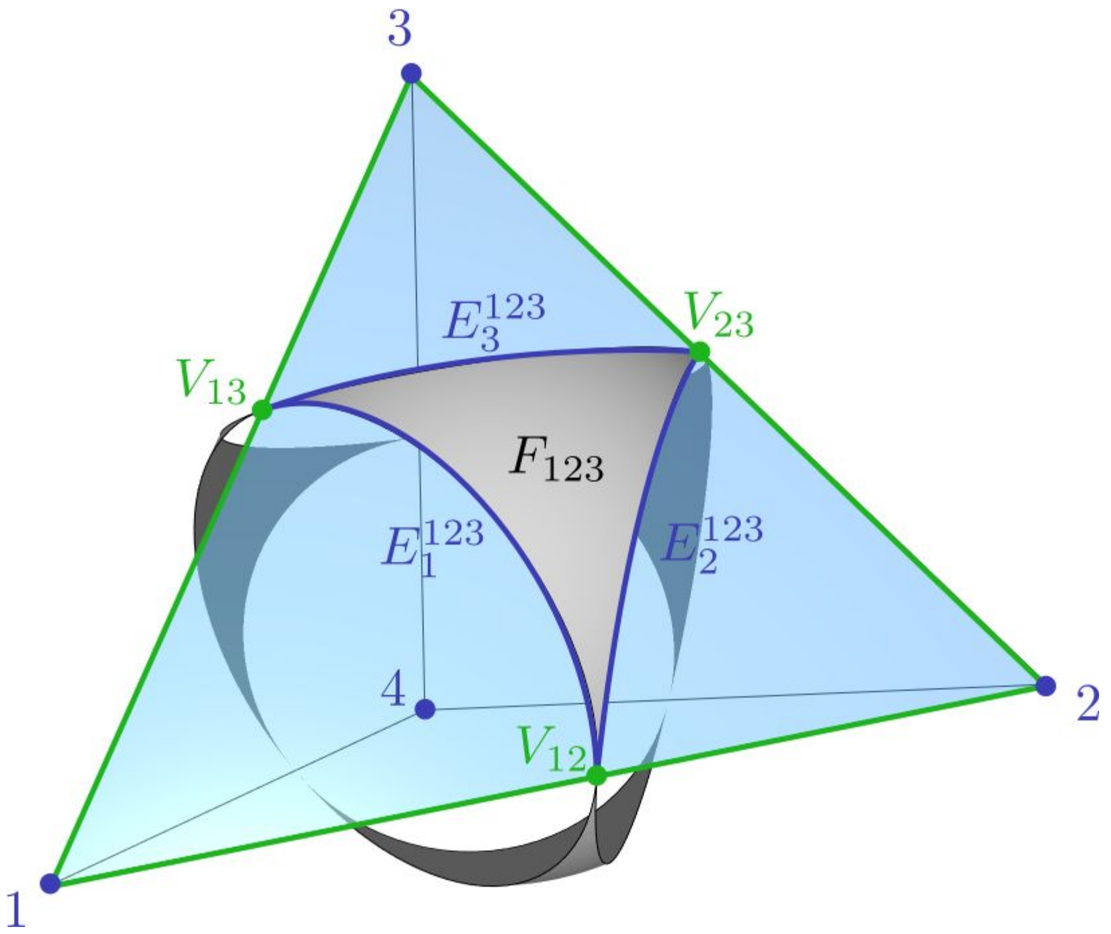
- États
  - Cellules topologiques



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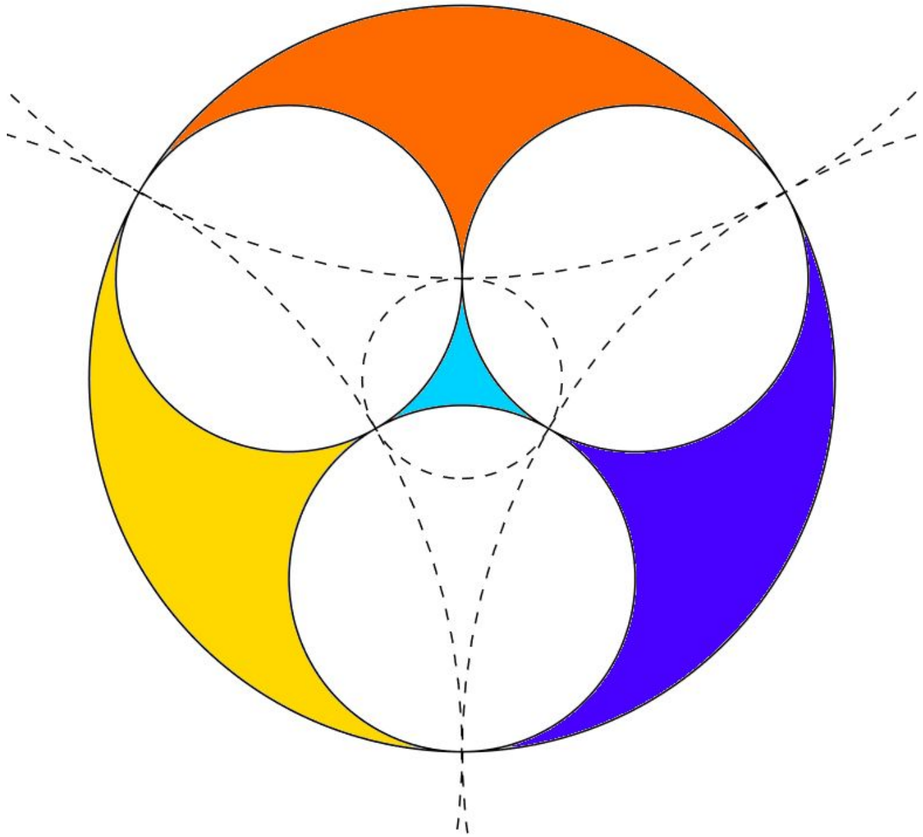
Automate

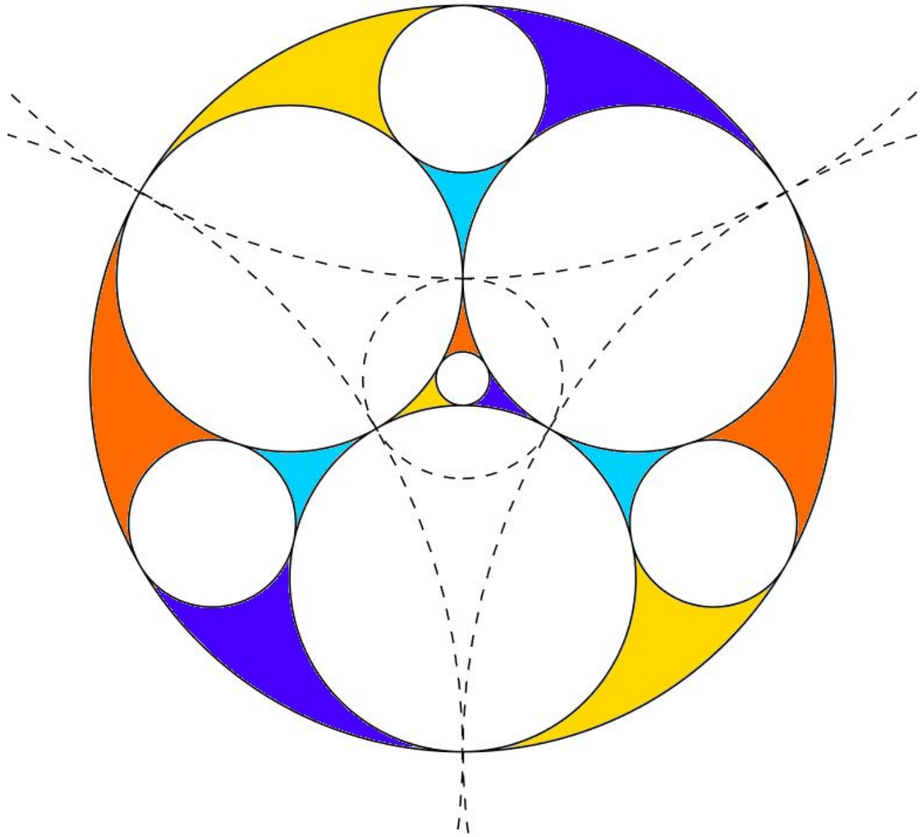
- États
  - Cellules topologiques
- Opérateurs de bord
  - Déduits à partir de la notation



Automate

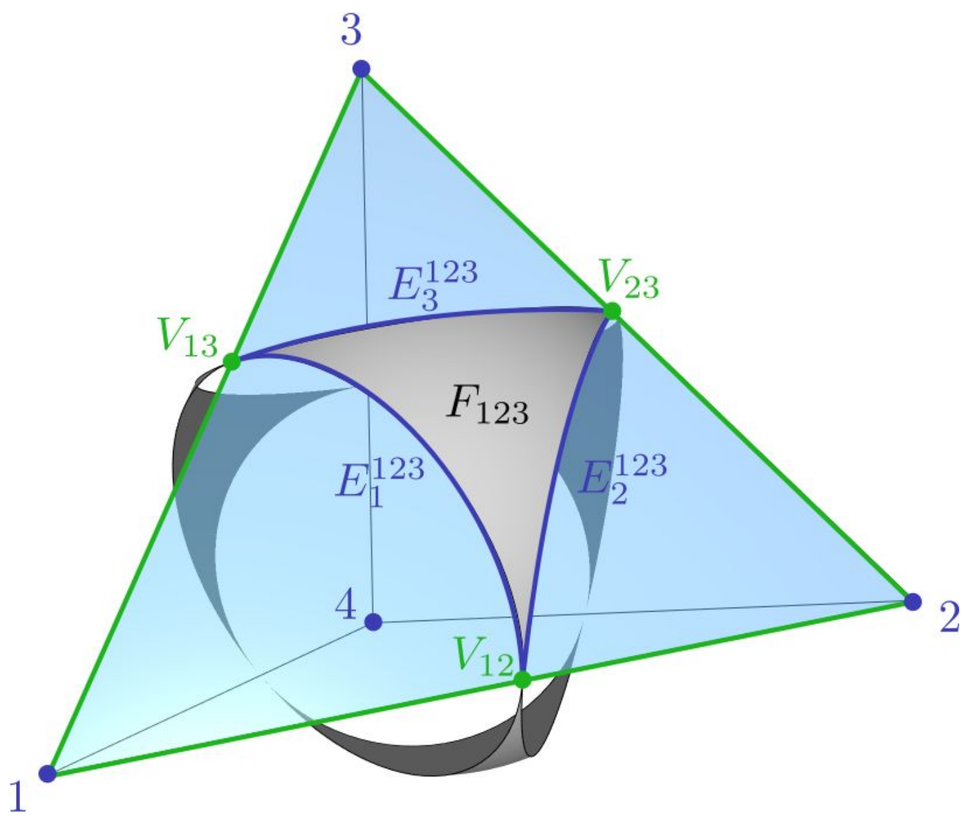
- Opérateurs de subdivision

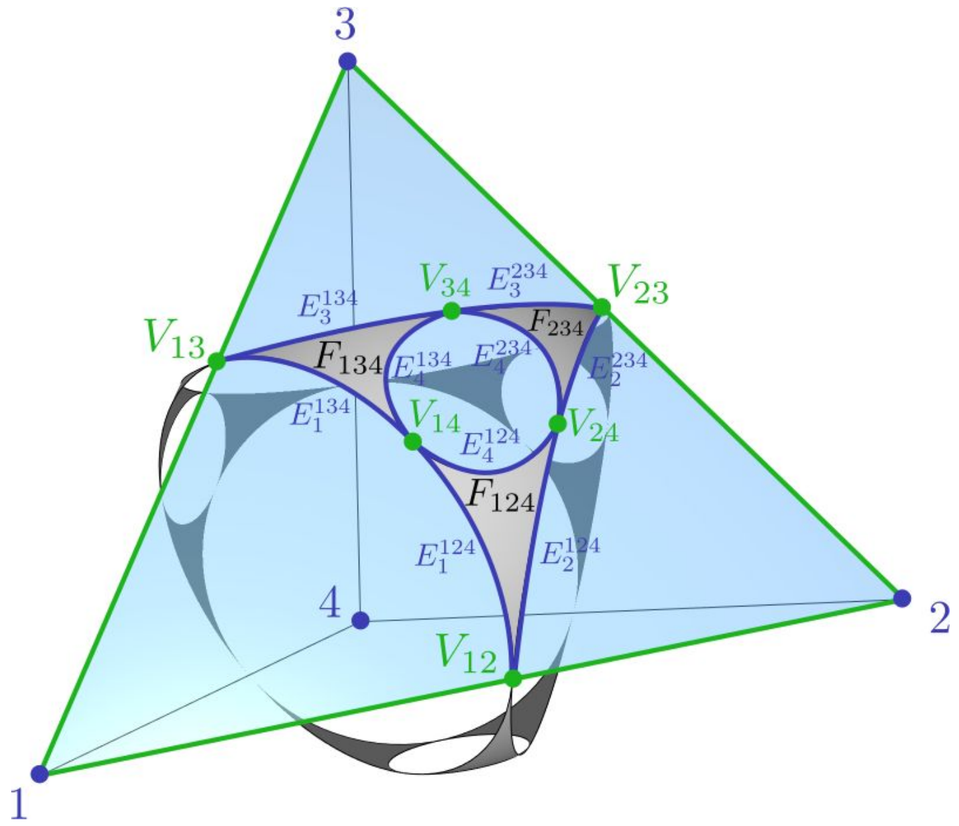




Automate

- Opérateurs de subdivision des faces
  - Déduits à partir du polyèdre

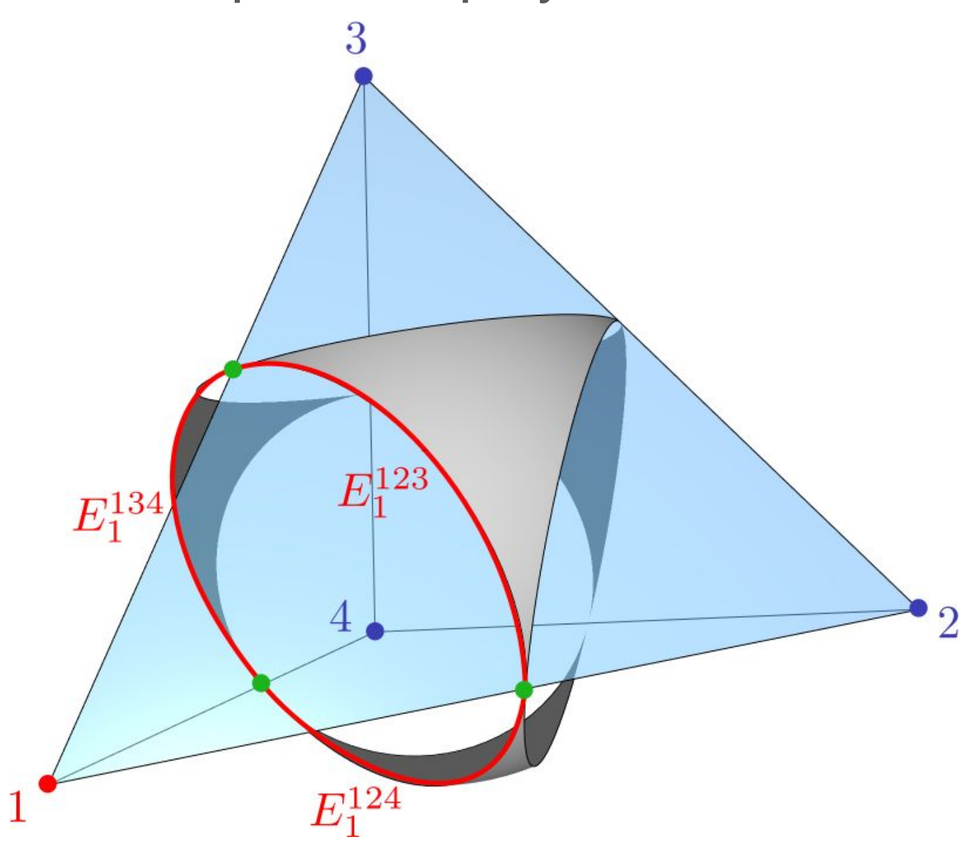


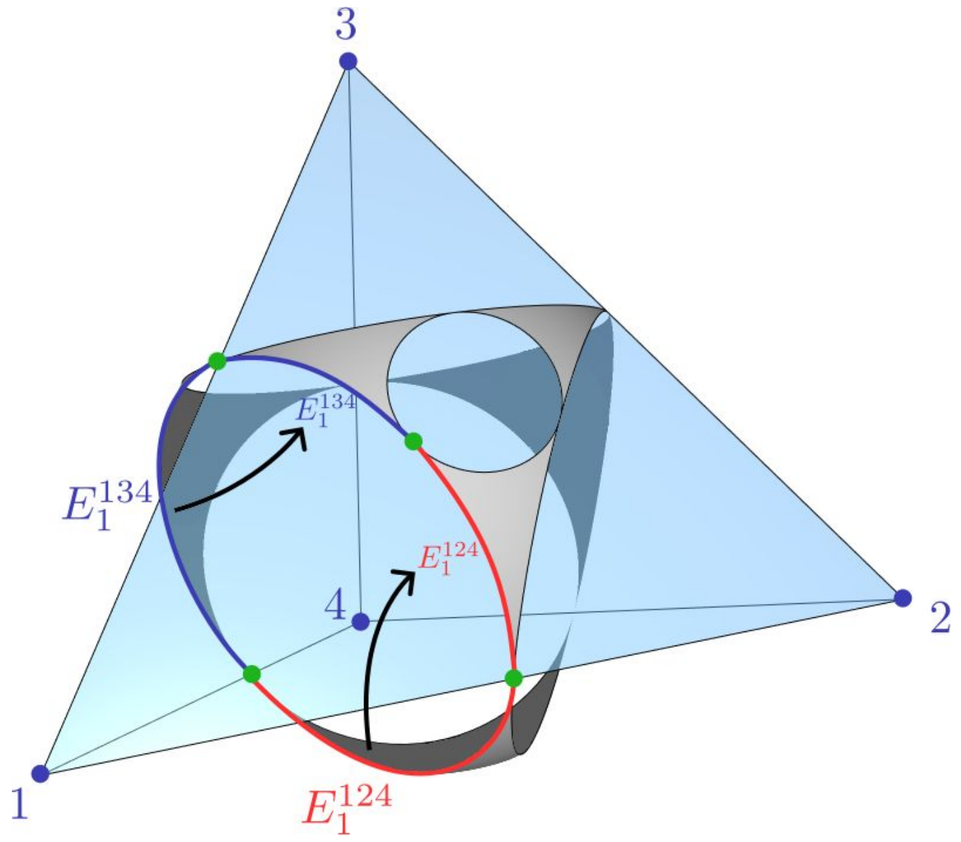


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Automate

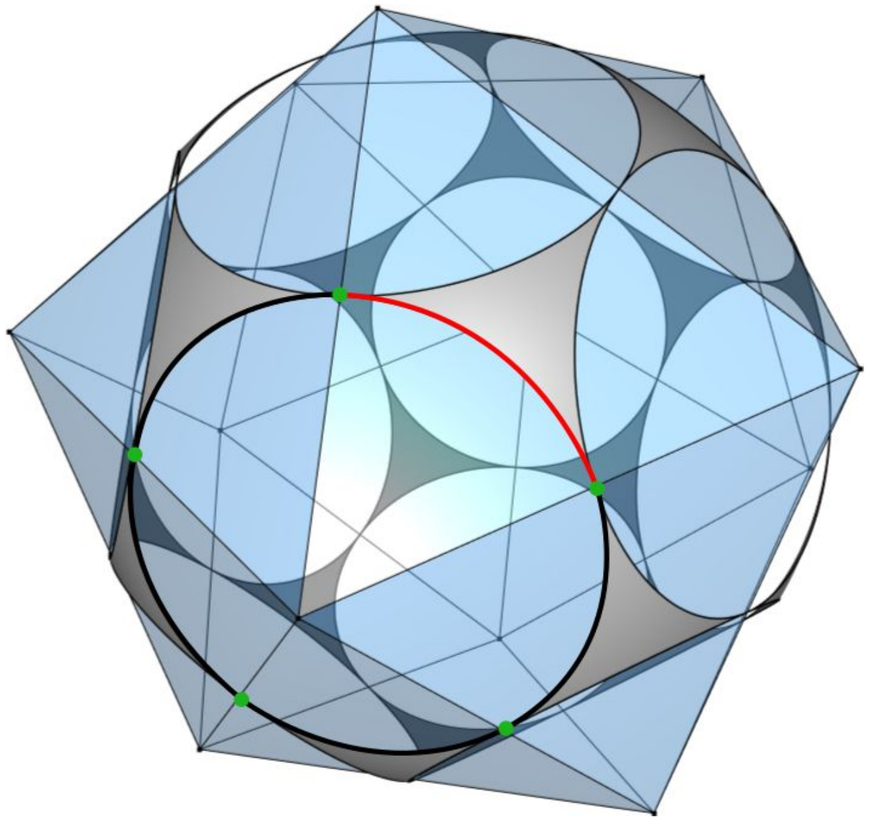
- Opérateurs de subdivision des arêtes
  - Déduits à partir du polyèdre

