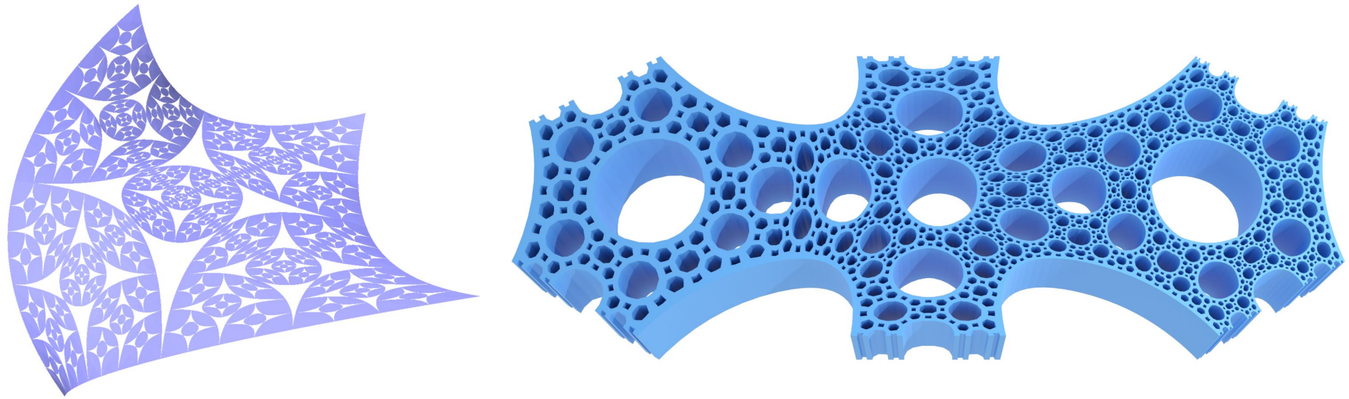
Conception automatique de structures lacunaires fractales



Boris Bordeaux[1], Christian Gentil[1], Lionel Garnier[1]
Iván Rasskin[2]

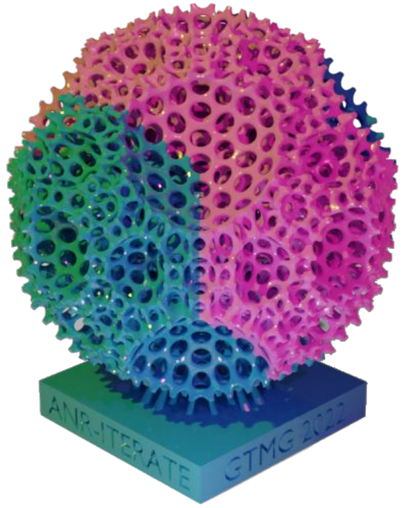
[1]LIB – Université Bourgogne Europe, Dijon, France
[2]LIS – Aix-Marseille Université, Marseille, France

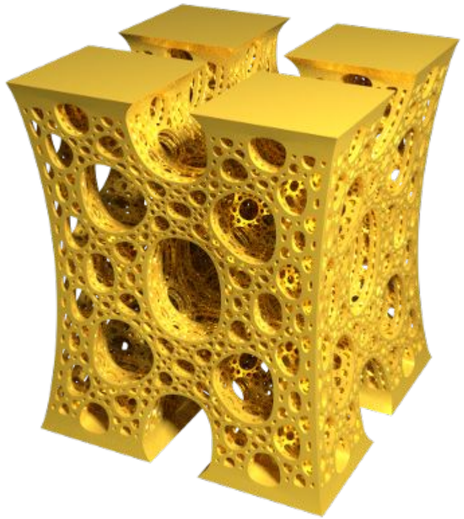
ANR ITERATE – ANR-20-CE46-0011 (2021-2025)

Projet ANR : ITERATE

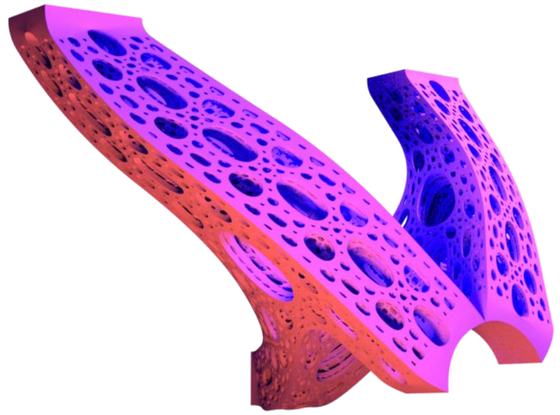
- Structures lacunaires fractales [GGS21]











Fractales : géométrie de la nature





[GGS21]





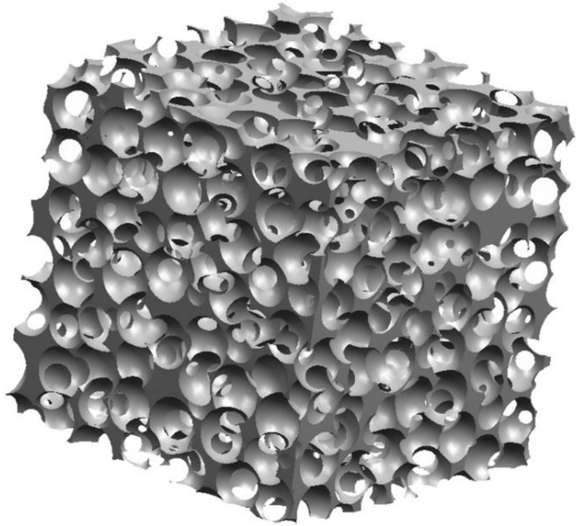
Plan

- État de l’art
- Problématique et démarche
- Conception automatique à partir des bords
- Conception automatique à partir de polyèdres
- Mesures sur les structures fractales
- Conclusion