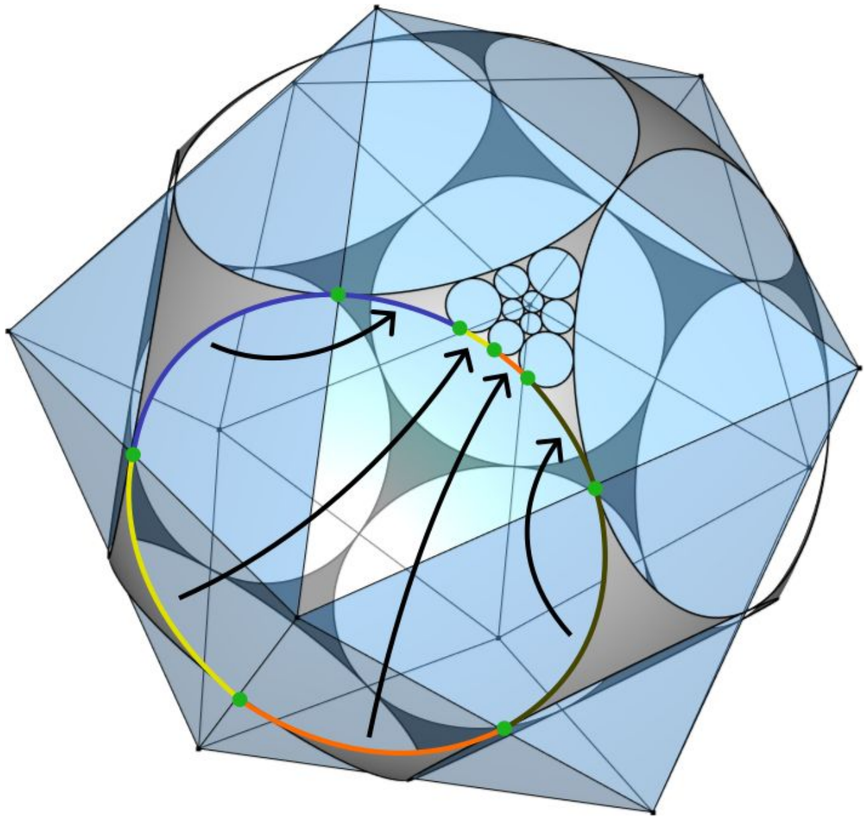




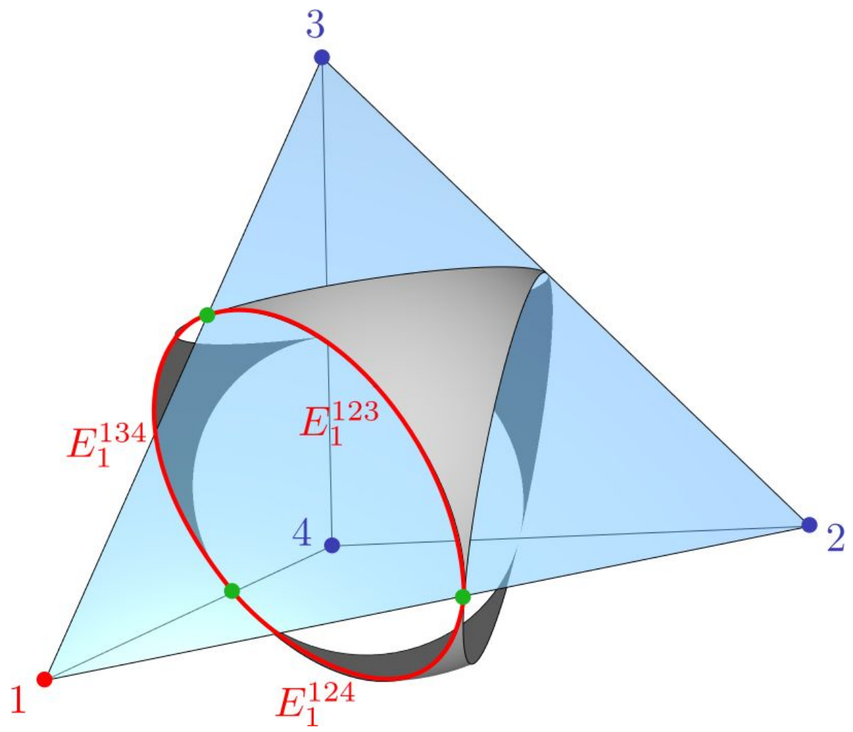
Automate

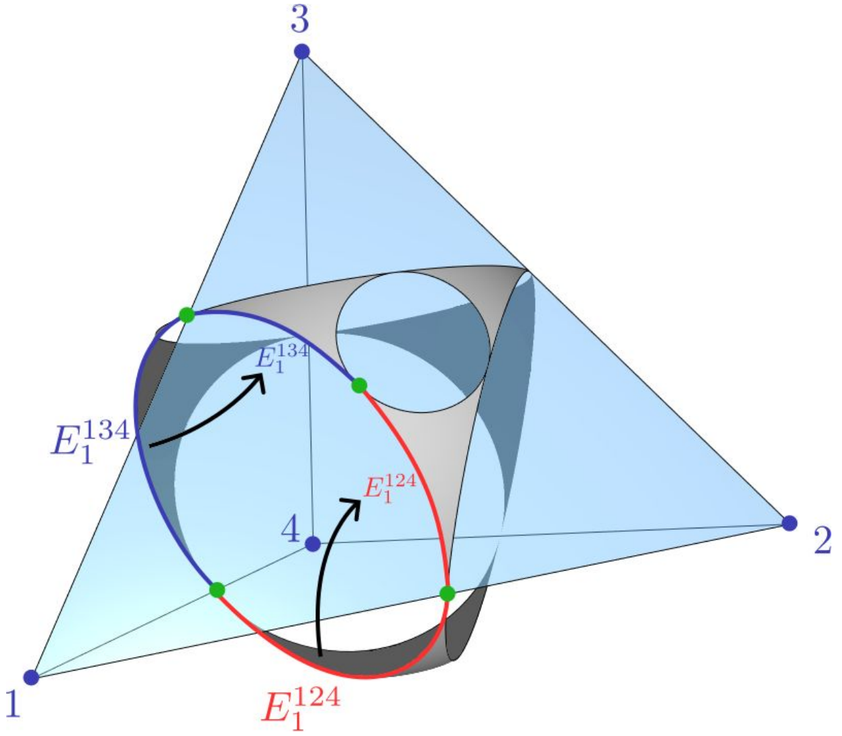
- Opérateurs de subdivision des arêtes
  - Déduits à partir du polyèdre

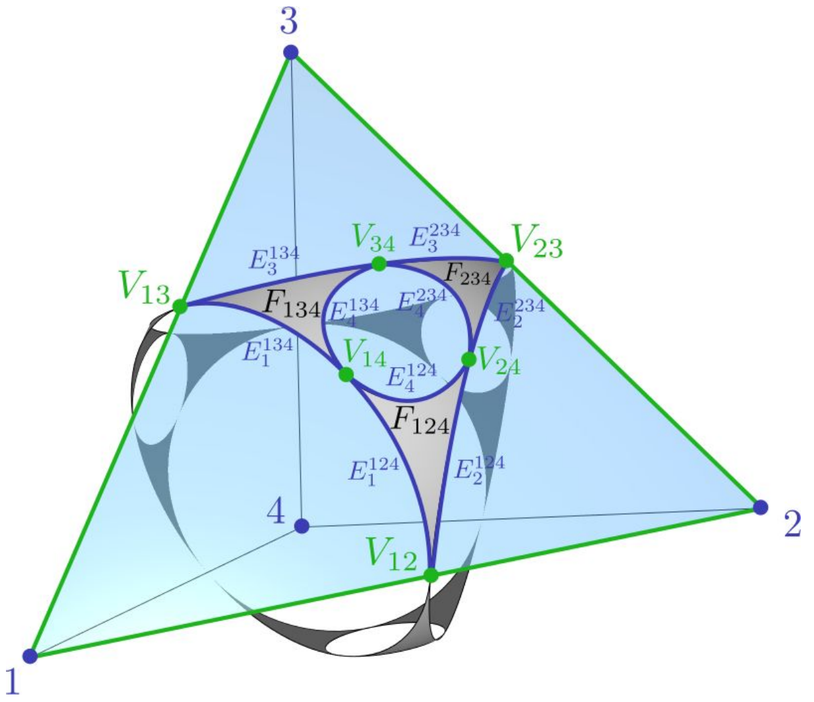




Contraintes d'incidence

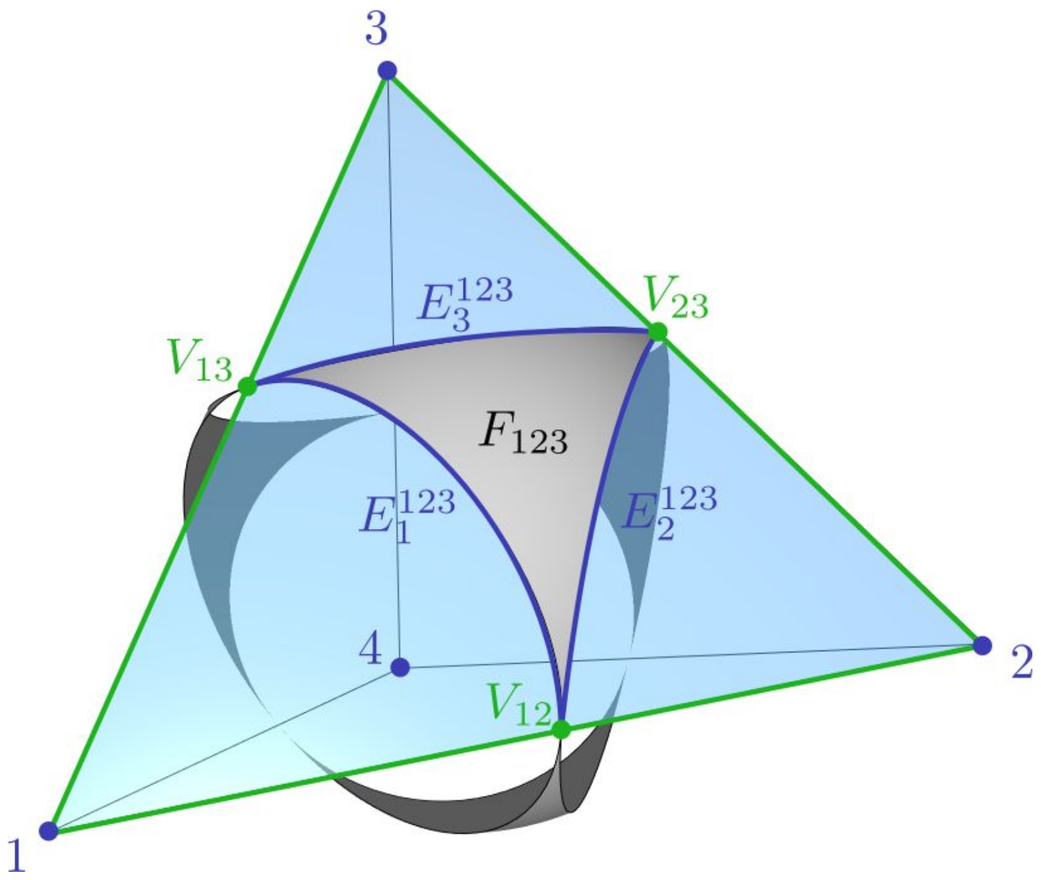


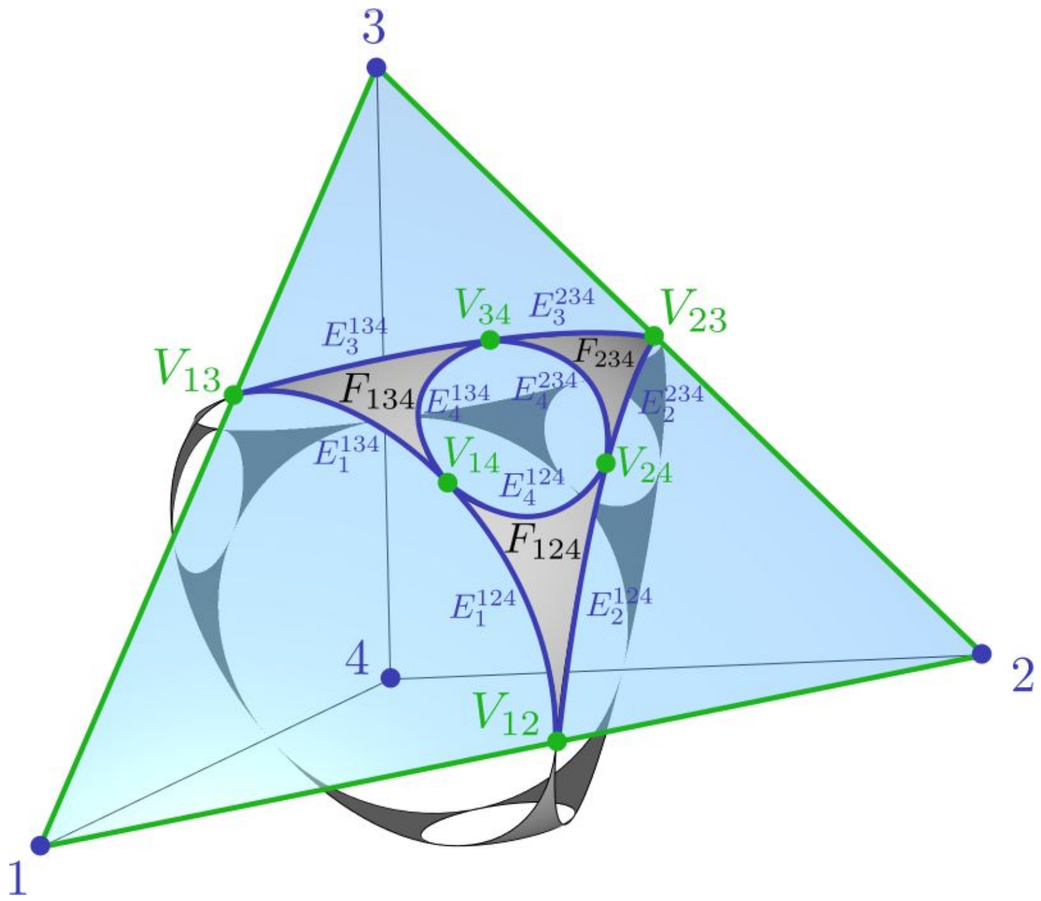




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Contraintes d’adjacence

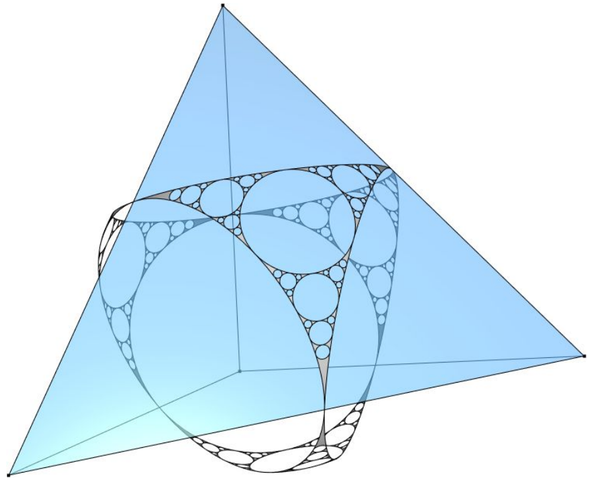


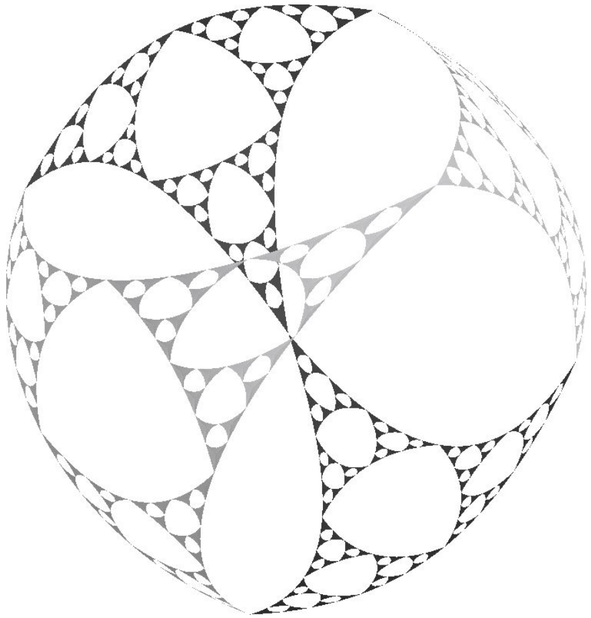


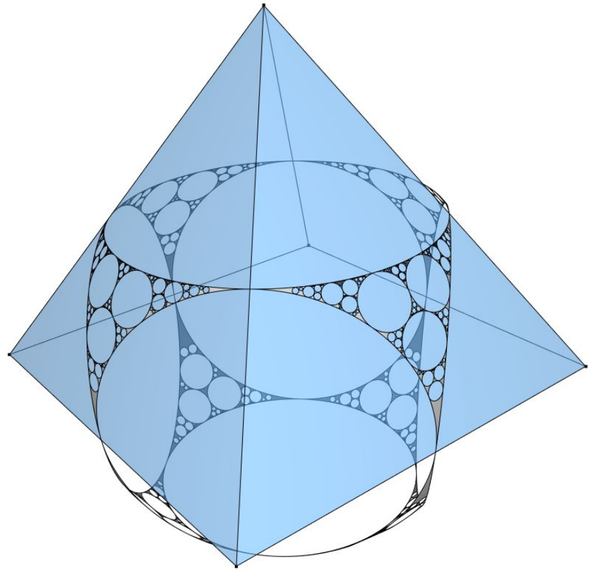
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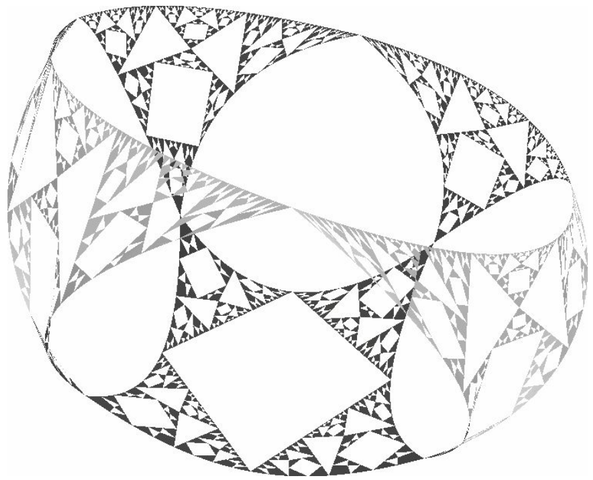
Contrôle de la lacunarité

Extraction à partir d’un polyèdre



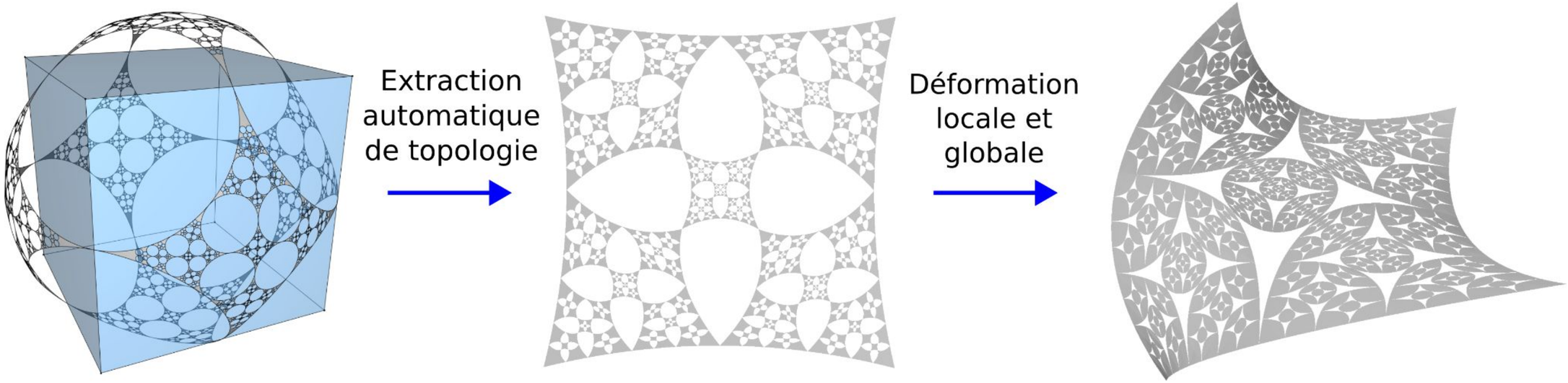




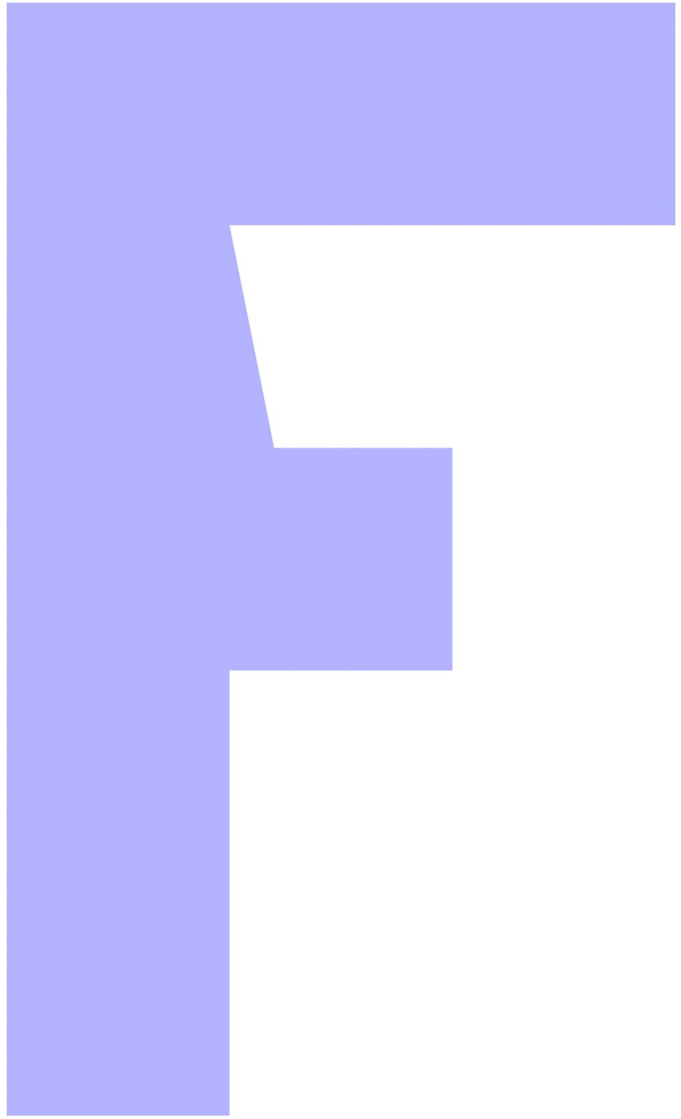


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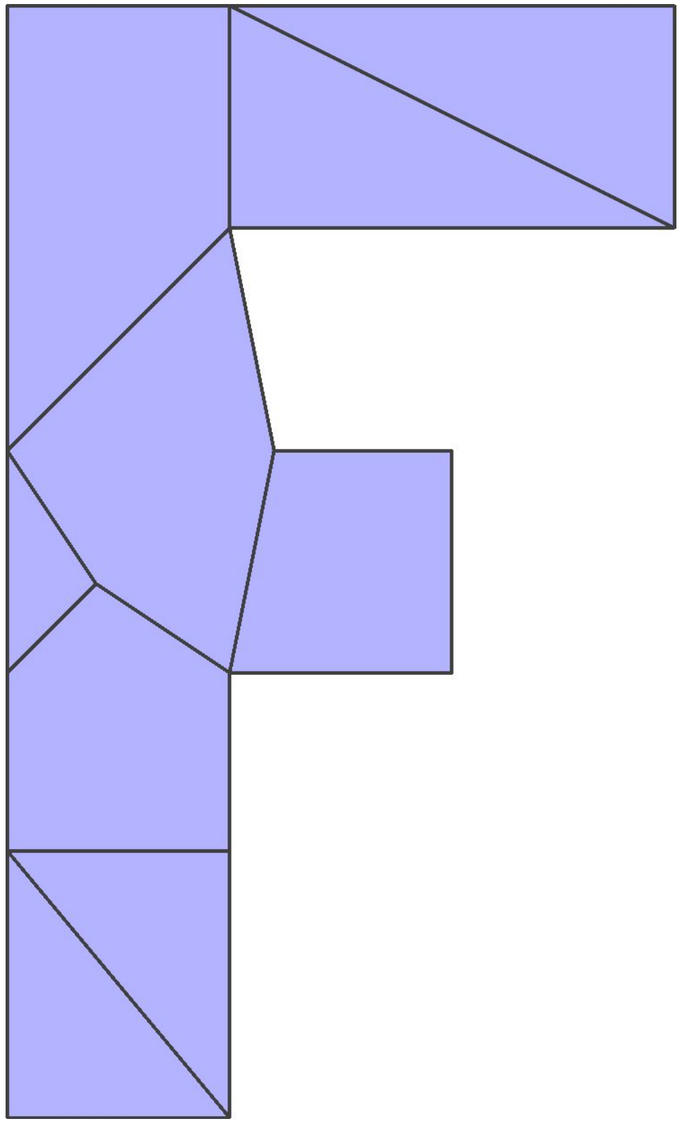
Extraction à partir d’un polyèdre



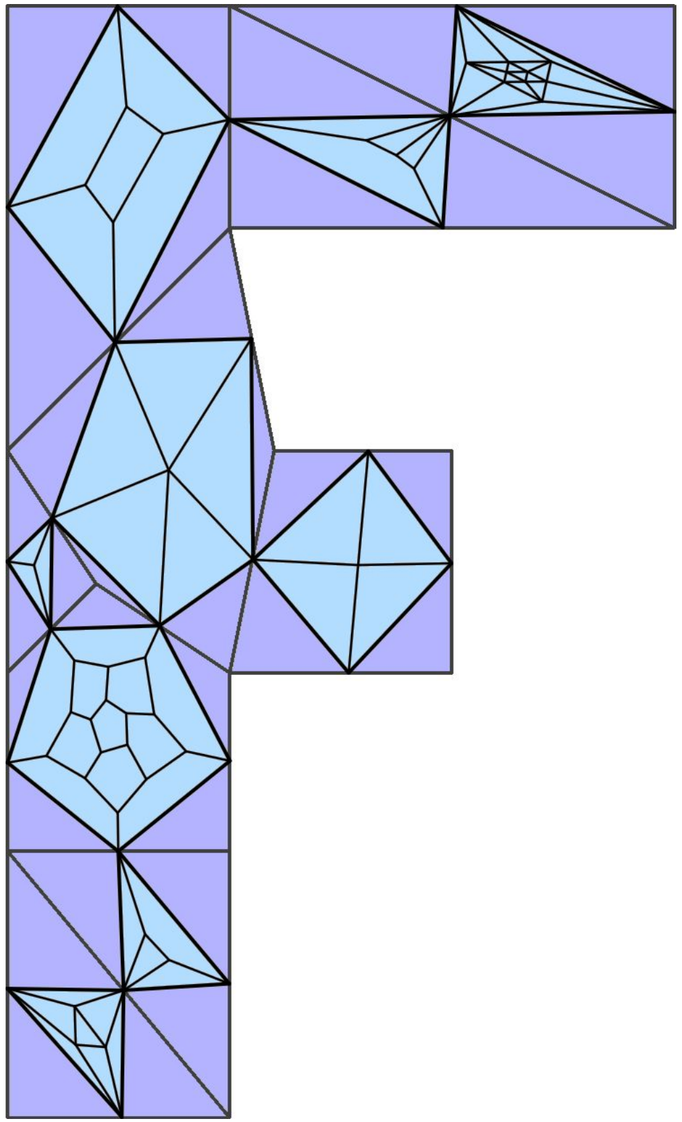
Assemblage



Assemblage

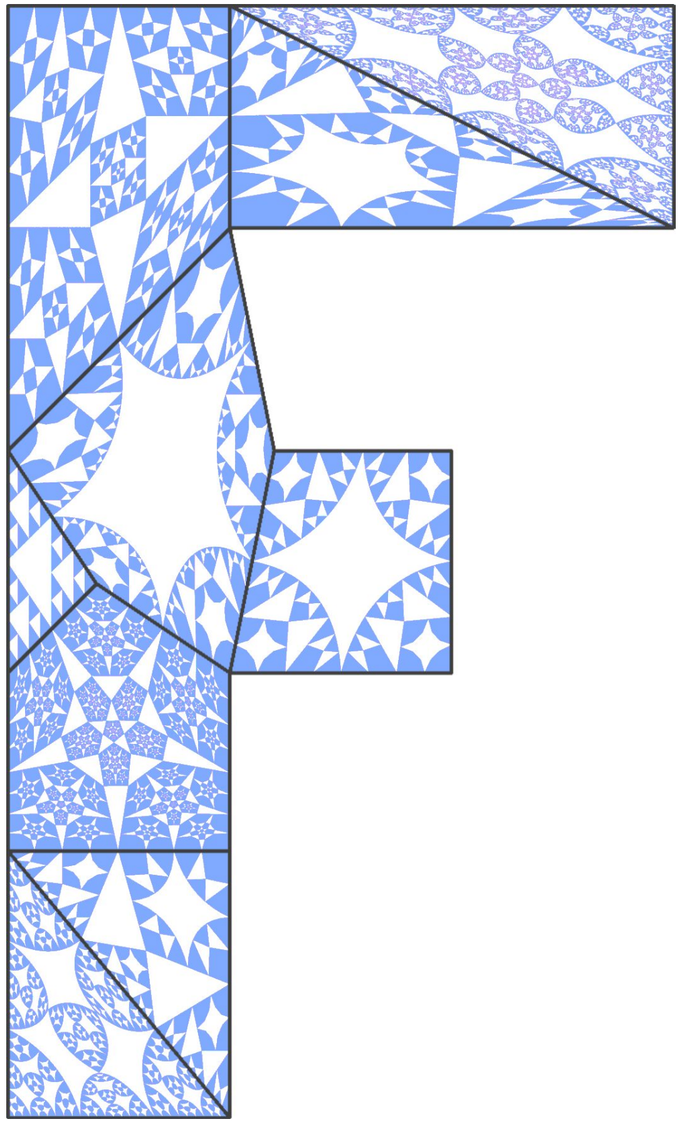


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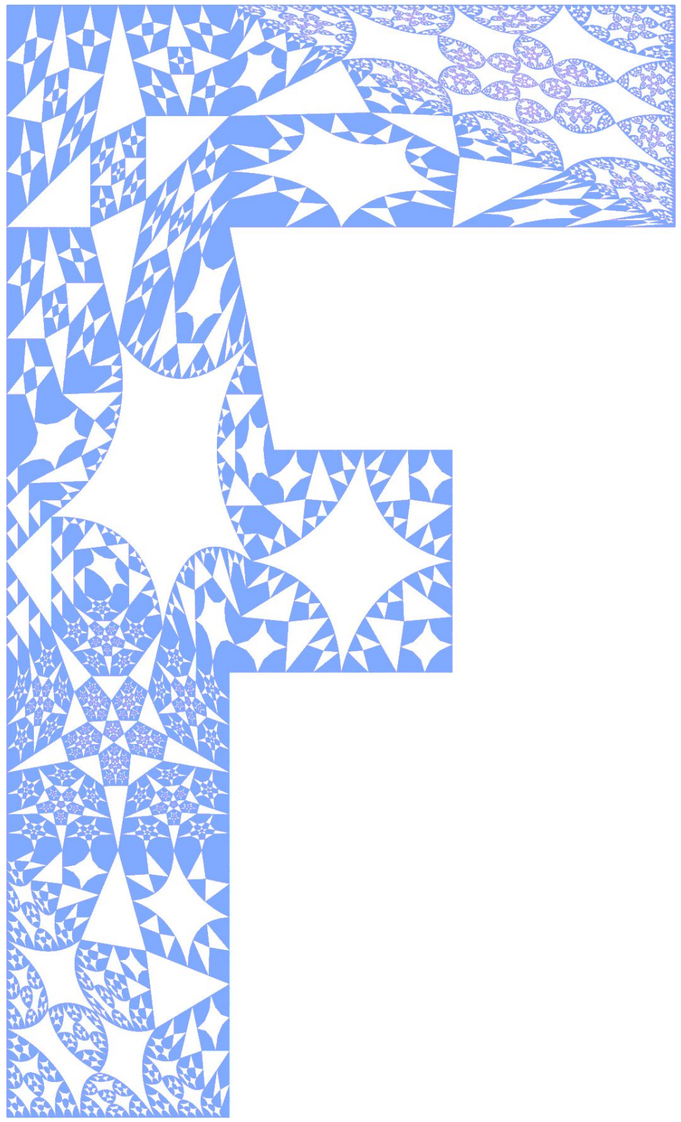


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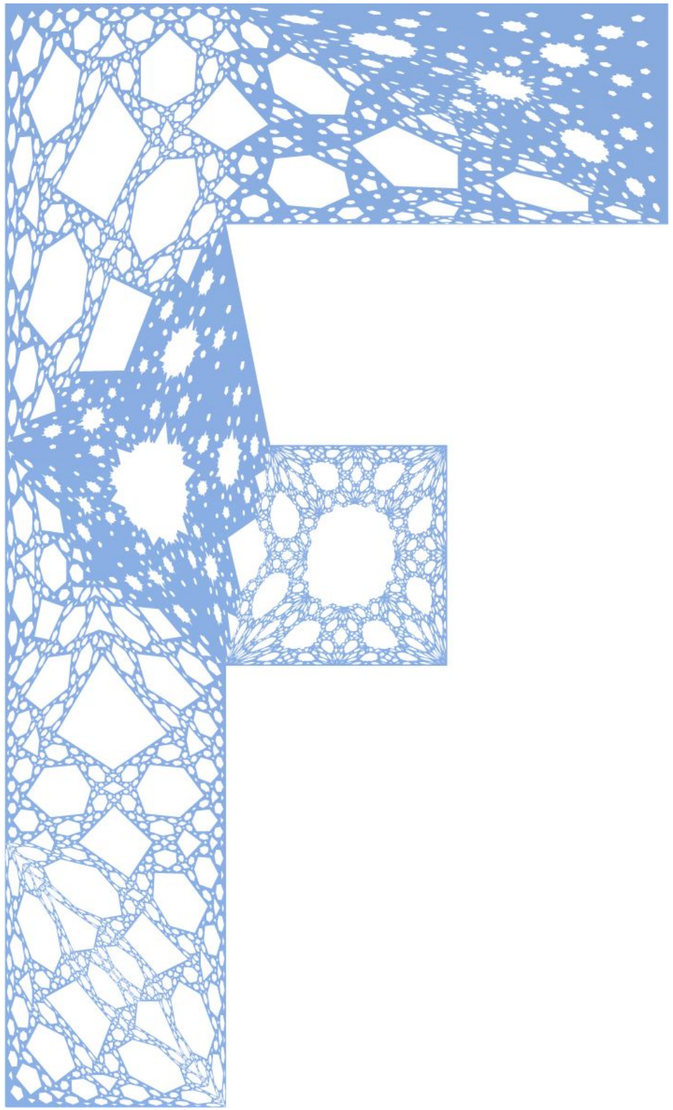
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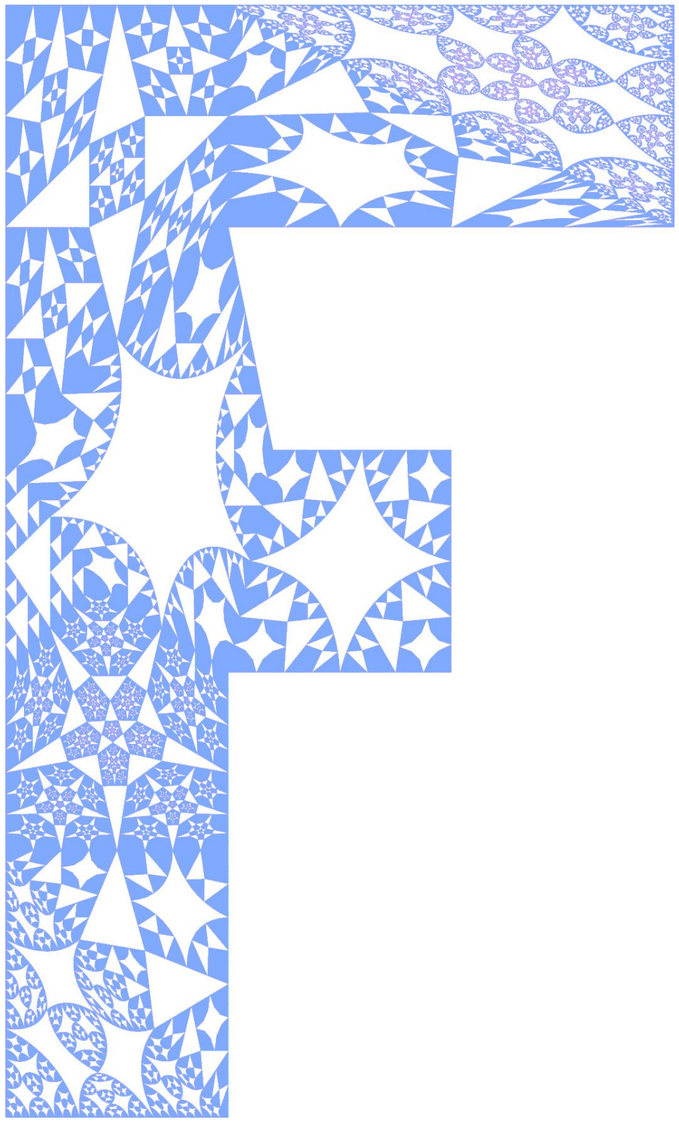


Assemblage



Assemblage

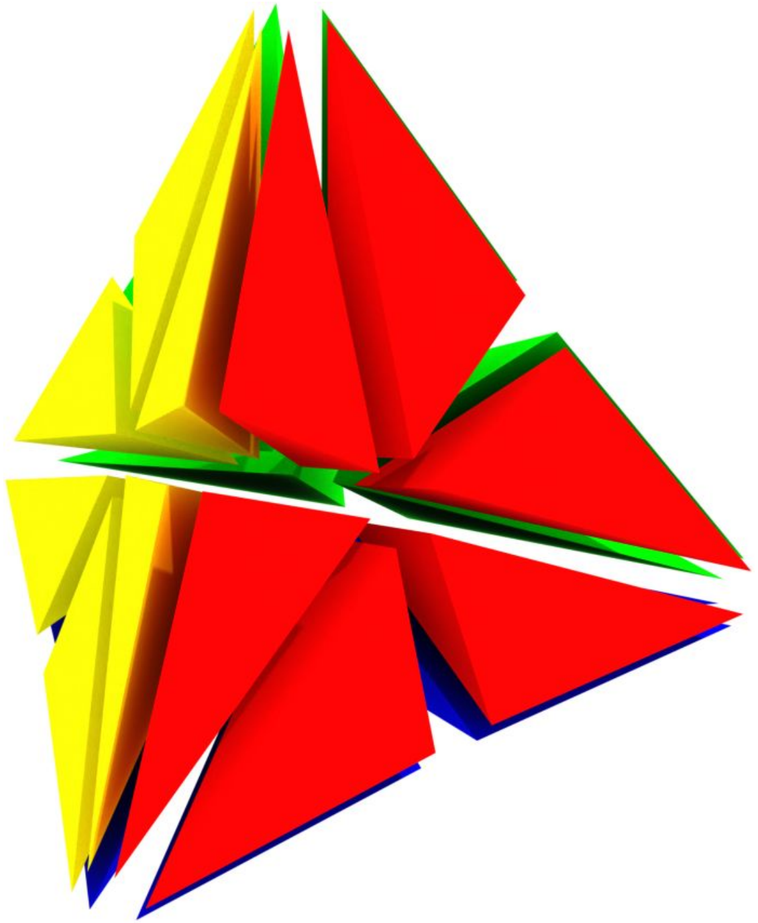




Conception automatique 3D à partir d’un polyèdre

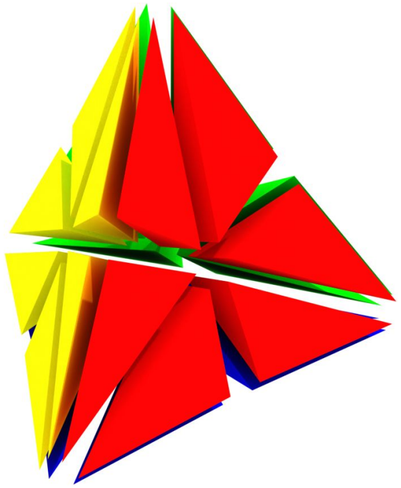
Subdivision d’un polyèdre

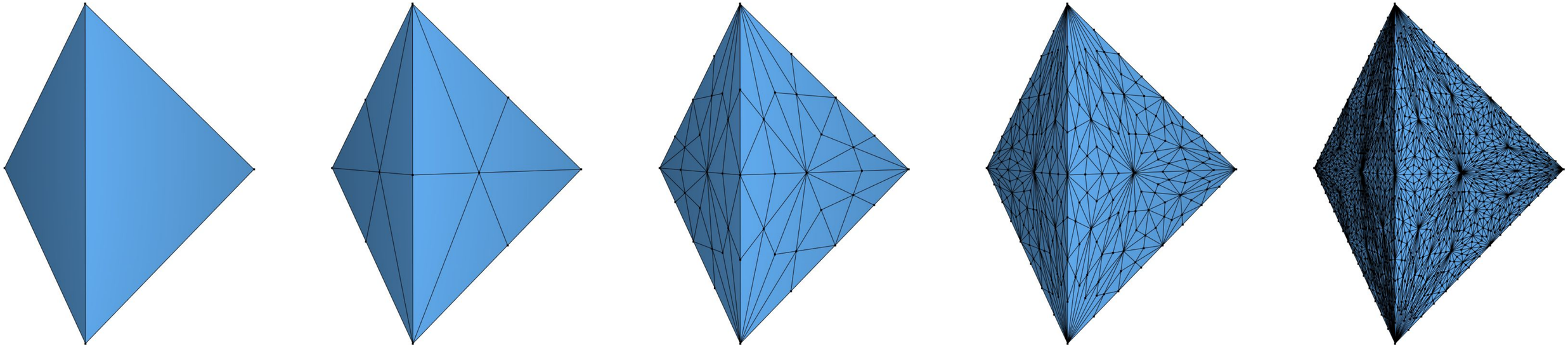
- Subdivision volumique du polyèdre
  - Subdivision barycentrique