État de l’art
Structures lacunaires

Structures lacunaires

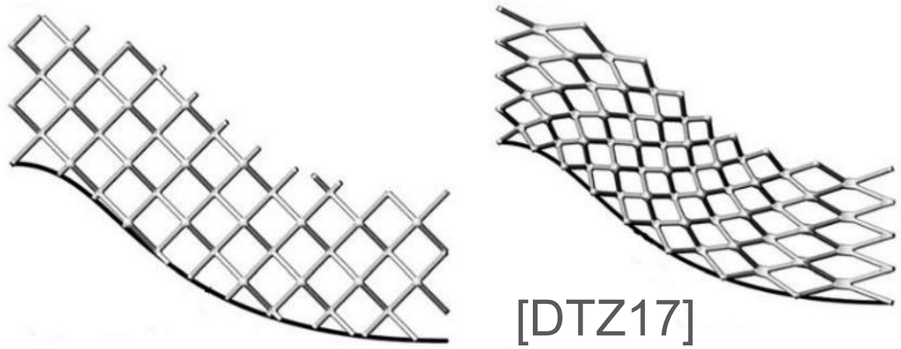
- Mousses
  - Aléatoire
- Structures lattices
  - Topologie régulière
  - Variation de géométrie
- Surfaces minimales
  - Triplement périodique
  - Géométrie contrainte



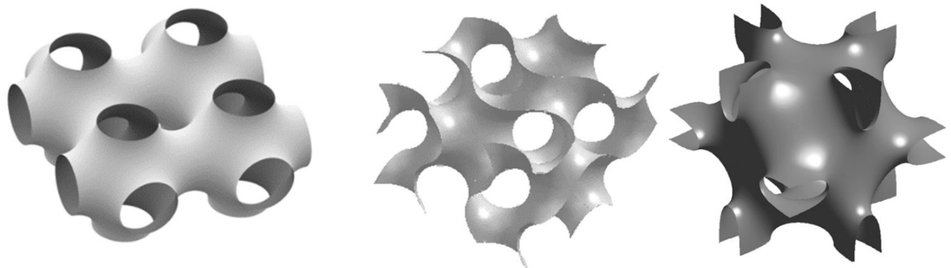
[KGE+07]



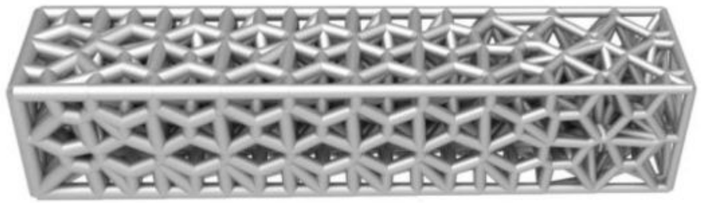
[LCL+21]



[DTZ17]



[BPFL19]

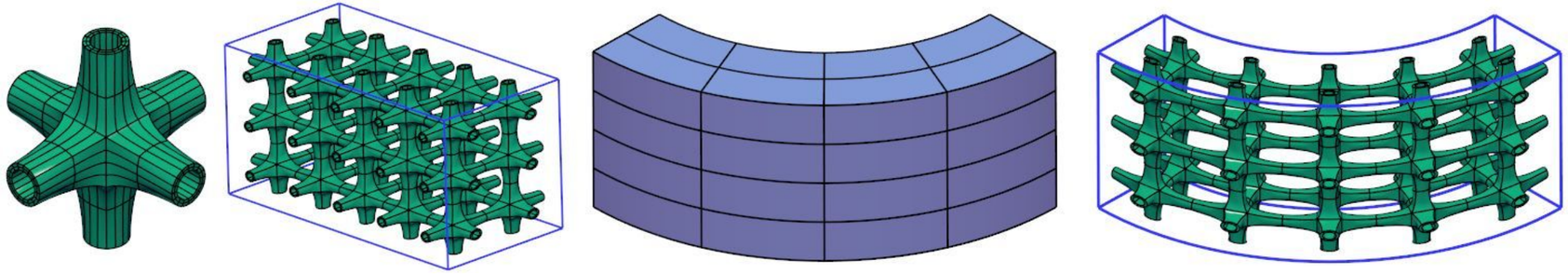


[LLXW20]

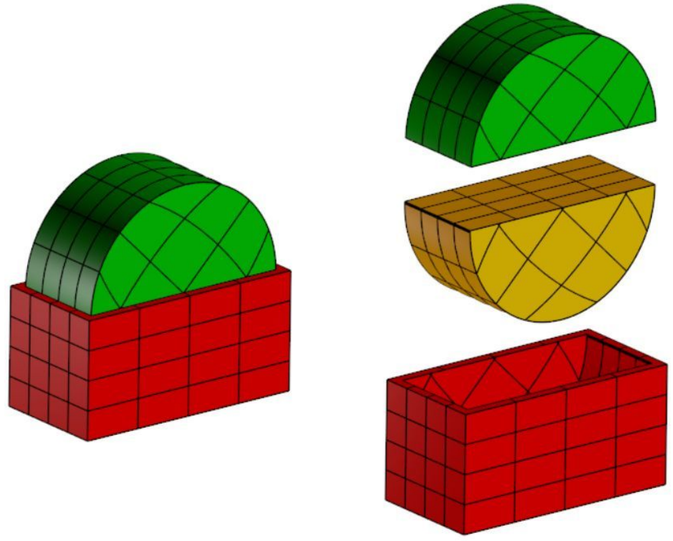
État de l’art
Méthodes récursives

Méthodes récursives

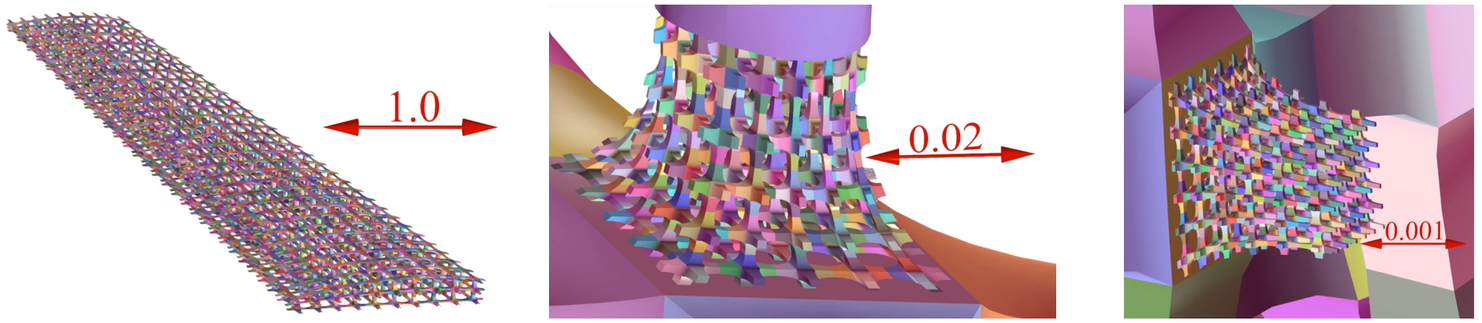
- Espace paramétrique
  - Motif répété
  - Gestion des intersections et des raccords
  - Sauts d’échelle importants