Subdivision d’un polyèdre

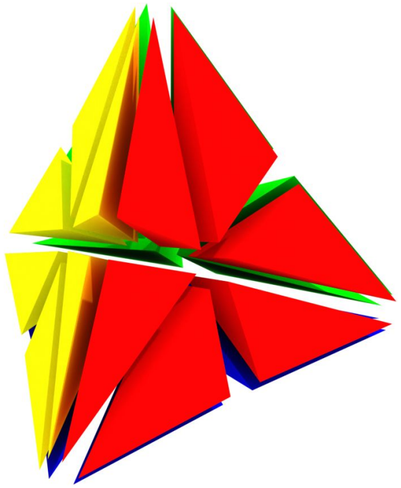
- Subdivision volumique du polyèdre
  - Subdivision barycentrique
  - Codage de la subdivision avec BC-IFS

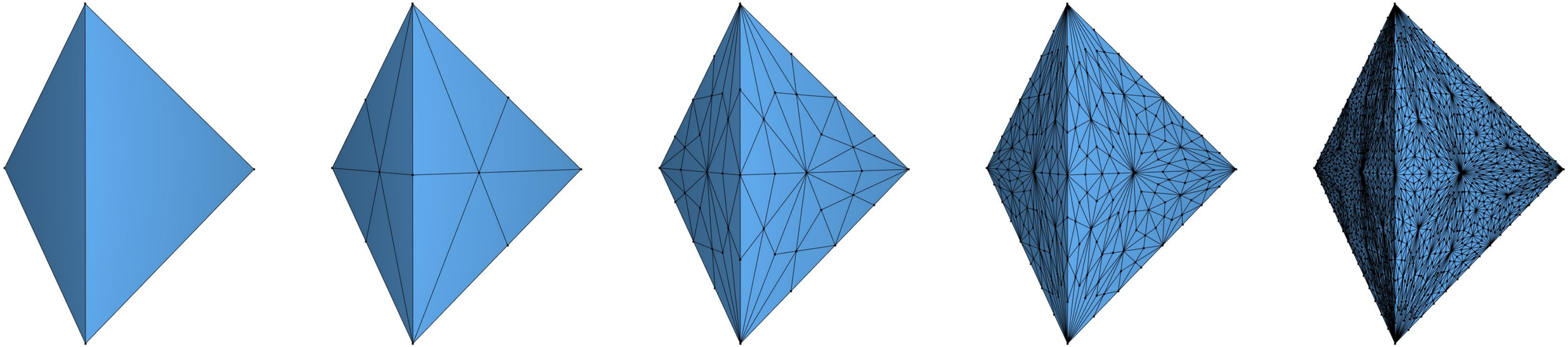




Subdivision d’un polyèdre

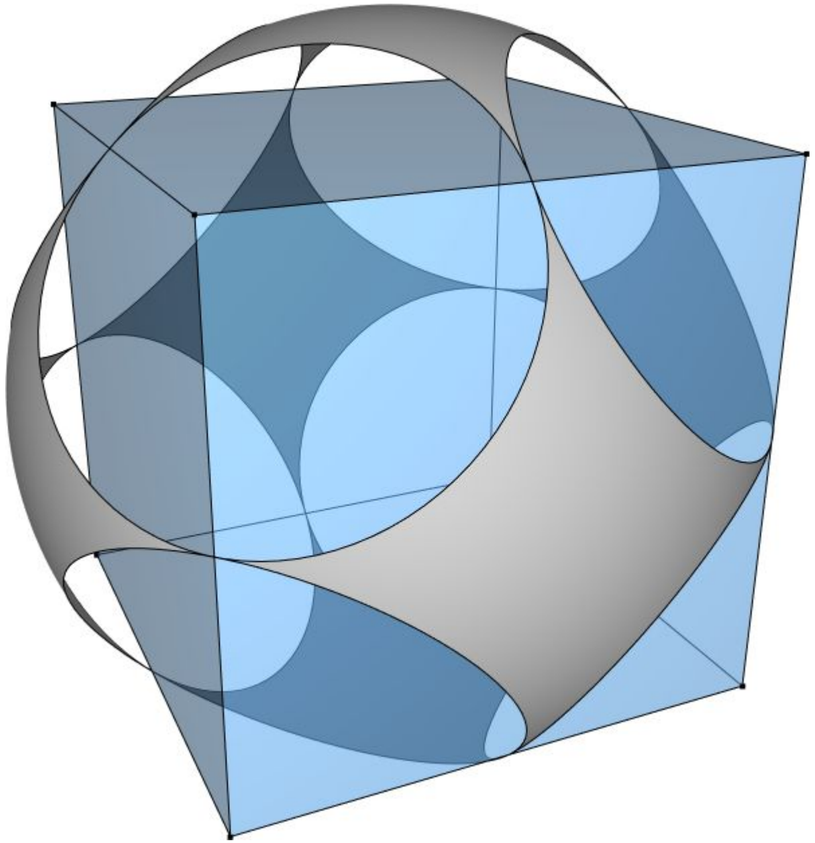
- Subdivision volumique du polyèdre
  - Subdivision barycentrique
  - Codage de la subdivision avec BC-IFS
  - Polyèdre fractal mais pas poreux





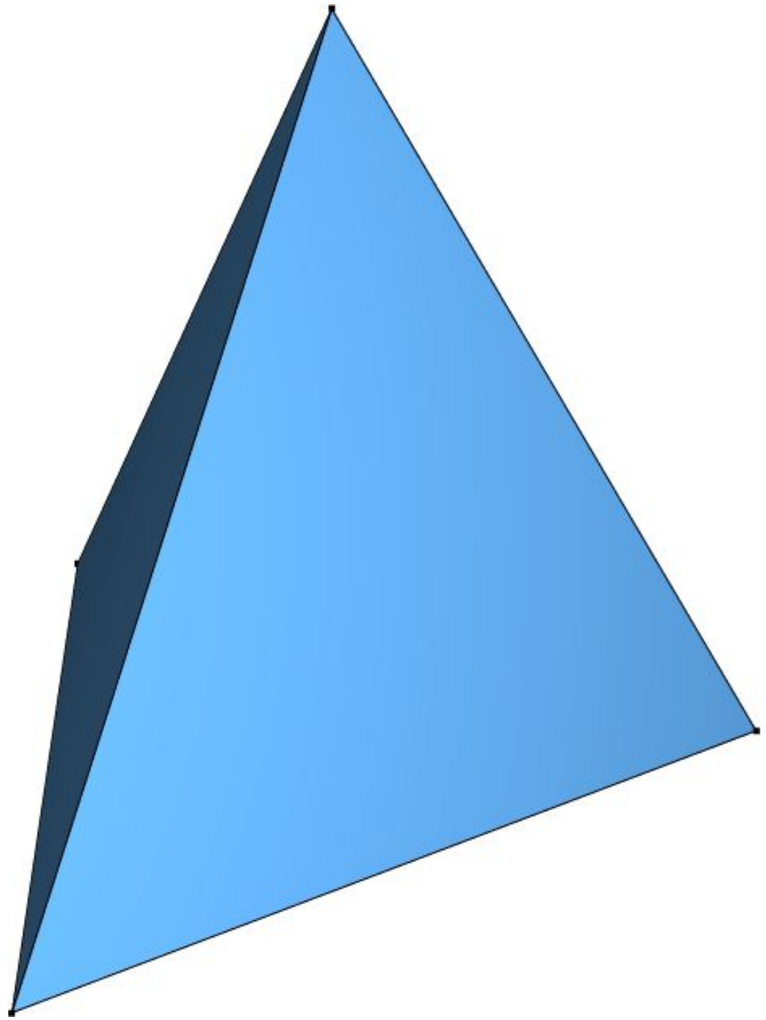
Subdivision d’un polyèdre

- Idée des empilements polyédriques de cercles
  - Sommet = lacune



Subdivision d’un polyèdre

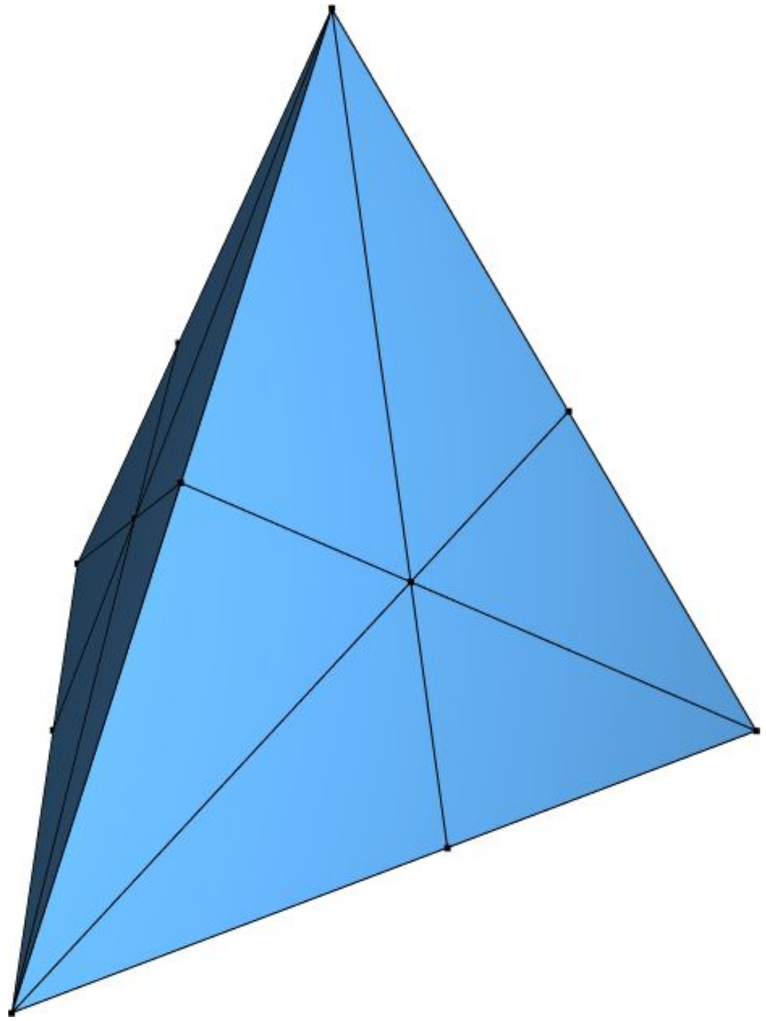
- Idée des empilements
  - Sommet = lacune

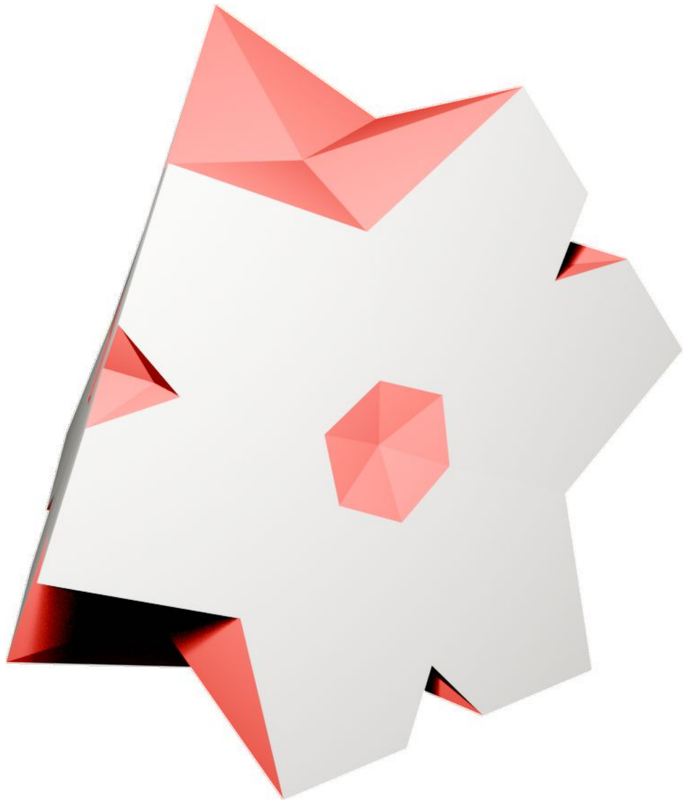




Subdivision d’un polyèdre

- Idée des empilements
  - Sommet = lacune

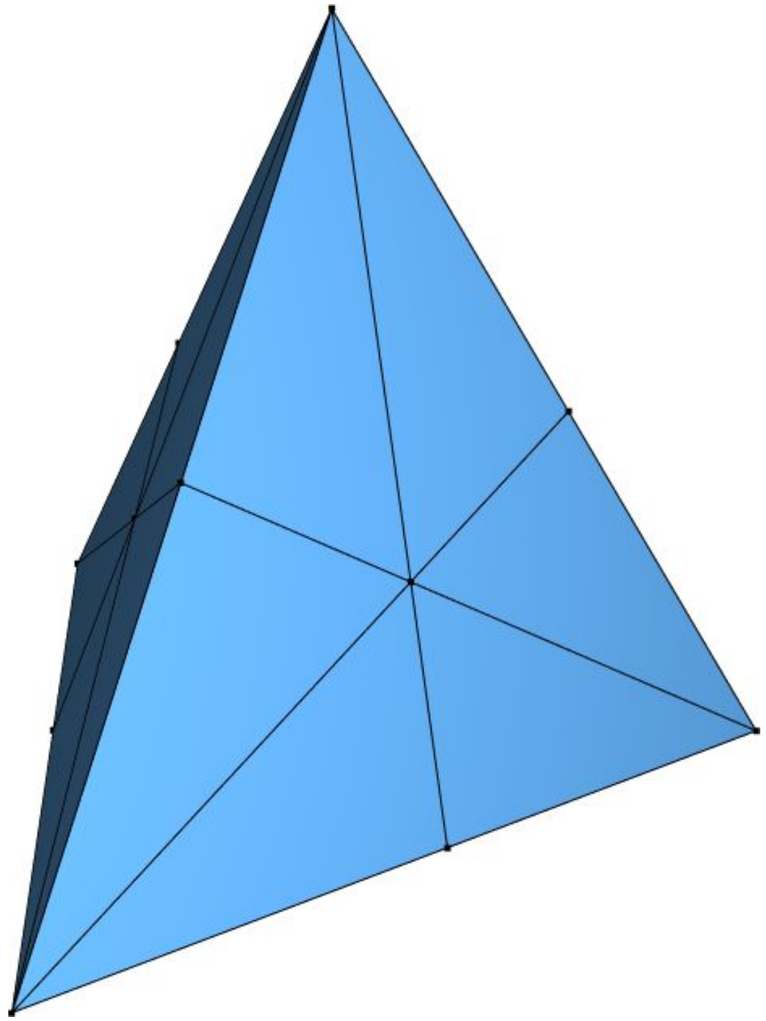


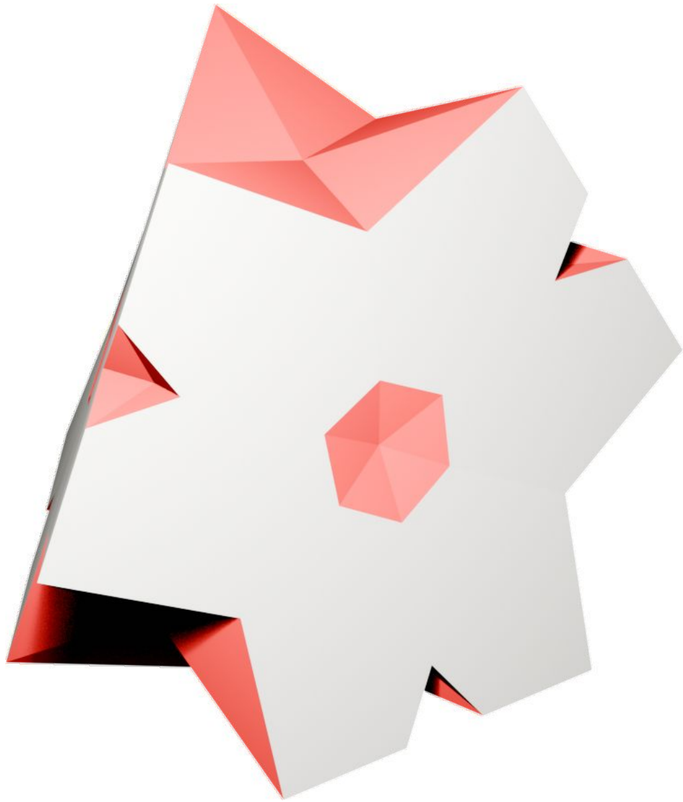


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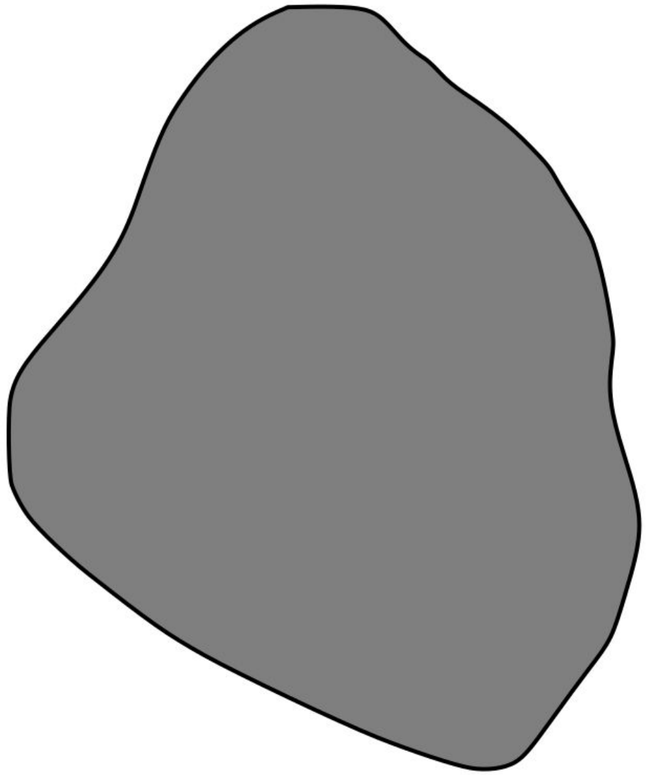
Subdivision d’un polyèdre

- Idée des empilements
  - Sommet = lacune
  - Relâchement de contraintes

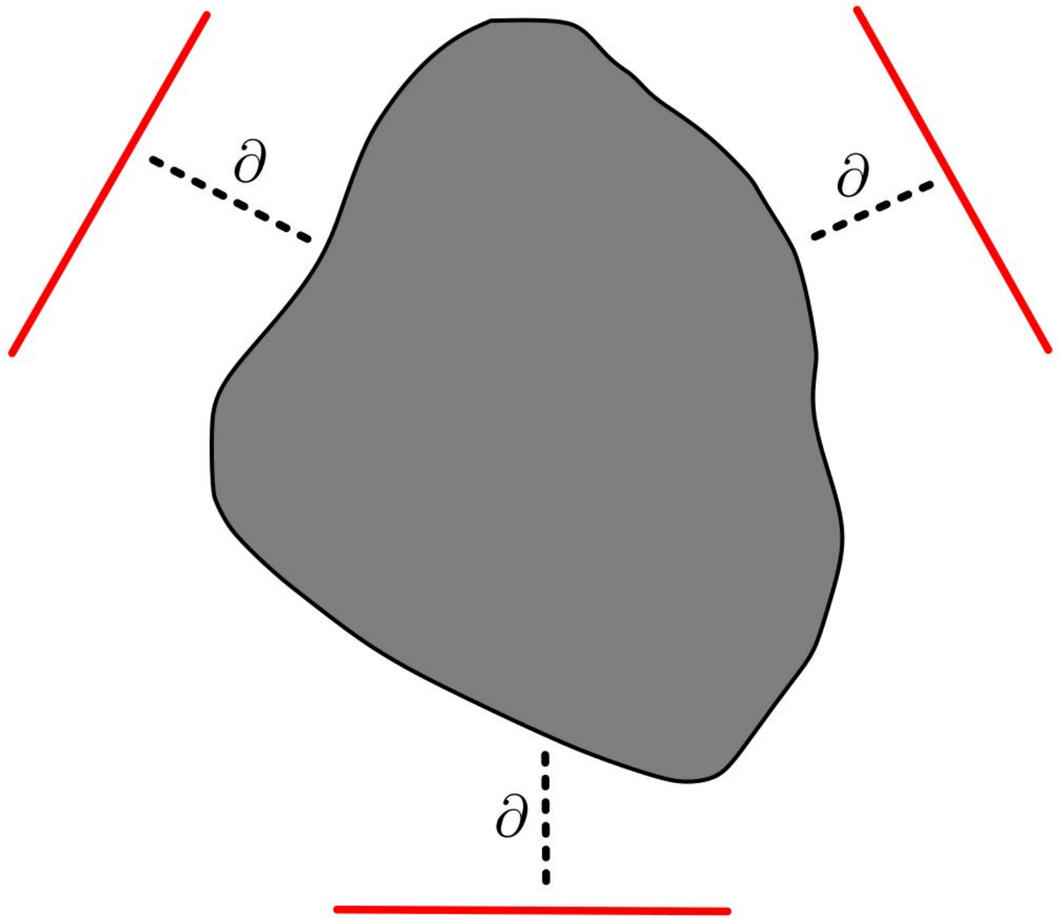




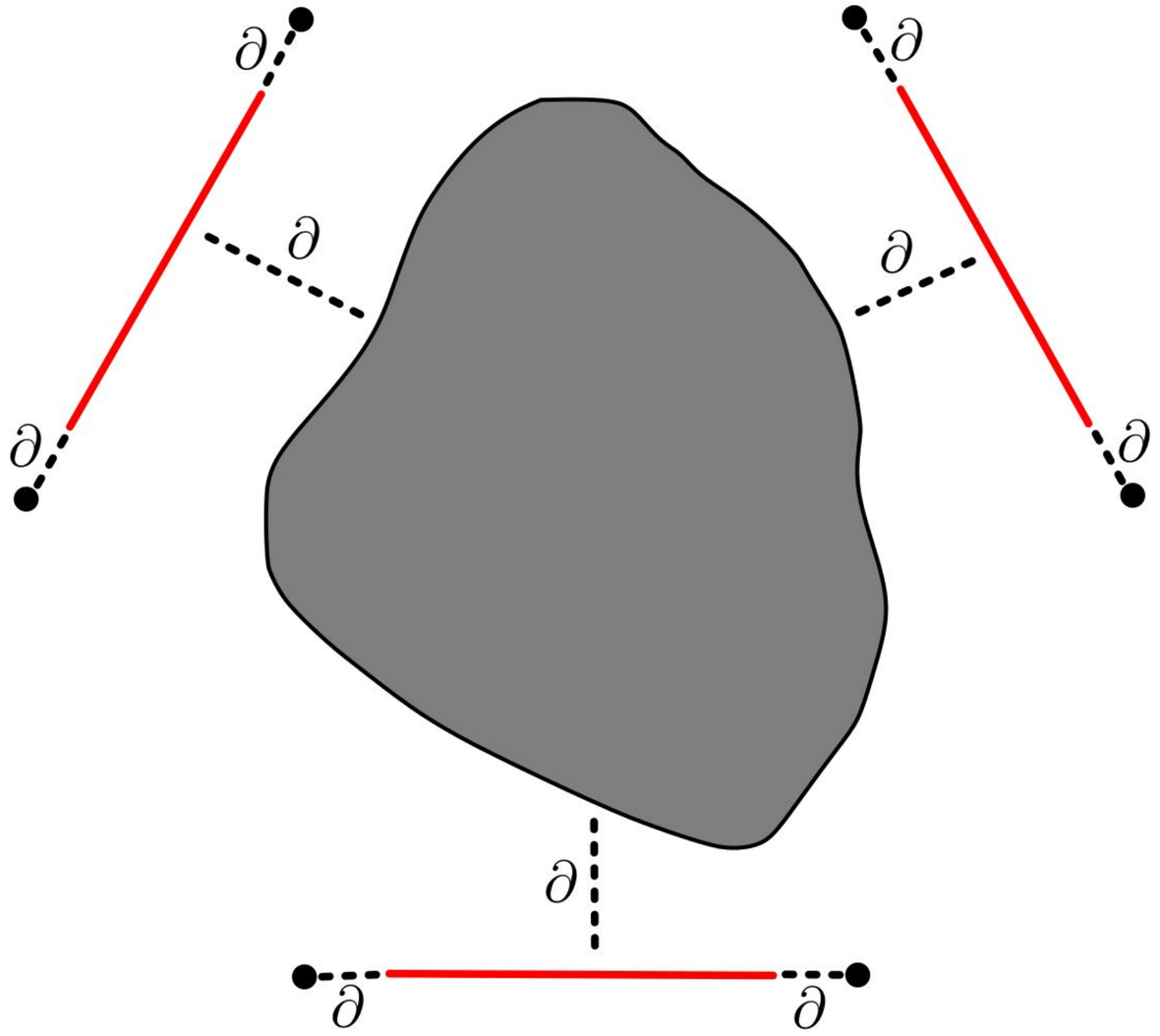
Modélisation avec BC-IFS



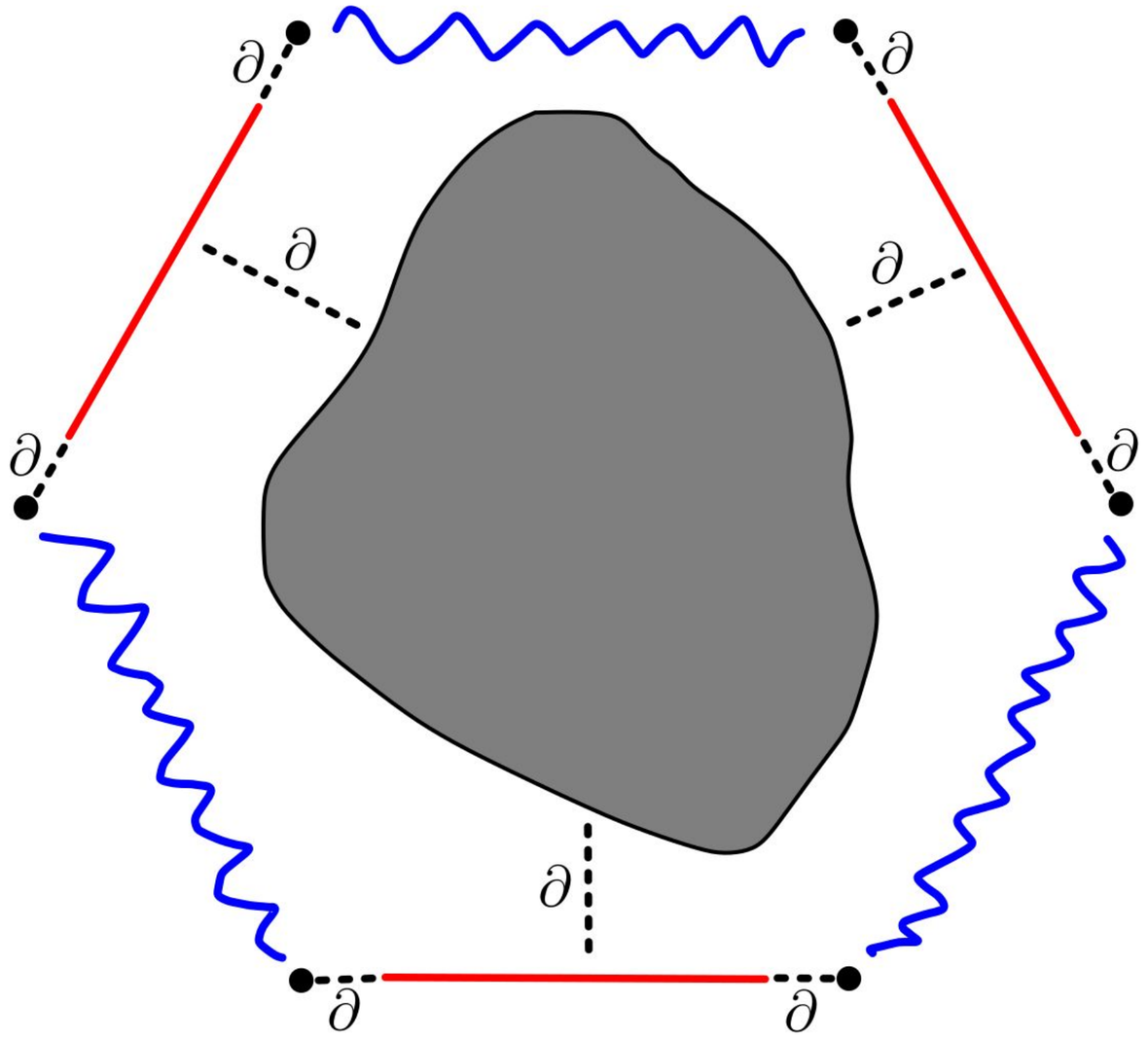
Modélisation avec BC-IFS



Modélisation avec BC-IFS

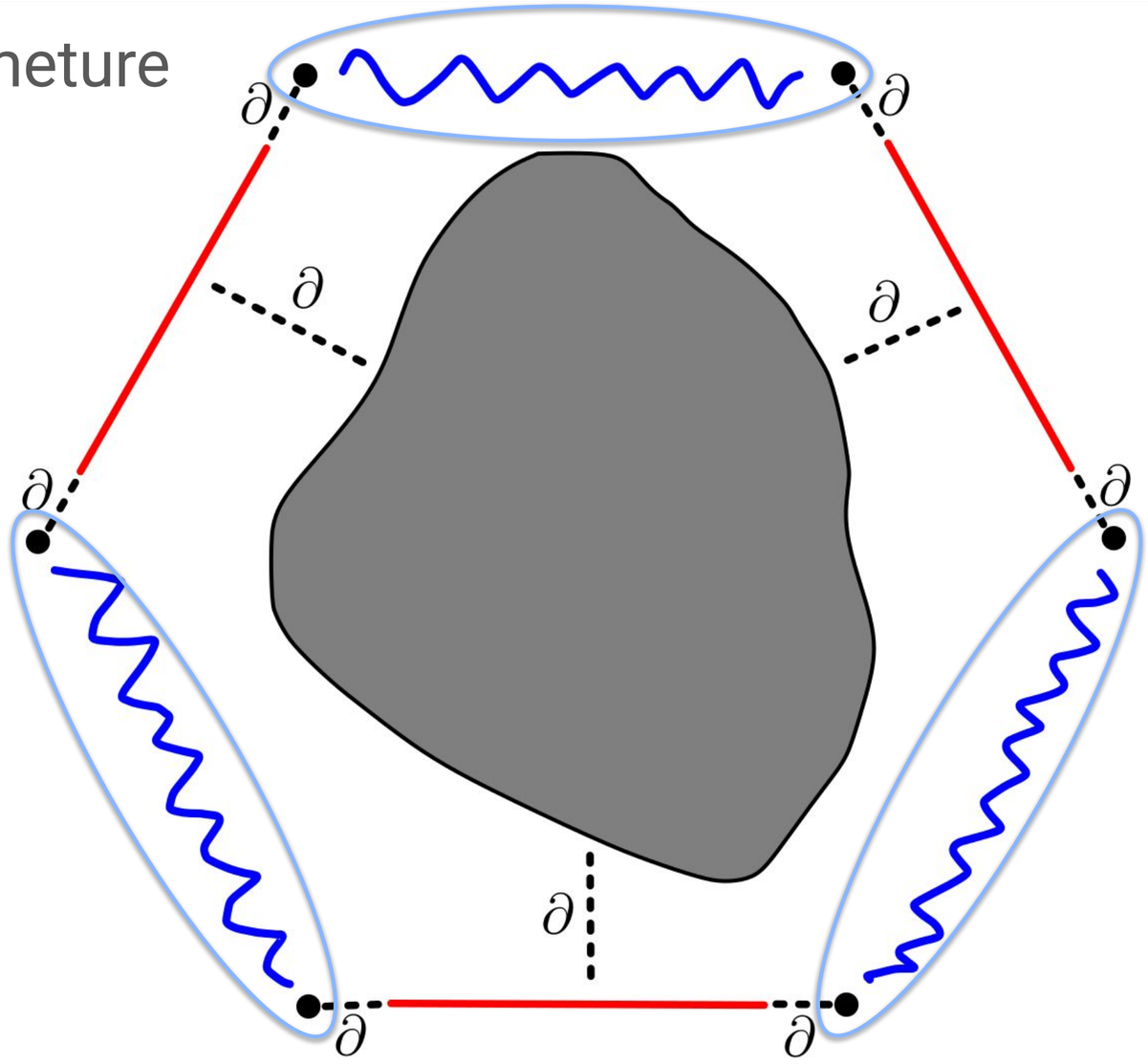


Modélisation avec BC-IFS



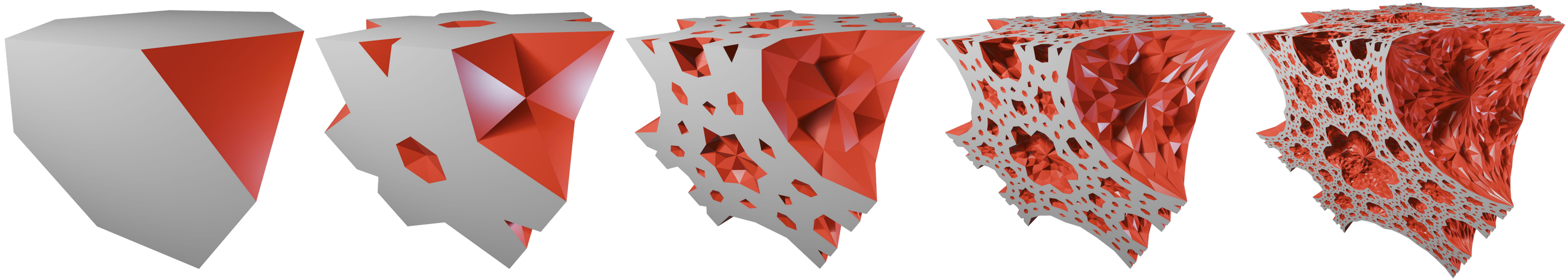
Modélisation avec BC-IFS

- Contraintes de fermeture



Subdivision d’un polyèdre

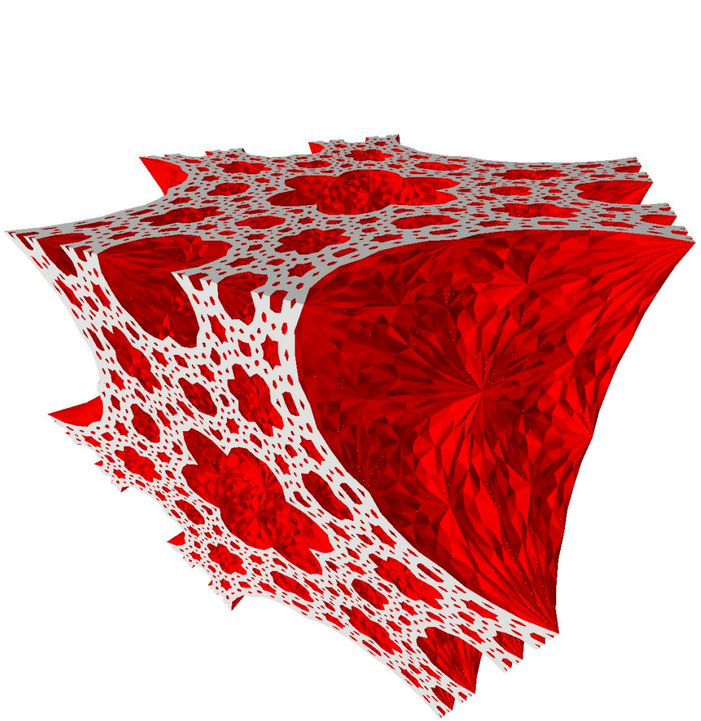
- Idée des empilements
  - Sommet = lacune
  - Relâchement de contraintes