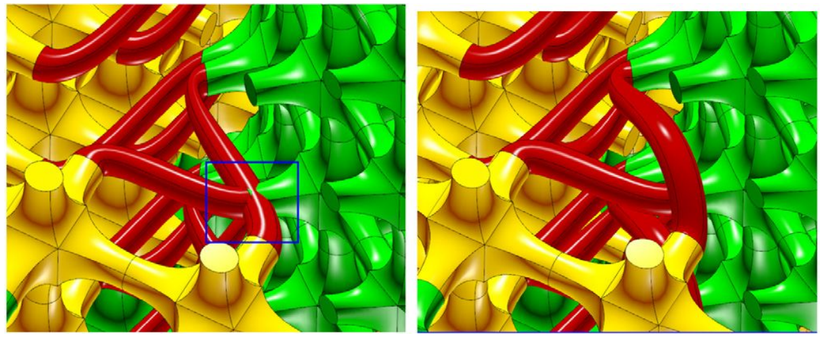
[Elb23]



[Elb23]



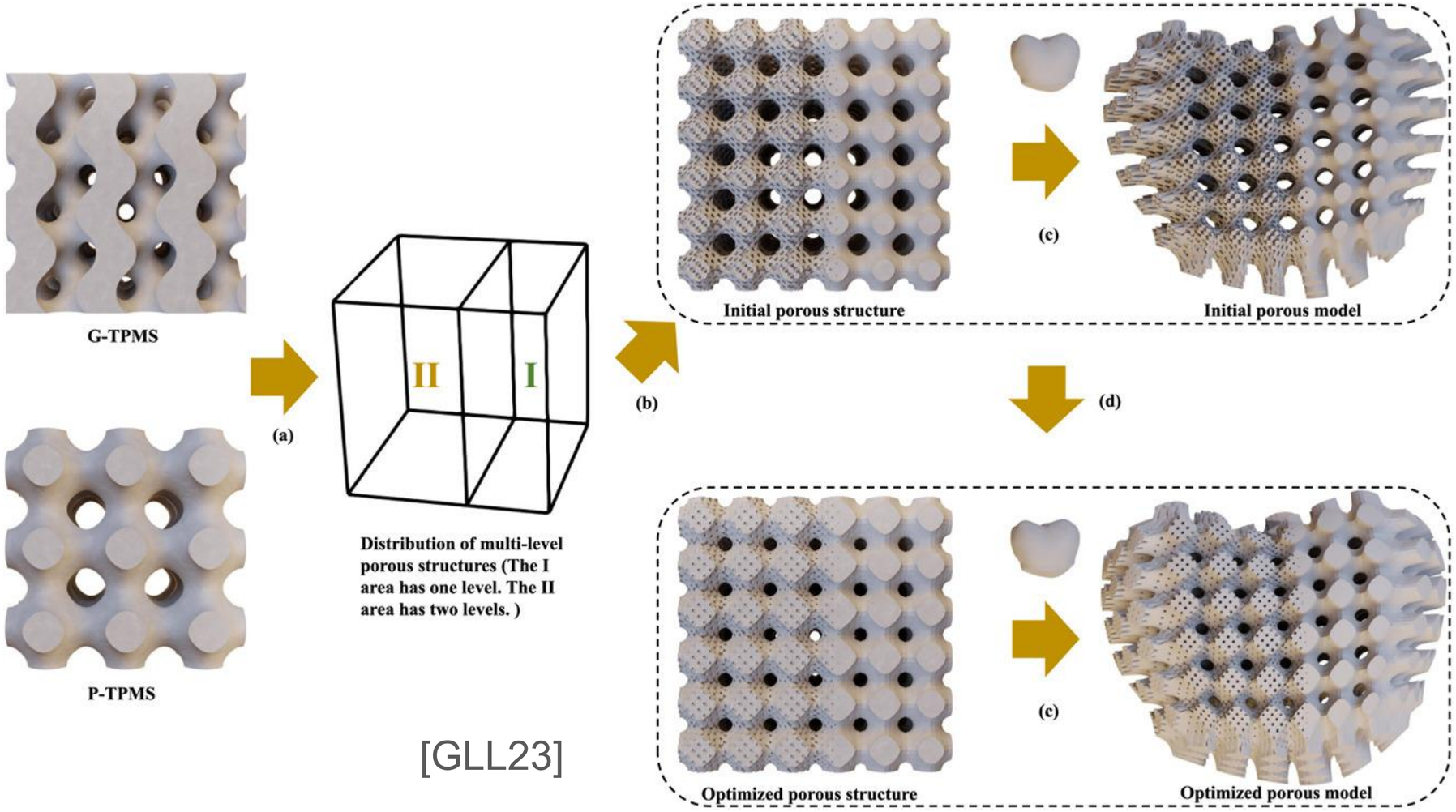
[Elb23]



[HE21]