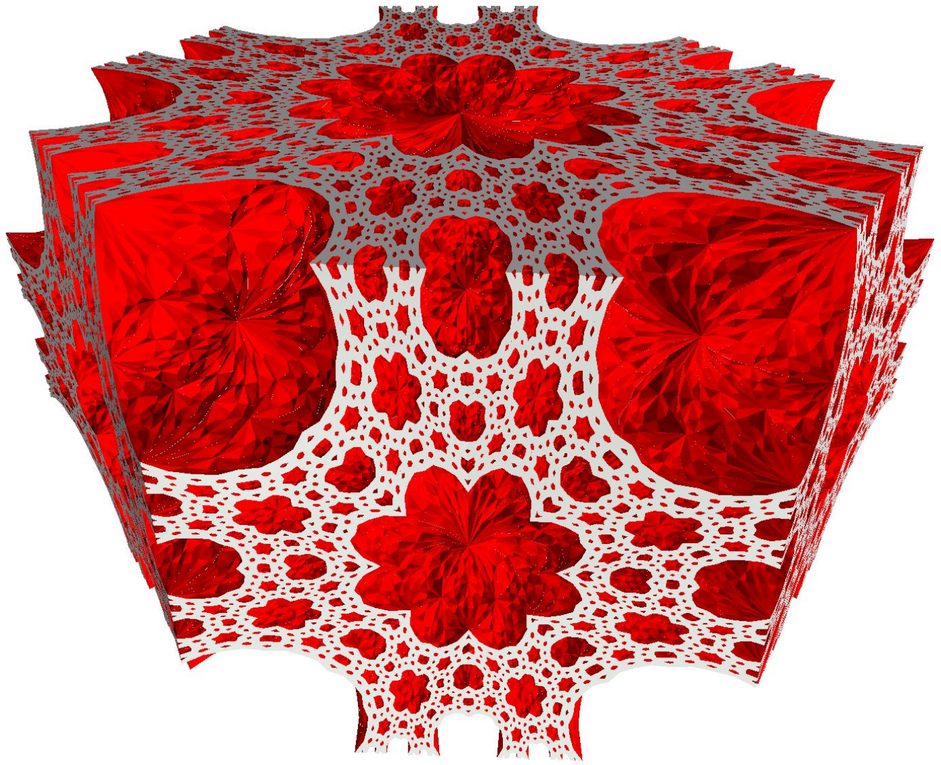


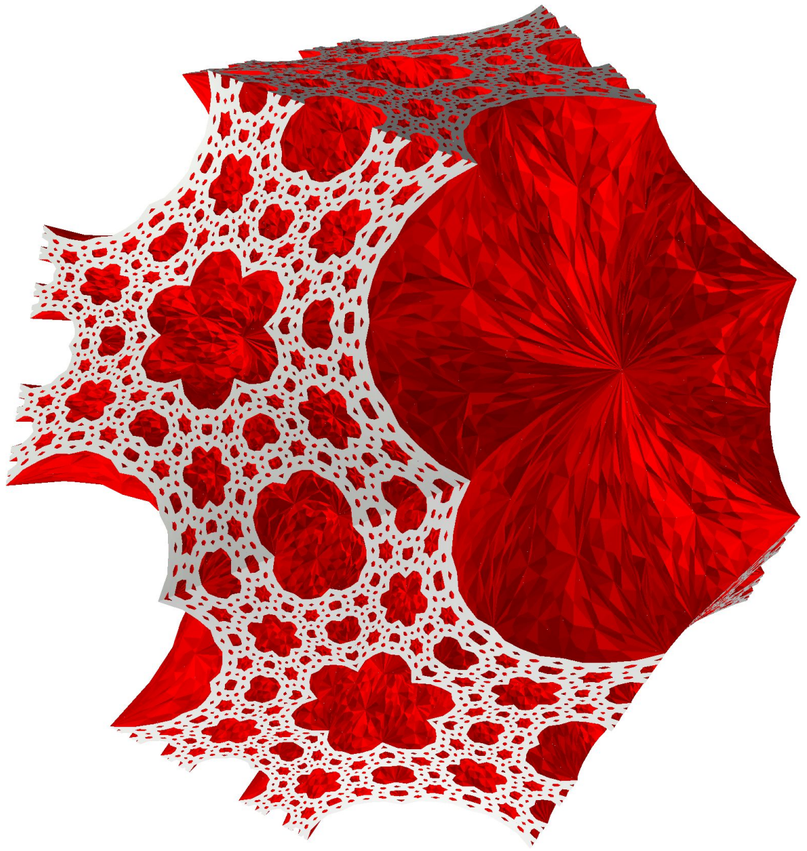
Contrôle de la lacunarité

Contrôle de la lacunarité

- Changer de polyèdre

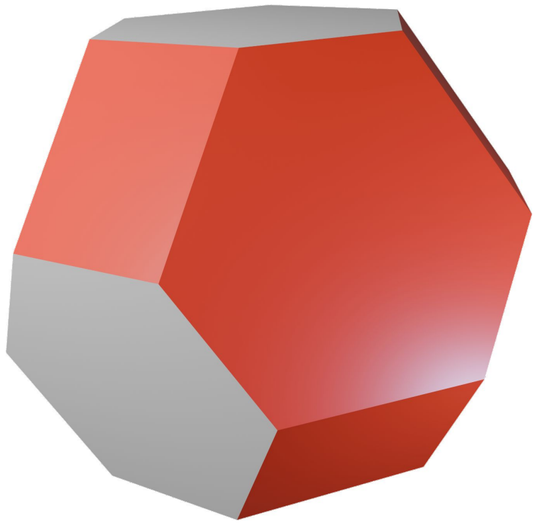


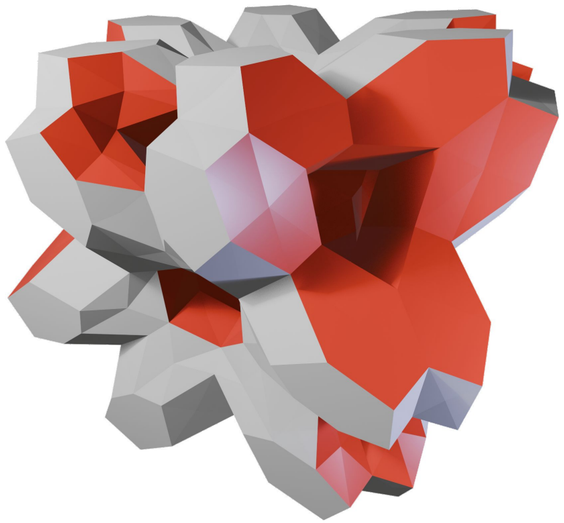


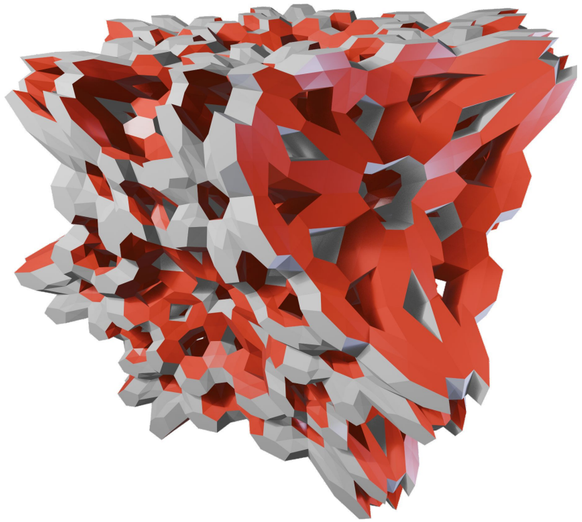


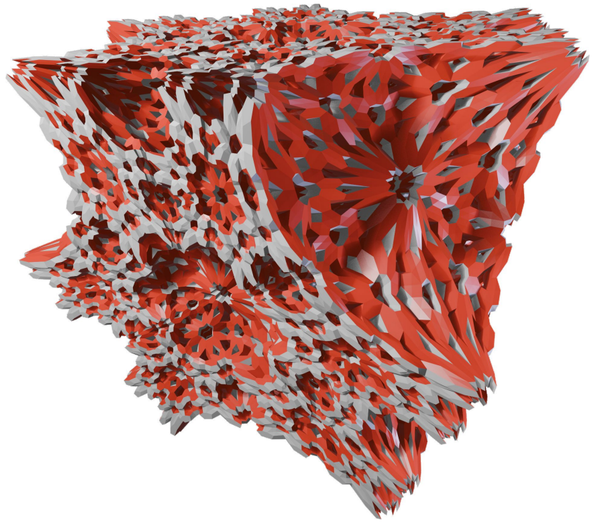
Contrôle de la lacunarité

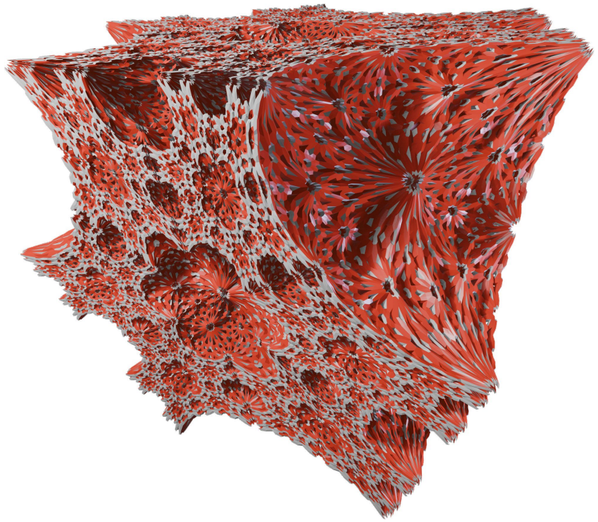
- Changer de polyèdre
- Changer de primitive





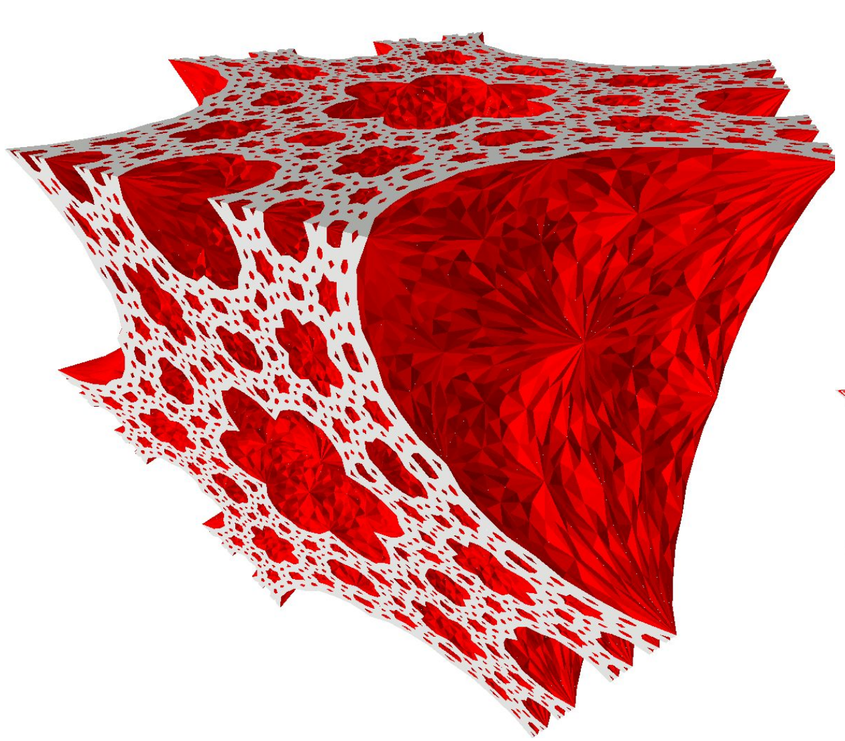


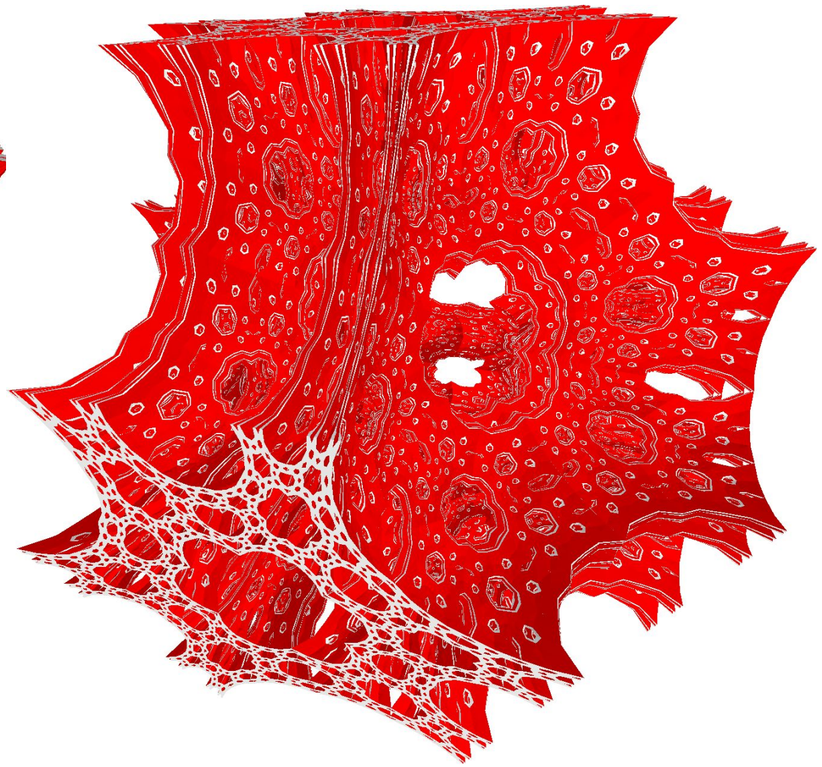




Contrôle de la lacunarité

- Changer de polyèdre
- Changer de primitive
- Relâcher des contraintes
  - Lacunes d’arêtes

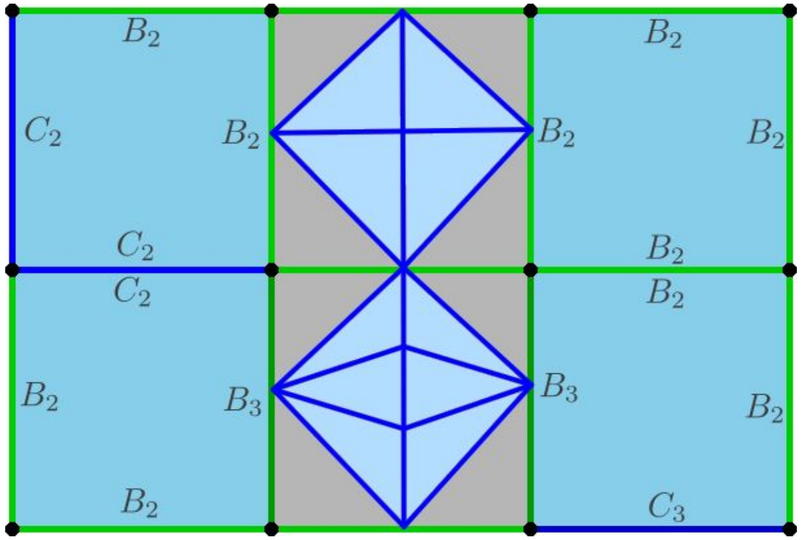


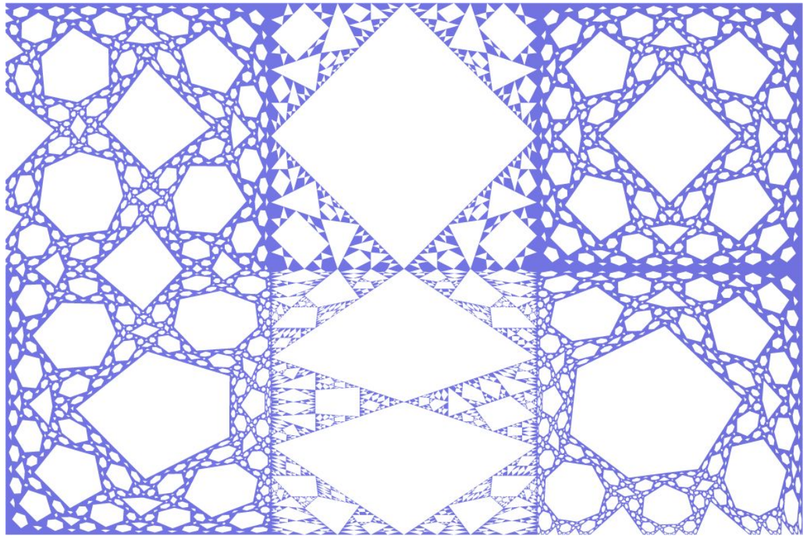


Conclusion

Contributions

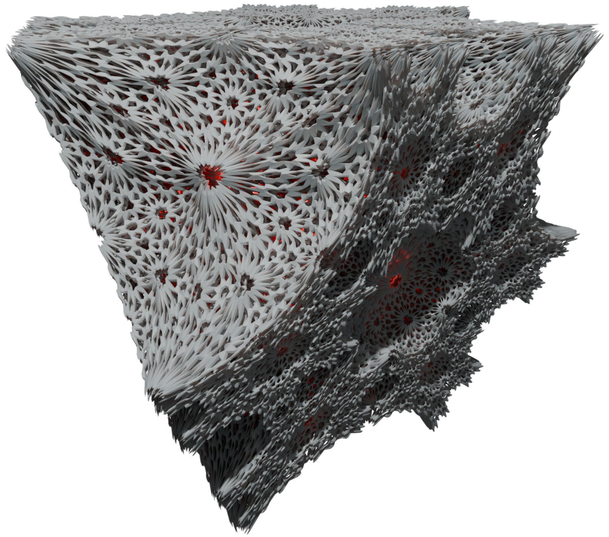
- 3 approches de conception automatique de structures lacunaires fractales
  - En 2D
    - À partir des bords
    - À partir d'un polyèdre

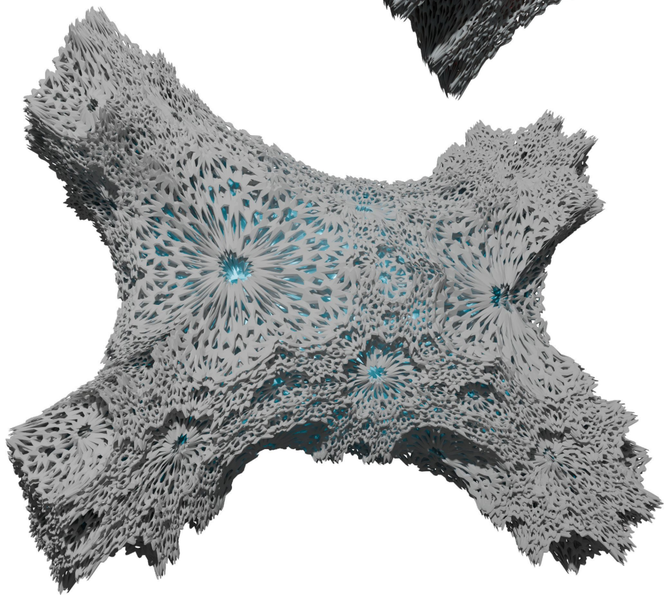


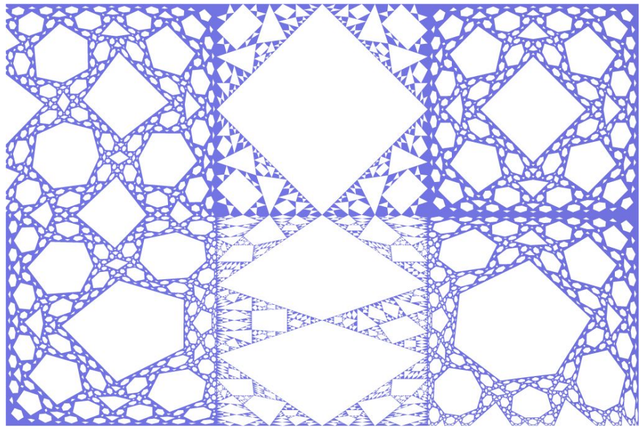


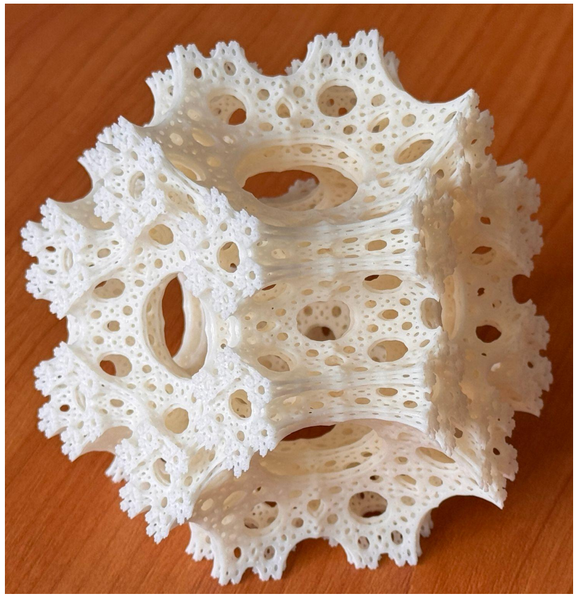
Contributions

- 3 approches de conception automatique de structures lacunaires fractales
  - En 2D
    - À partir des bords
    - À partir d’un polyèdre
  - En 3D
    - À partir d’un polyèdre



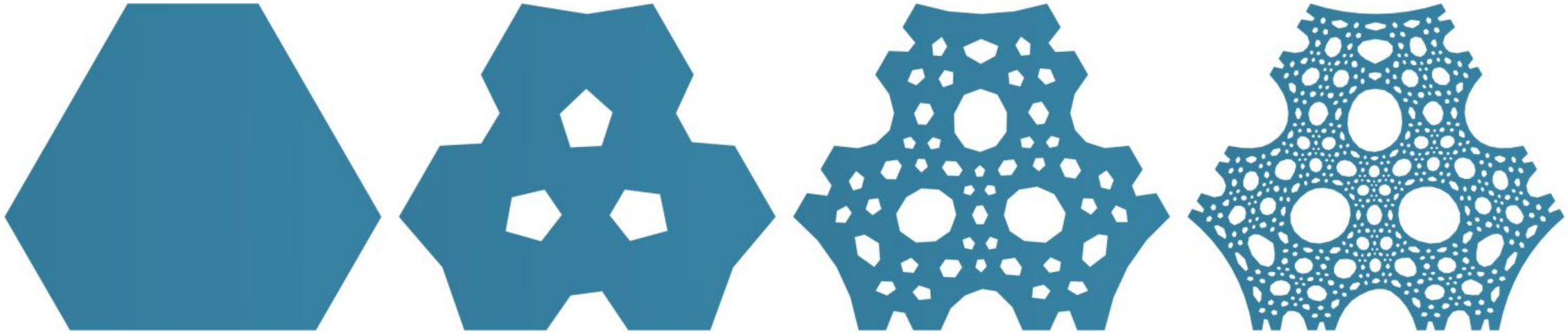






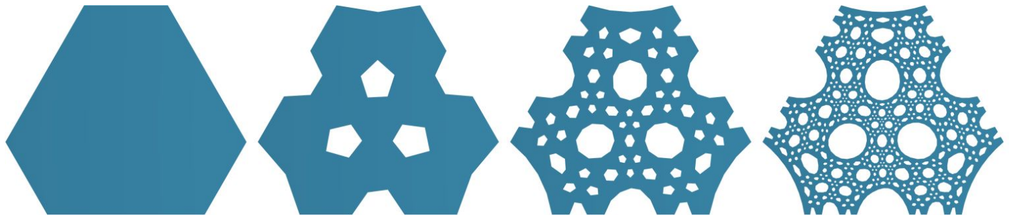
Travaux futurs

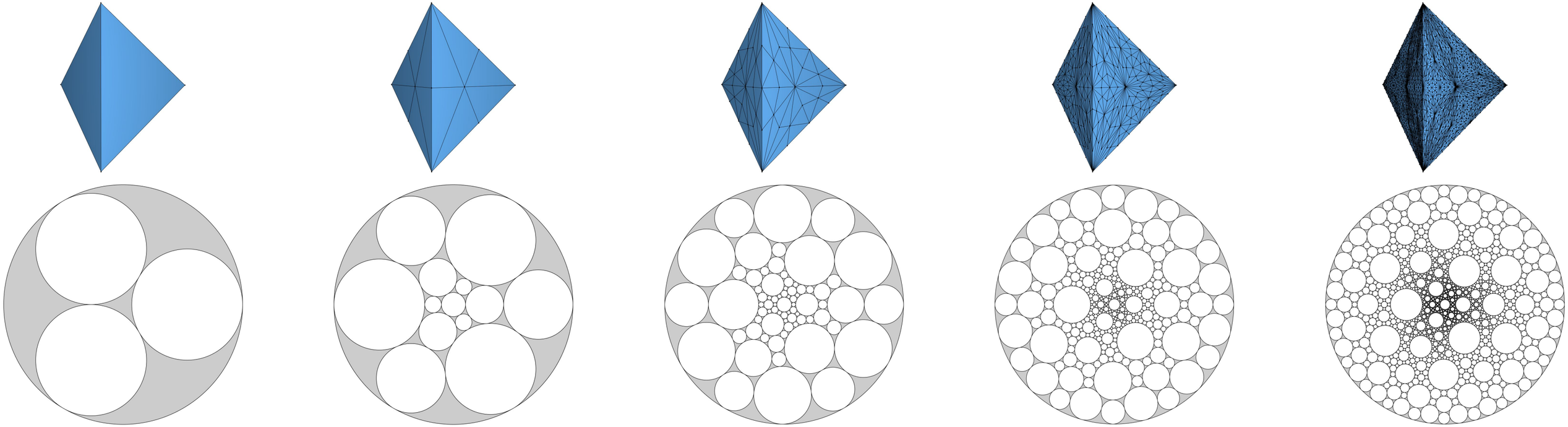
- Autres processus en 2D
  - Plusieurs lacunes centrales