Méthodes récursives

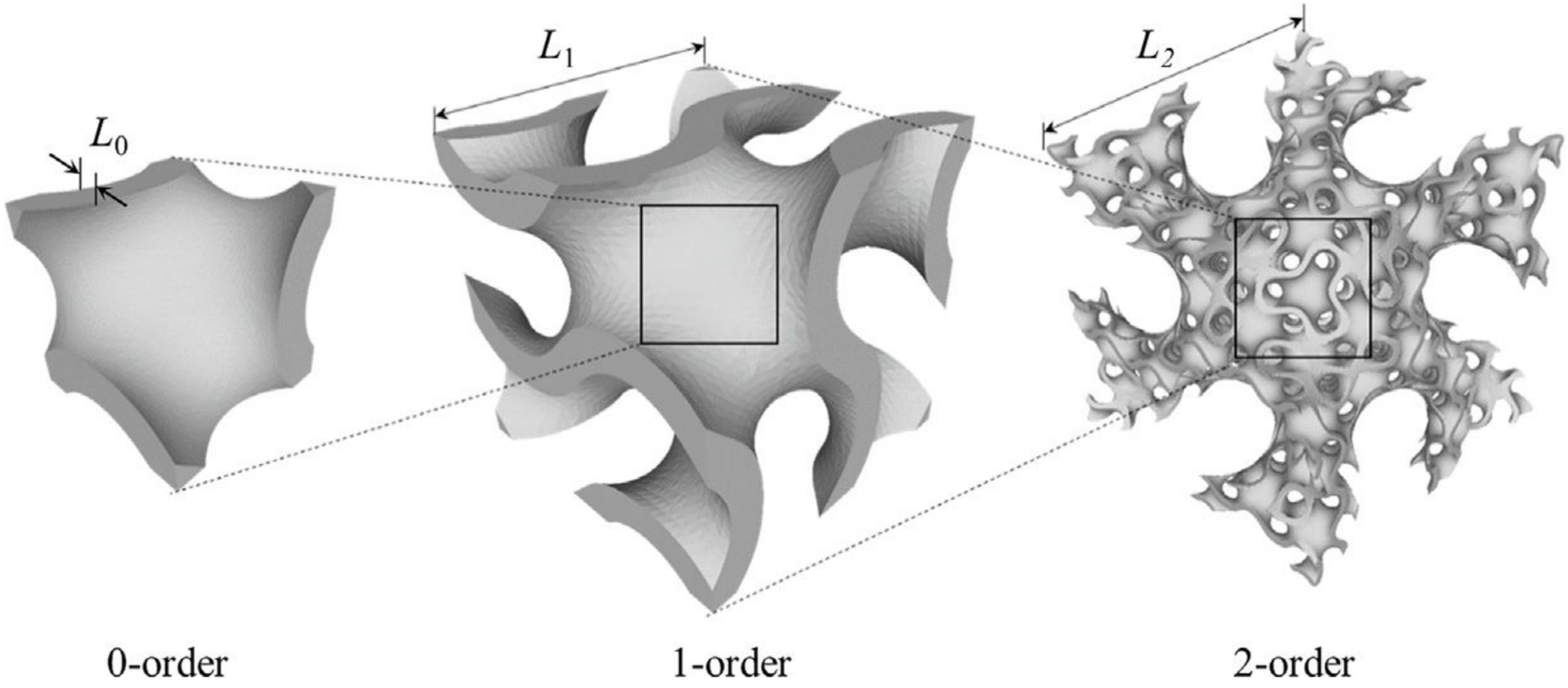
- Surfaces minimales triplement périodiques
  - Fonctions implicites
  - Paramétrables



[GLL23]

Méthodes récursives

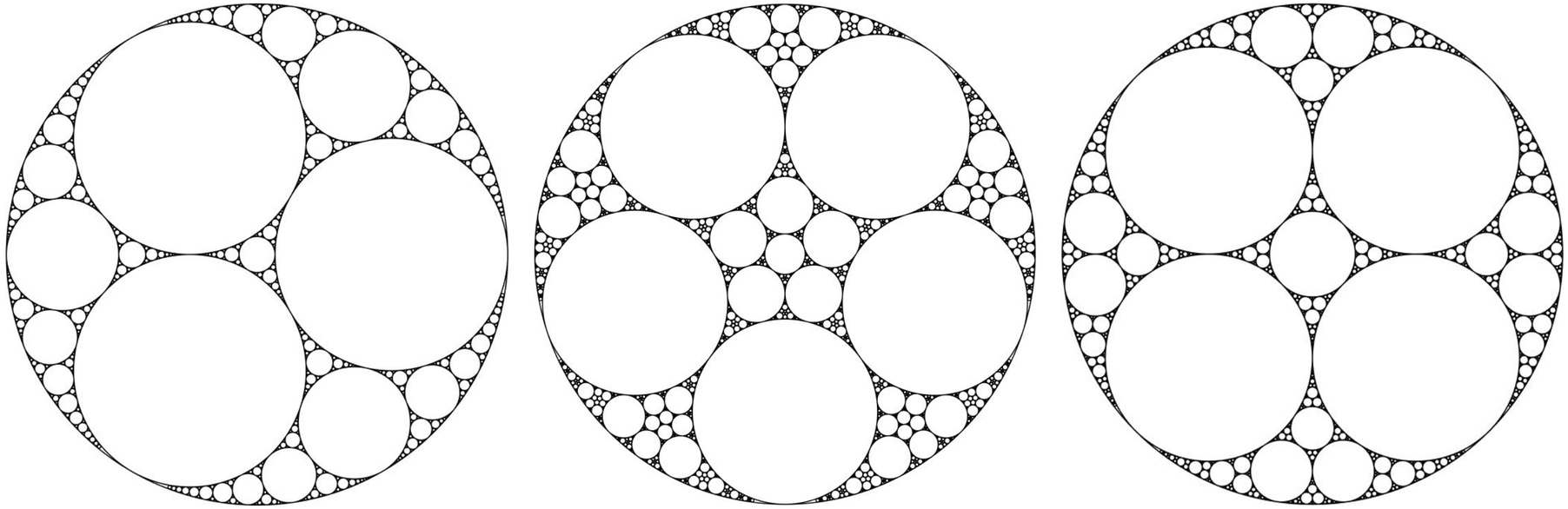
- Hiérarchie de surfaces minimales triplement périodiques

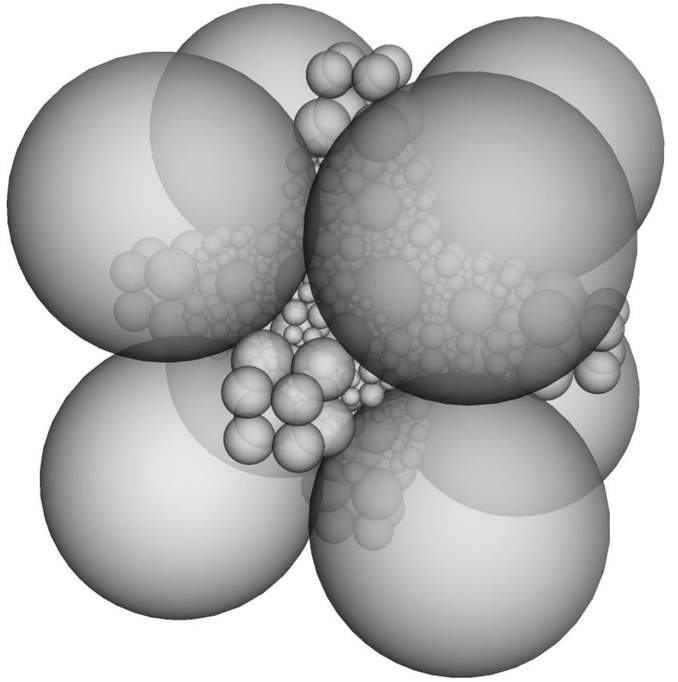


[ZHWF21]