Travaux futurs

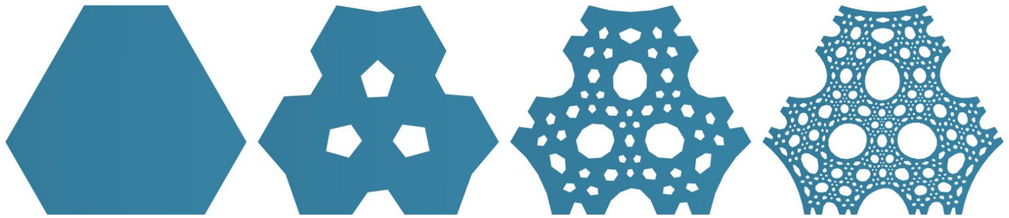
- Autres processus en 2D
  - Plusieurs lacunes centrales
  - Autres types d’empilements

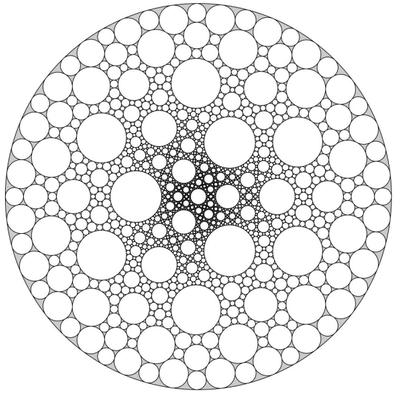


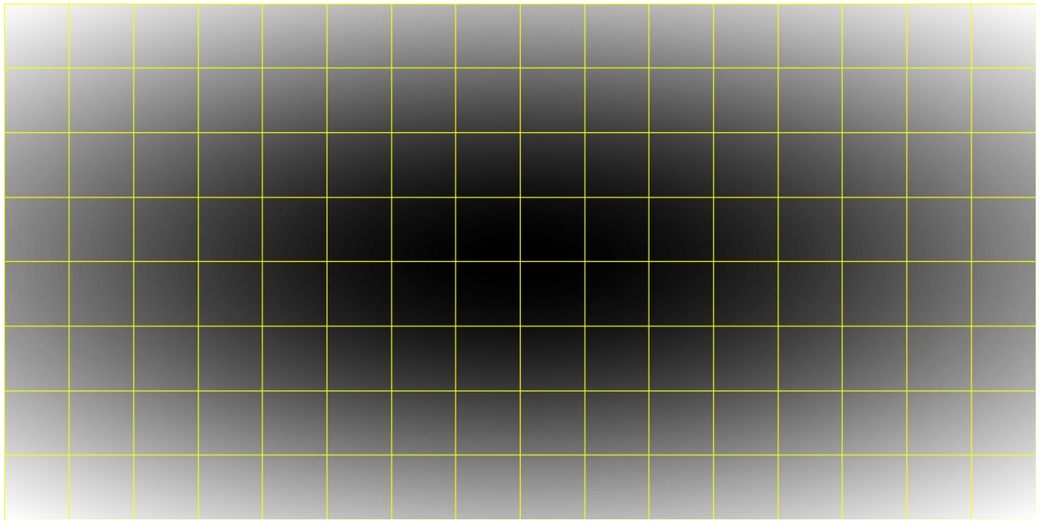


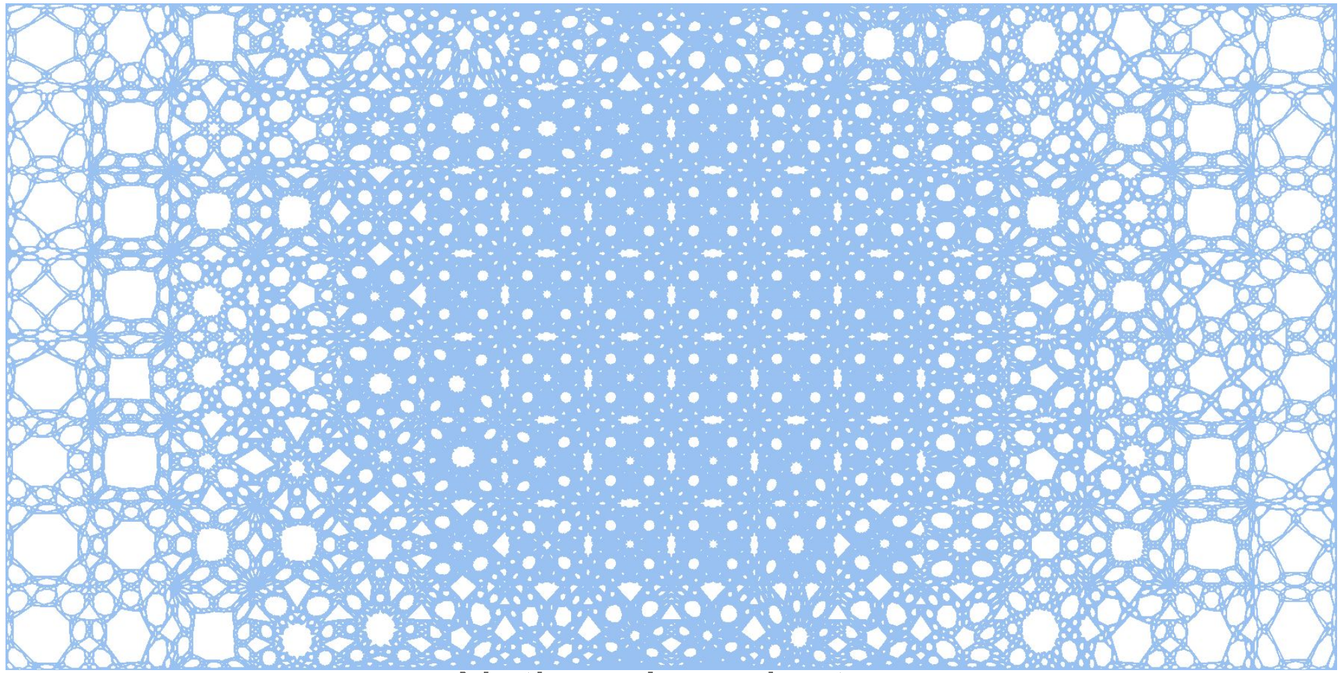
Travaux futurs

- Autres processus en 2D
  - Plusieurs lacunes centrales
  - Autres types d'empilements
- Optimisation topologique
  - Adaptation automatique de la topologie





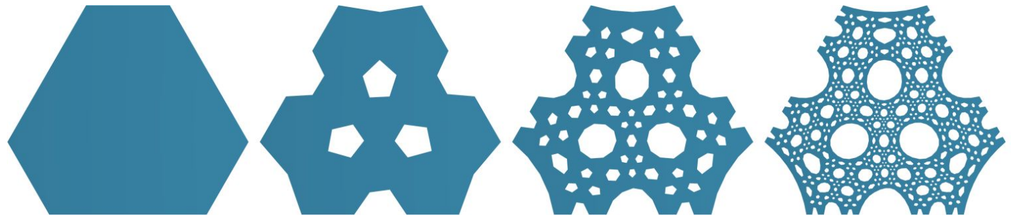


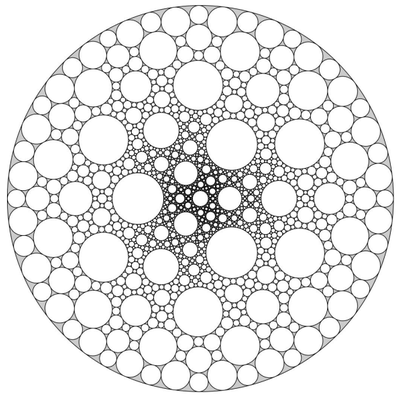


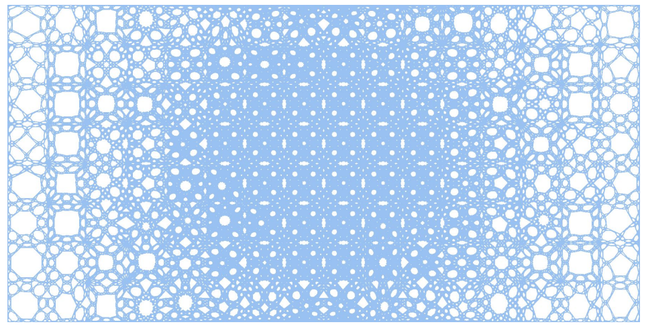
Nathan Jacquinet

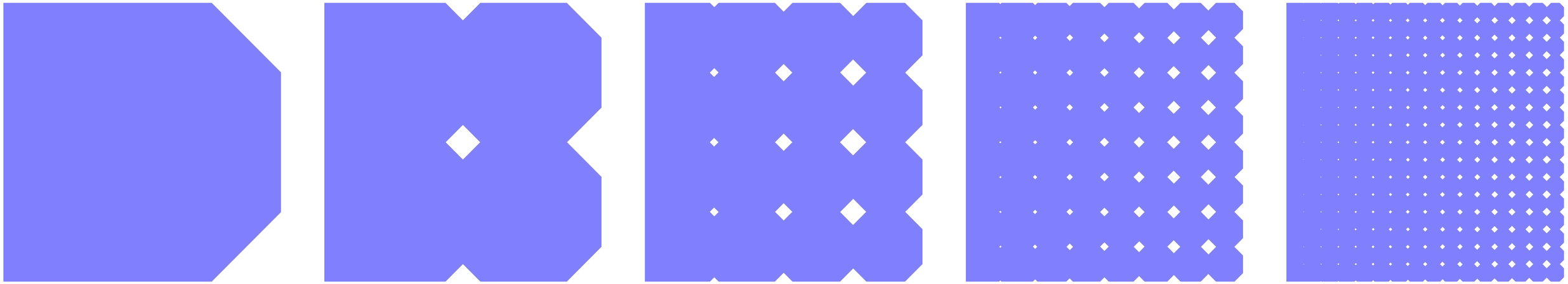
Travaux futurs

- Autres processus en 2D
  - Plusieurs lacunes centrales
  - Autres types d'empilements
- Optimisation topologique
  - Adaptation automatique de la topologie
- Variation de la primitive



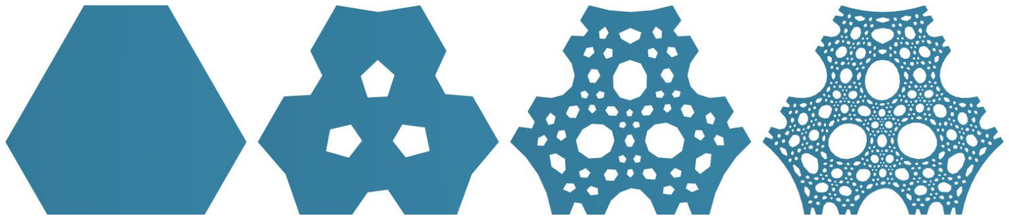


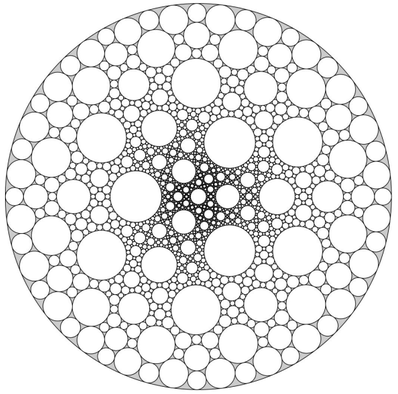


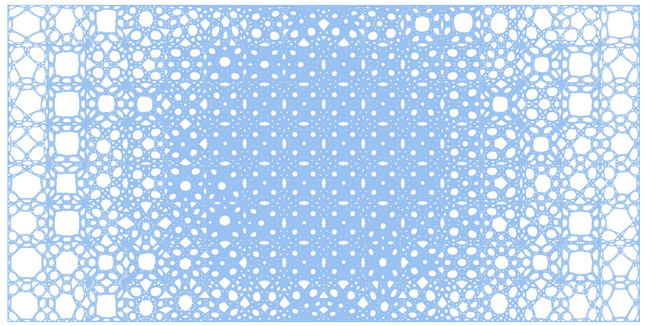


Travaux futurs

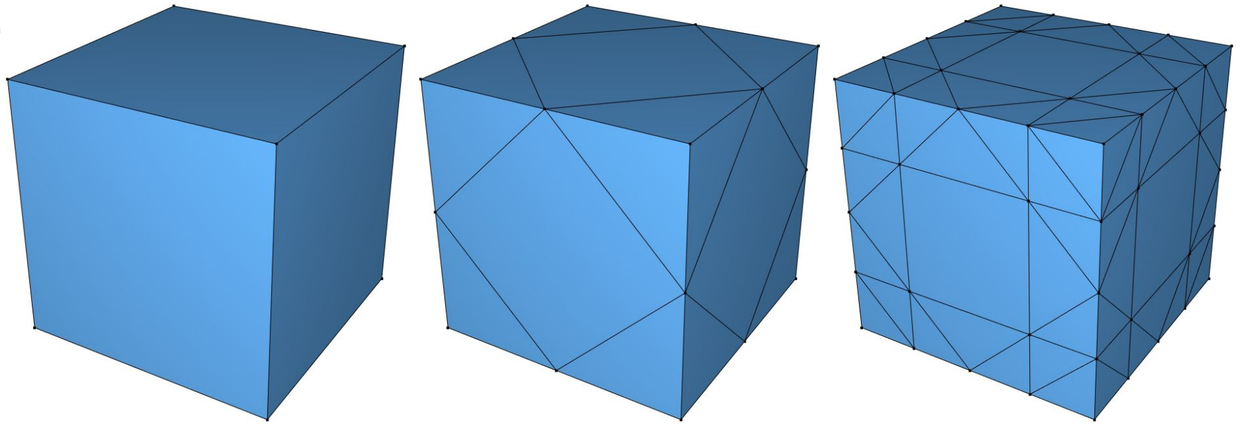
- Autres processus en 2D
  - Plusieurs lacunes centrales
  - Autres types d'empilements
- Optimisation topologique
  - Adaptation automatique de la topologie
- Variation de la primitive
- Études d'autres subdivisions de polyèdre
  - Encoder les règles dans le modèle BC-IFS
  - Formalisation des règles de subdivision





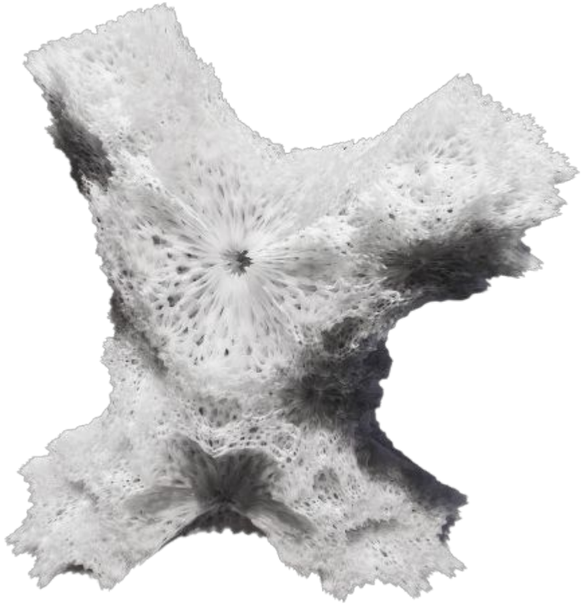


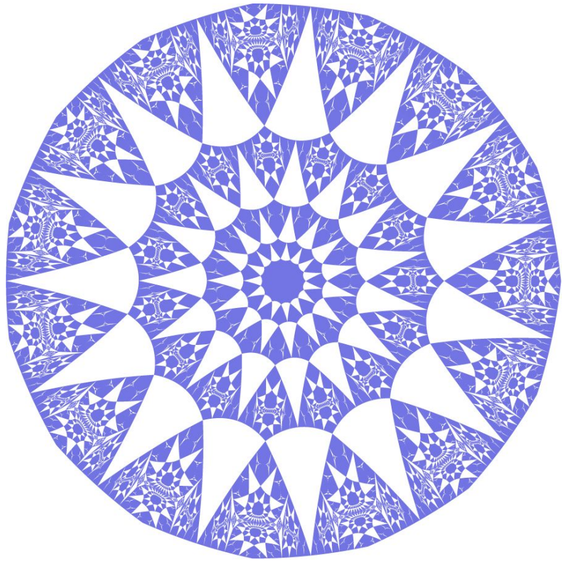




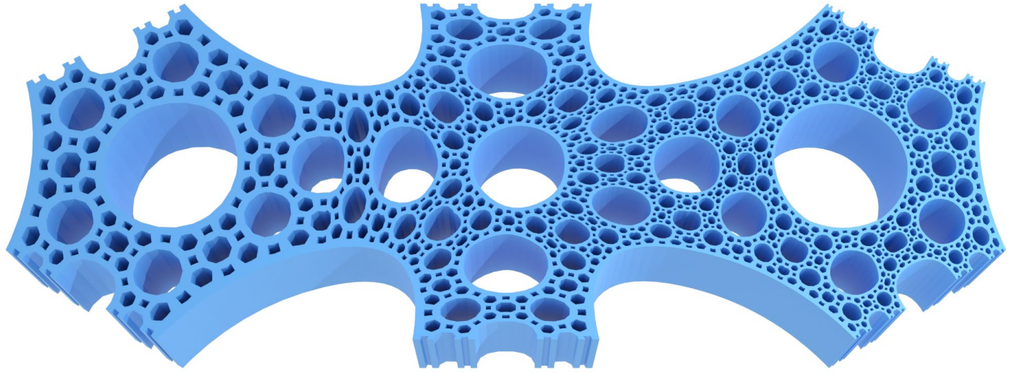
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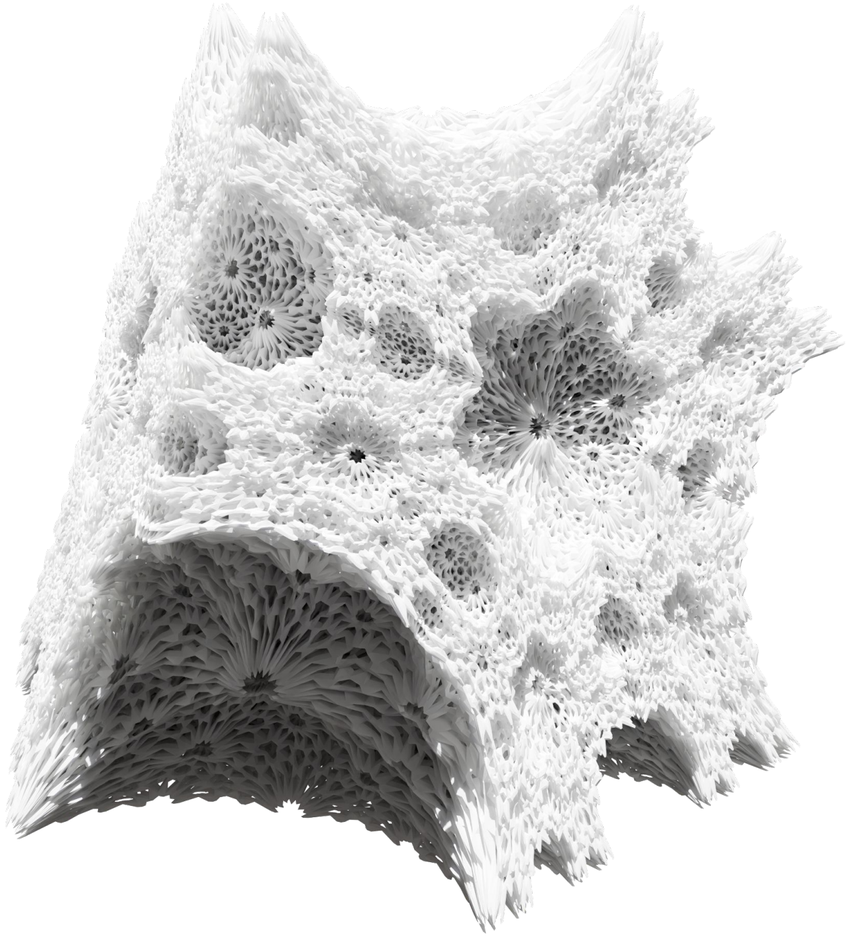
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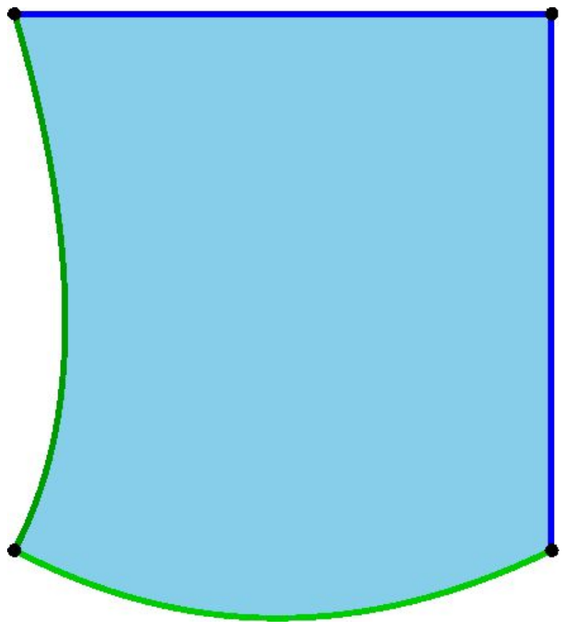


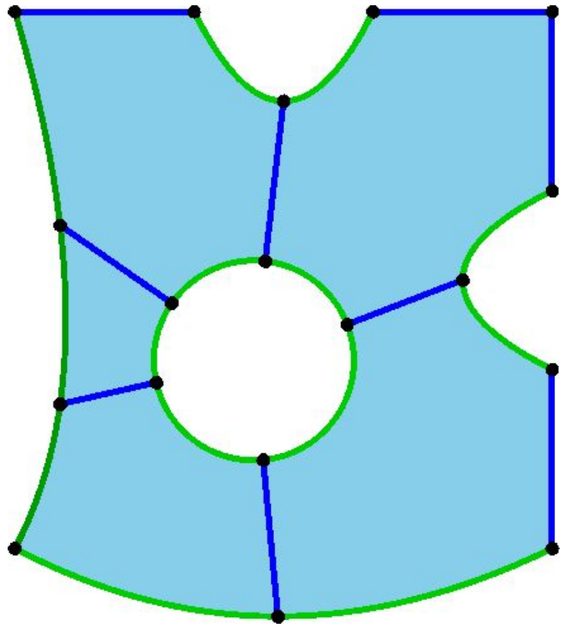
Merci pour votre attention !