Méthodes récursives

- Empilements cristallographiques de sphères [KN19]
  - Inversions
  - Généralisable





Iván Rasskin

État de l’art
Modèle fractal

Modèle fractal

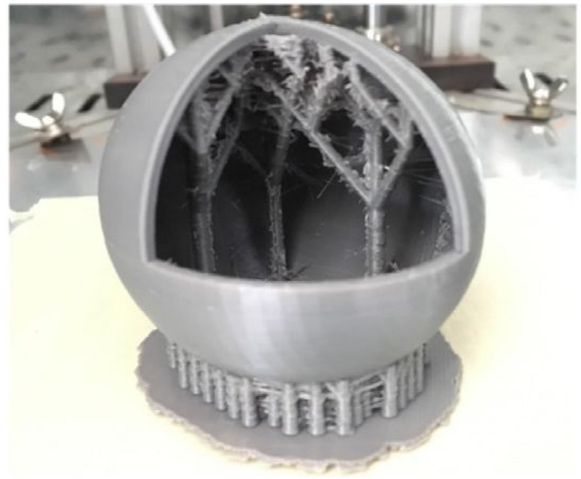
- L-Systems [Pru86]
  - Structures arborescentes



[PMKL01]



[PCFH18]



[ZLRL19]

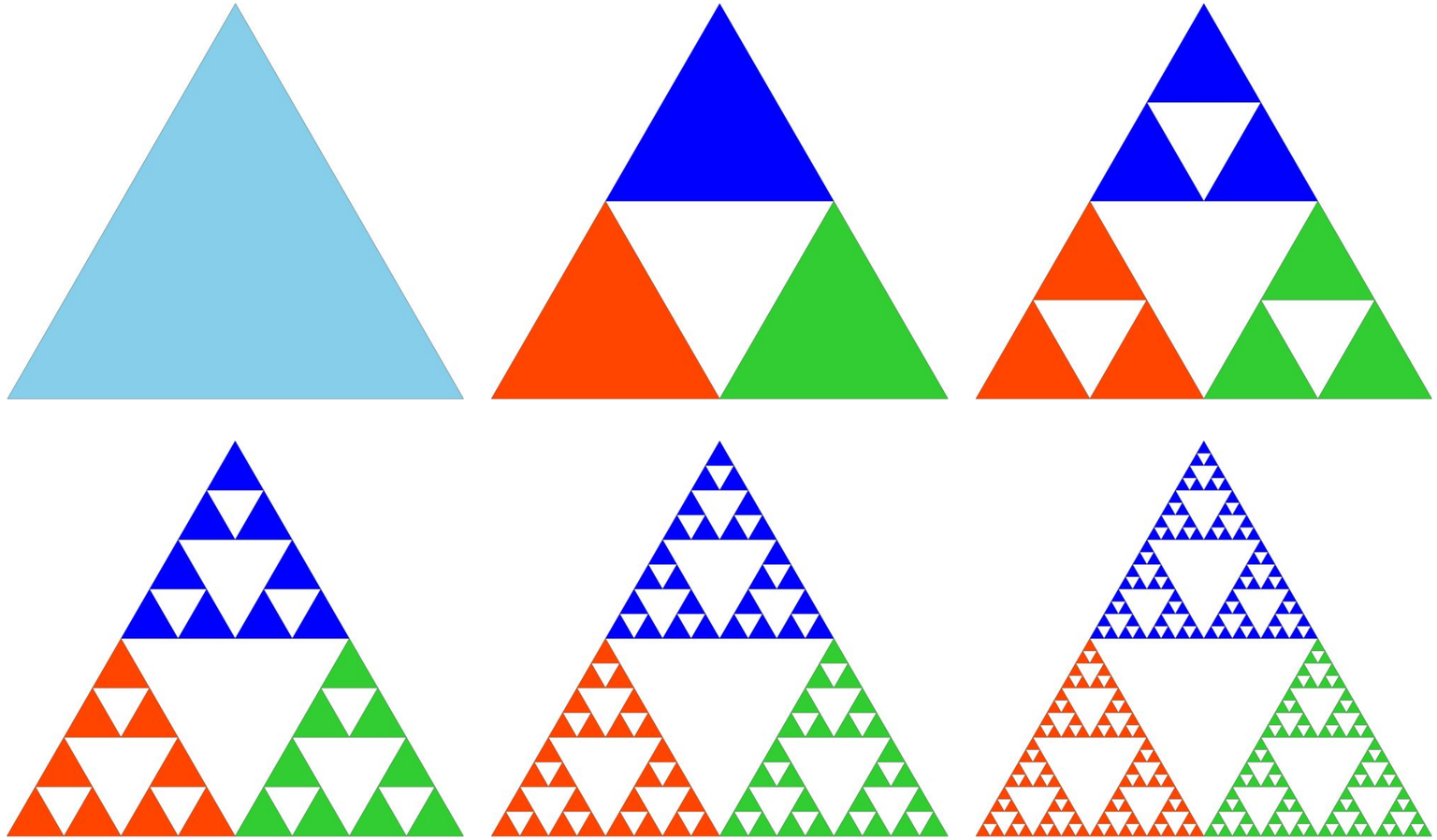
Modèle fractal

- Iterated Function System [Hut81, BD85]
  - Transformations contractantes

$$T_2 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$T_0 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \quad T_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

$$S = T_0(S) \cup T_1(S) \cup T_2(S)$$



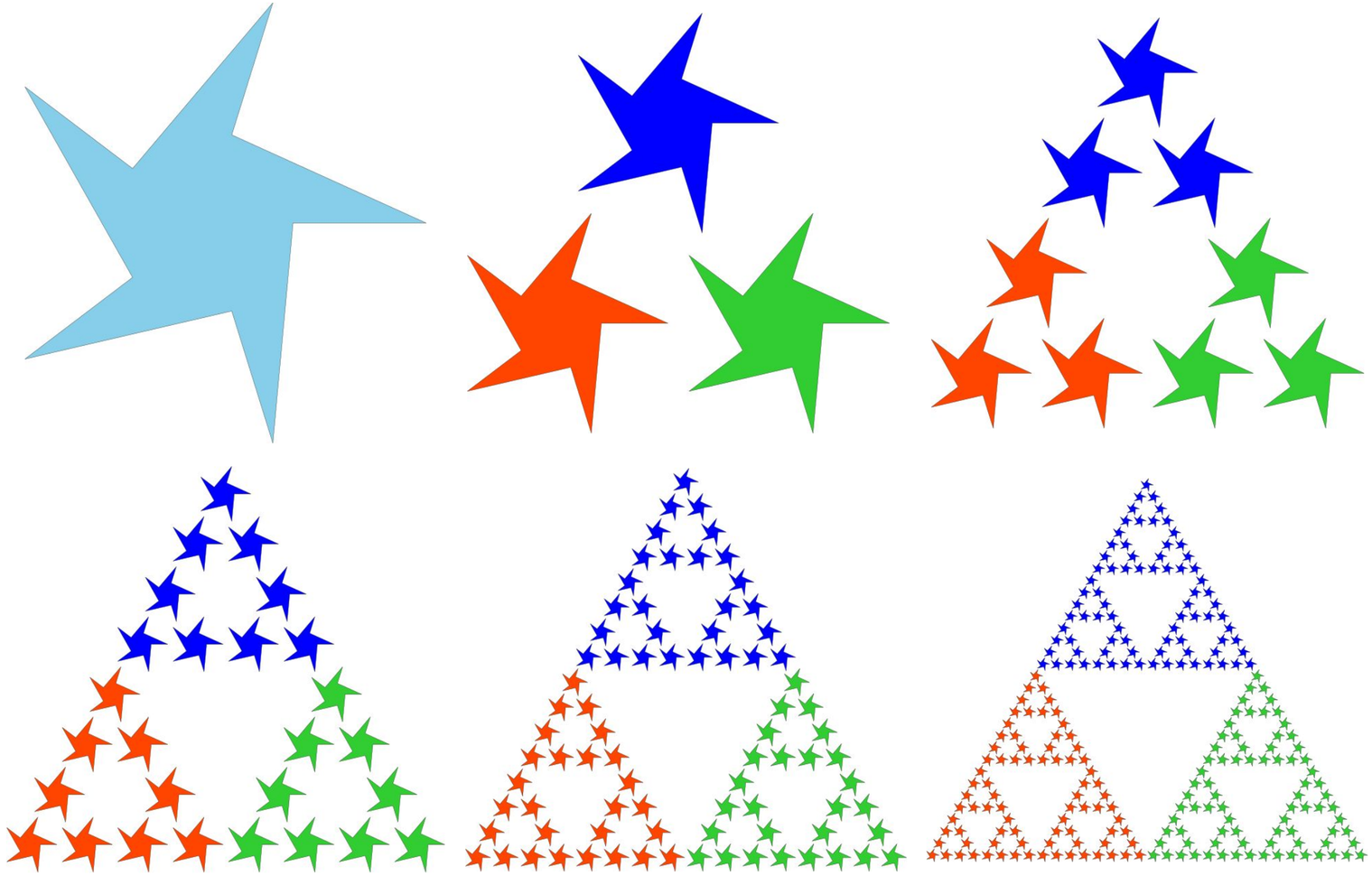
Modèle fractal

- Iterated Function System [Hut81, BD85]
  - Transformations contractantes
  - Peu importe la primitive

$$T_2 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

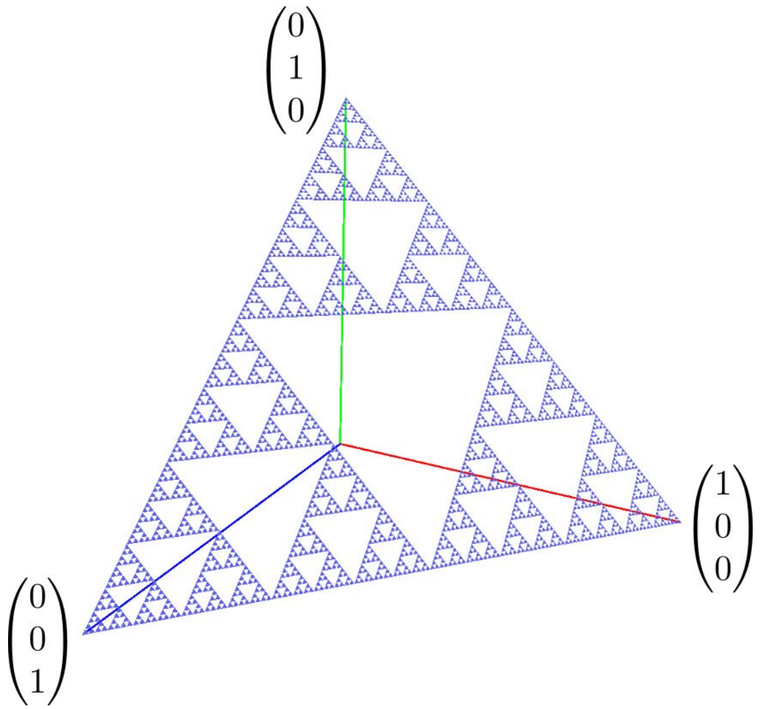
$$T_0 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \quad T_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