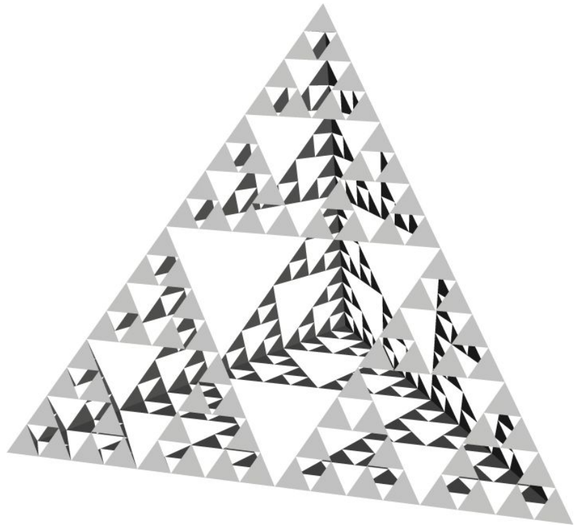
Bonus : Convergence du processus de subdivision

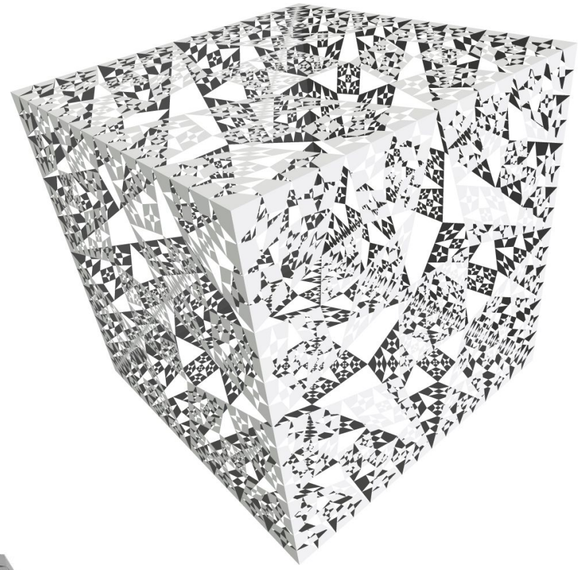


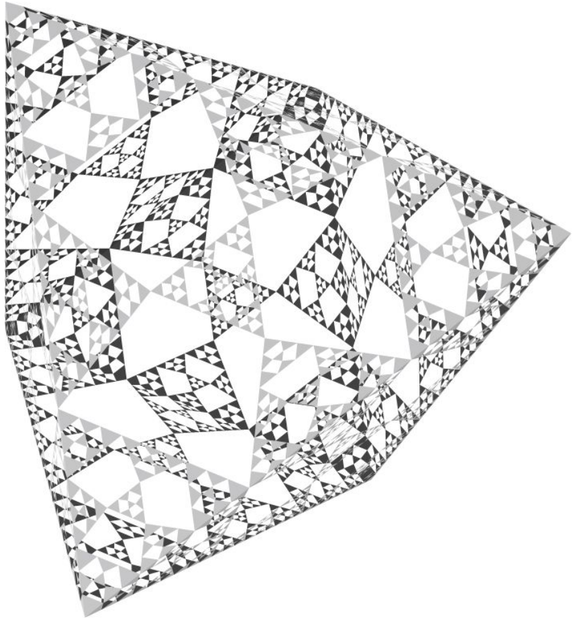


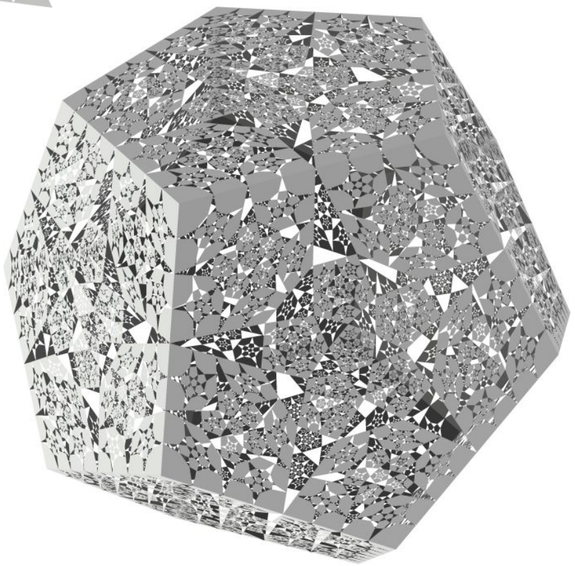
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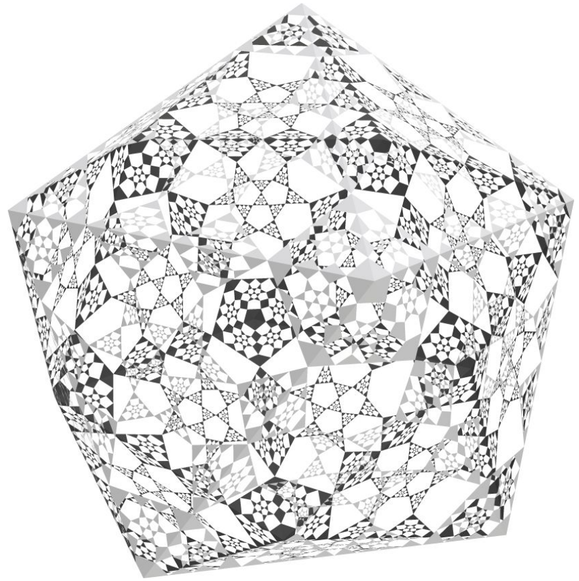
Bonus : Polyèdres fractals



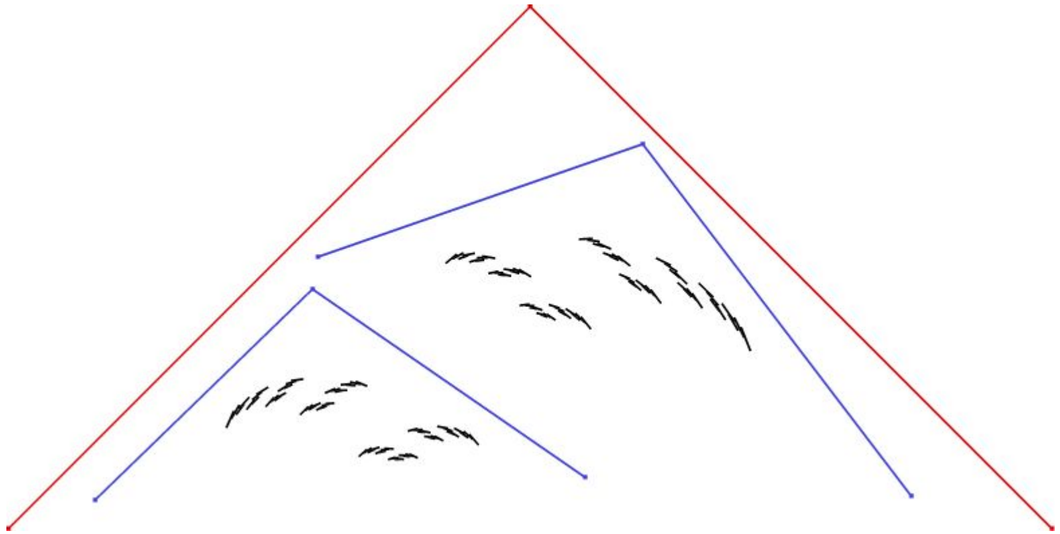


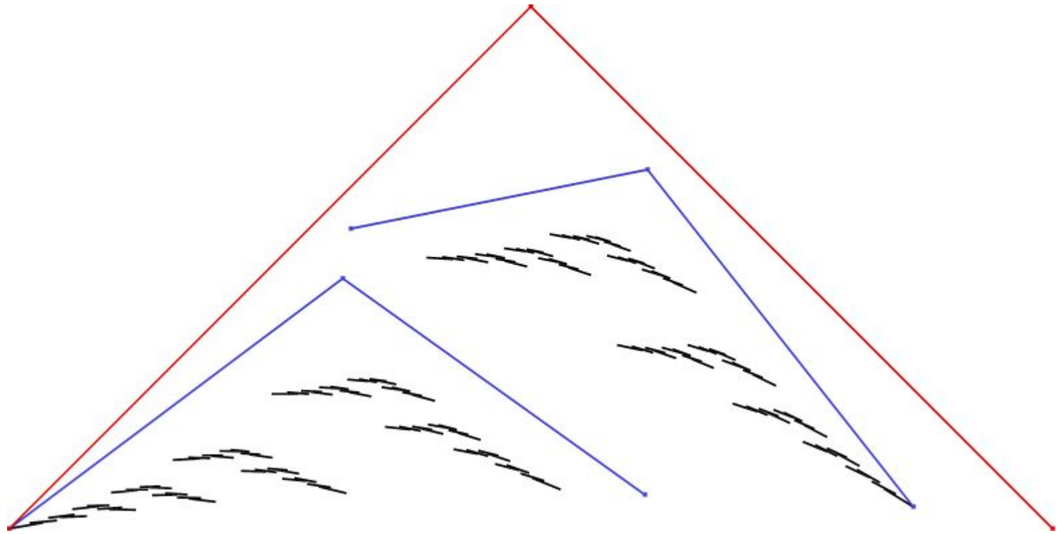


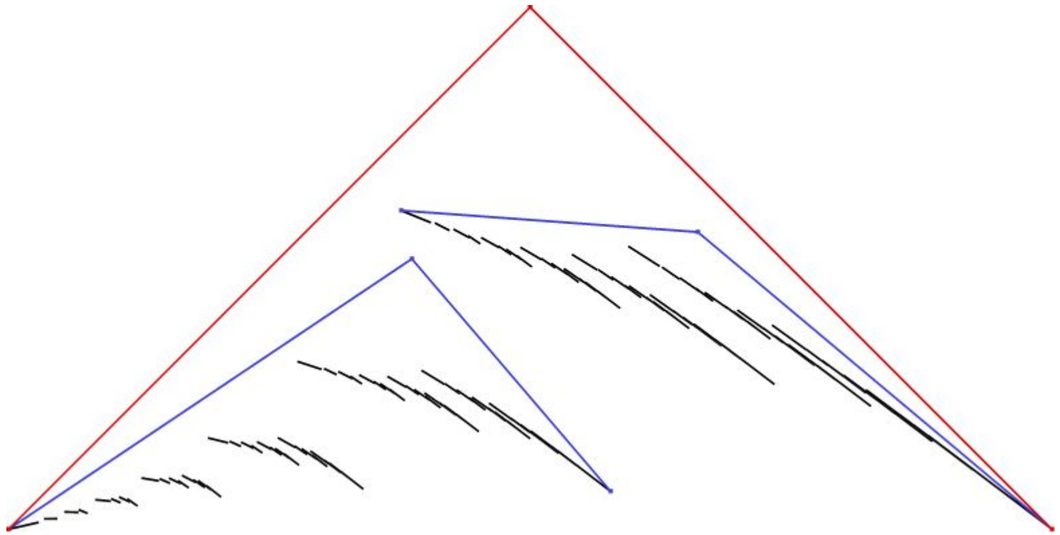


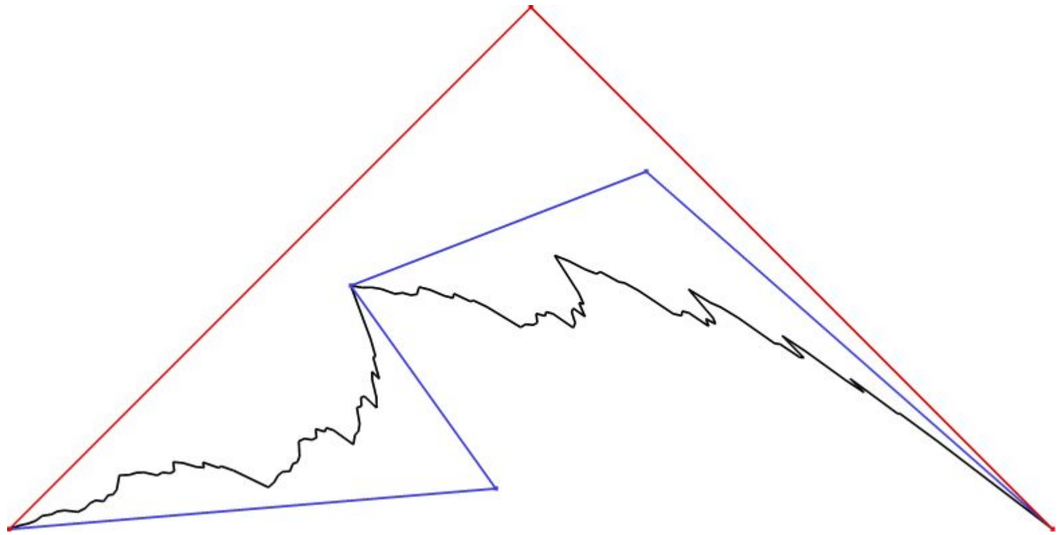


Bonus : Illustration des contraintes



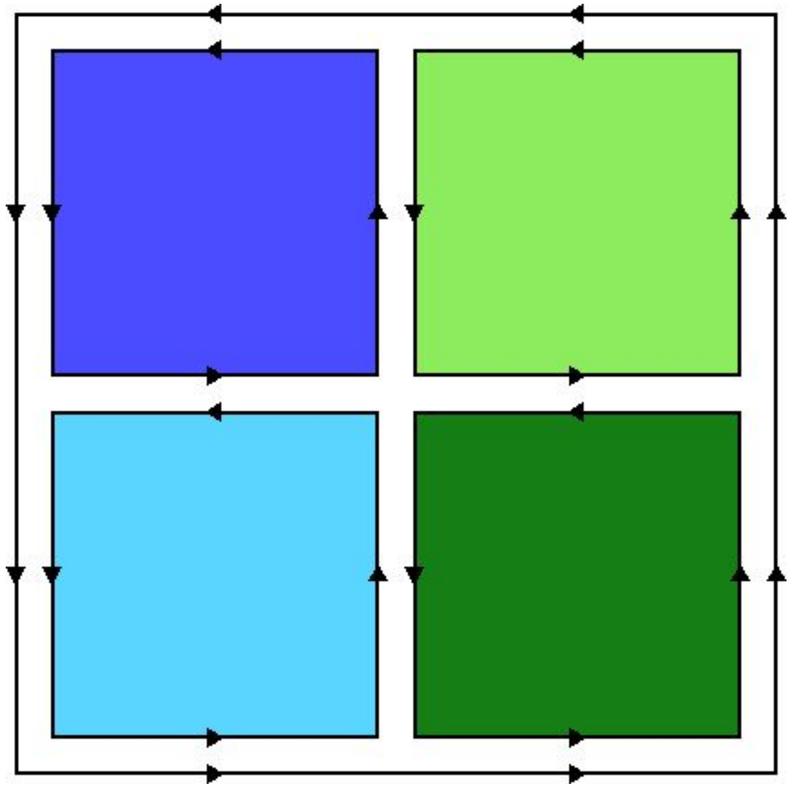


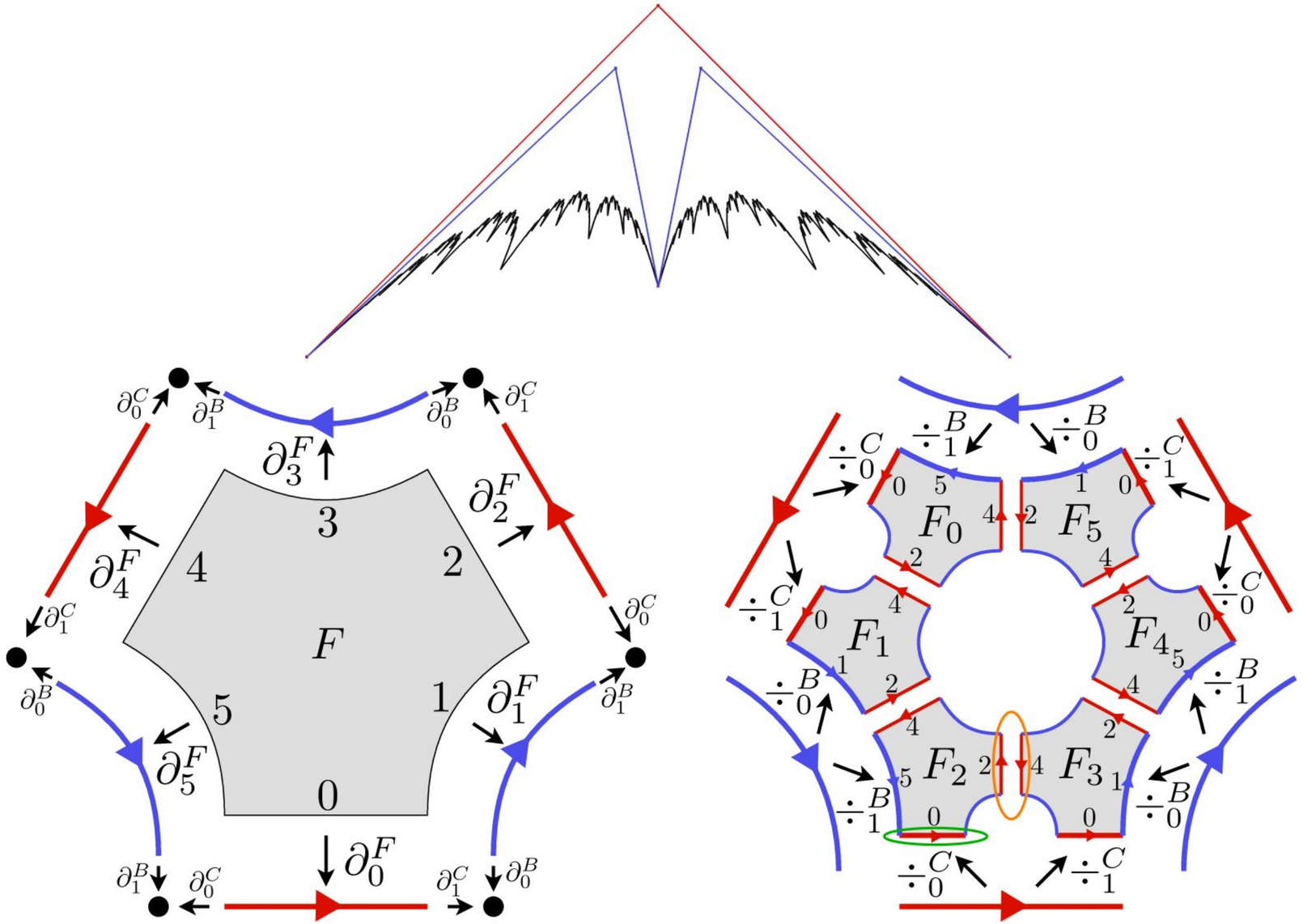




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Bonus : Orientation et symétrie





Bonus : Lacunes de primitive