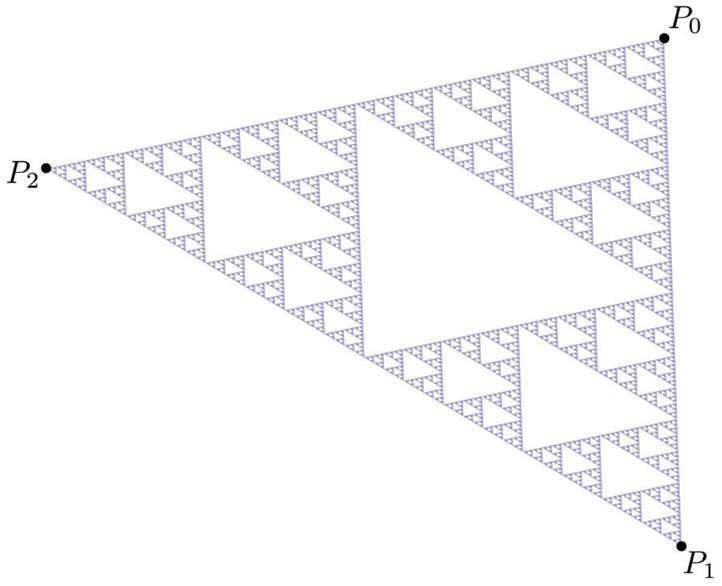
$$S = T_0(S) \cup T_1(S) \cup T_2(S)$$

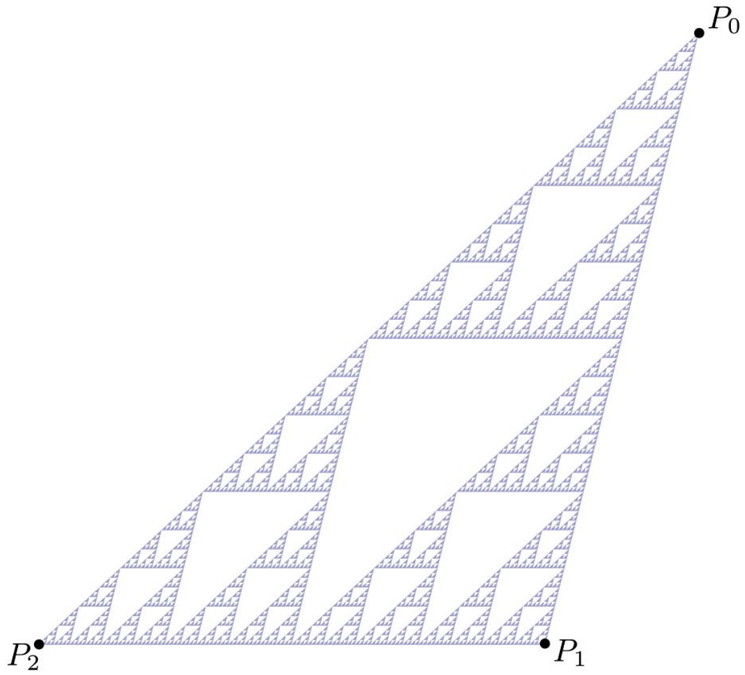


Modèle fractal

- Iterated Function System [Hut81, BD85]
  - Transformations contractantes
  - Peu importe la primitive
  - Espace barycentrique

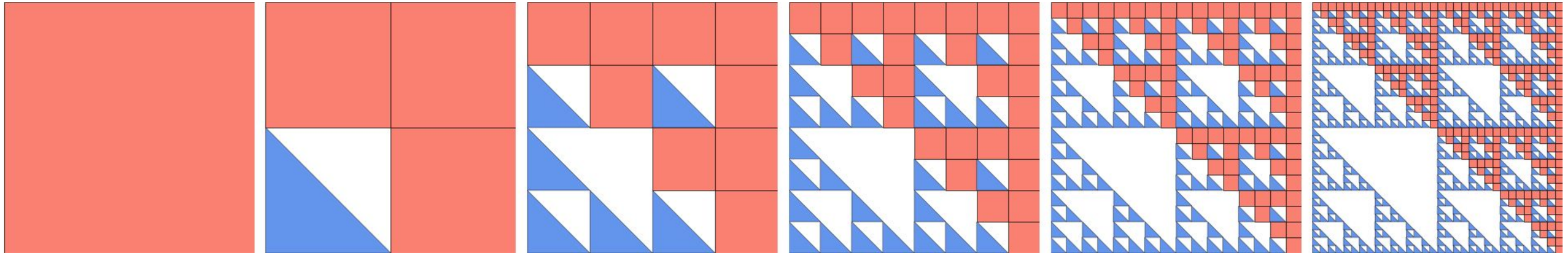


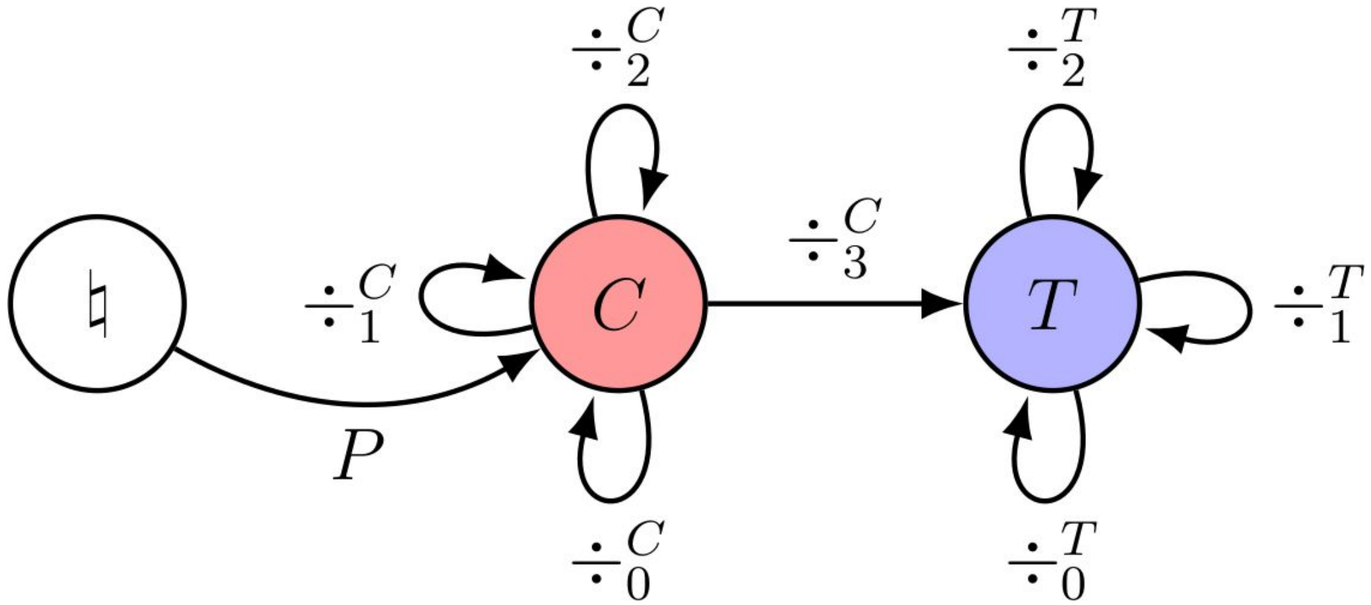




Modèle fractal

- Controlled-IFS

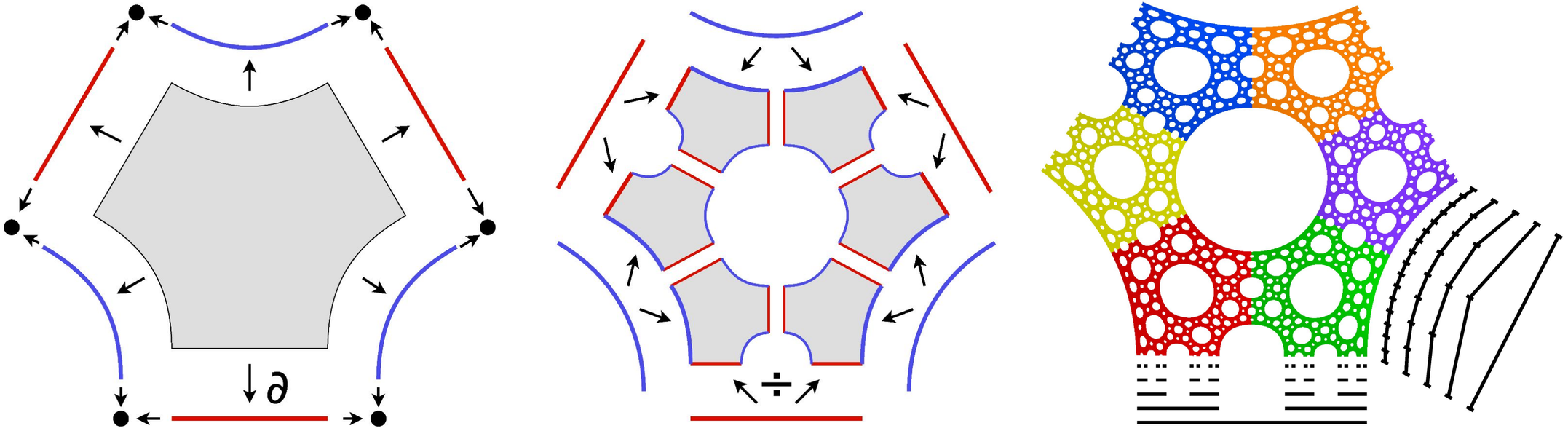




[GGS21]

Modèle fractal

- Boundary Controlled IFS
  - Décomposition cellulaire



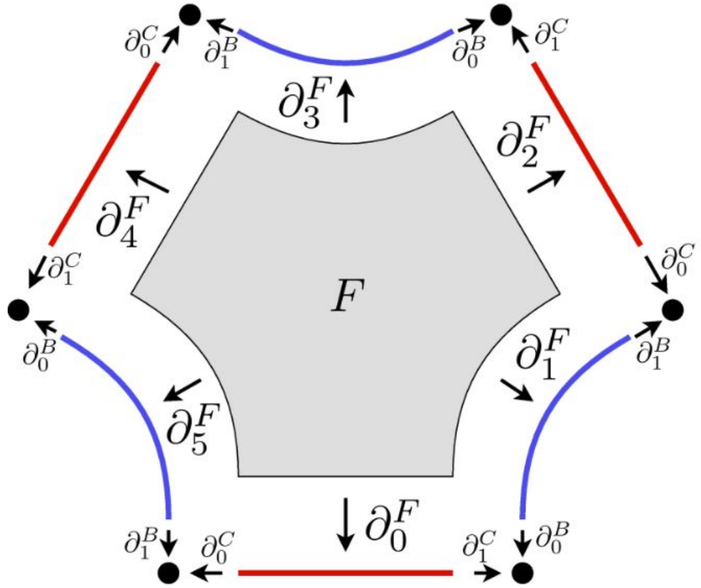
[GGS21]

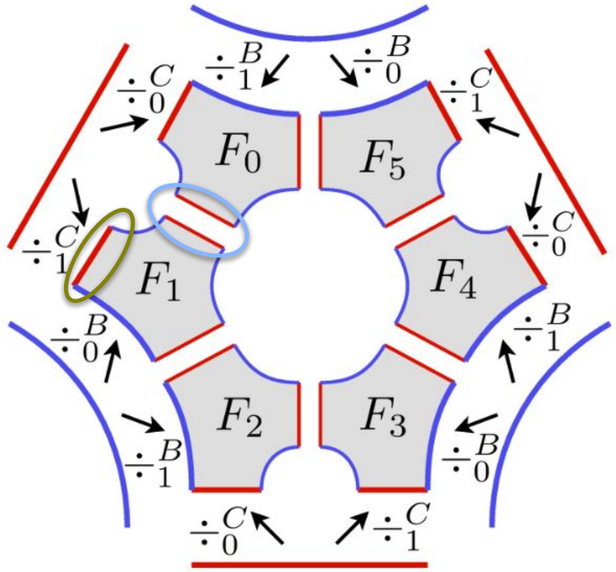
Modèle fractal

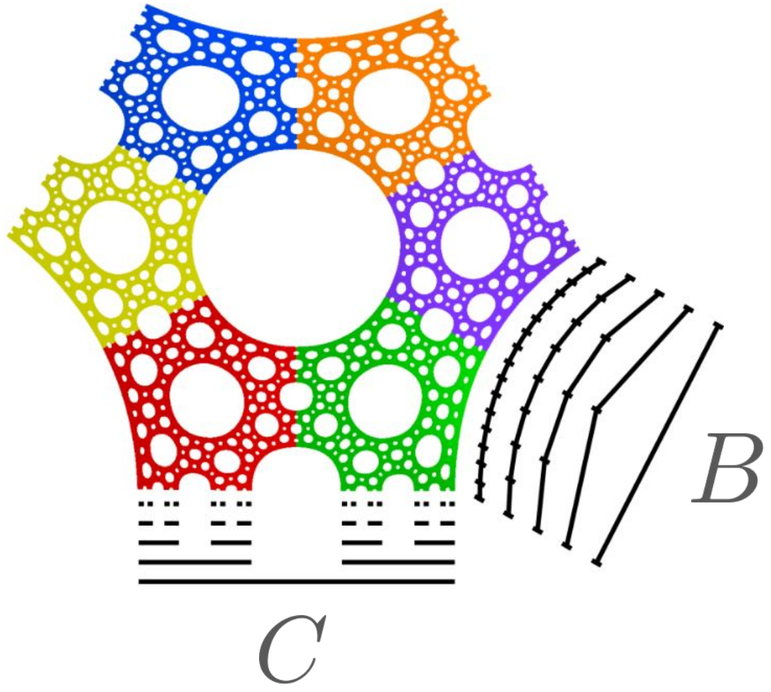
- BC-IFS
  - Décomposition cellulaire
  - Contraintes d'incidence et d'adjacence

$$\partial_4^F \div_1^C = \div_1^F \partial_0^F$$

$$\div_0^F \partial_2^F = \div_1^F \partial_4^F$$





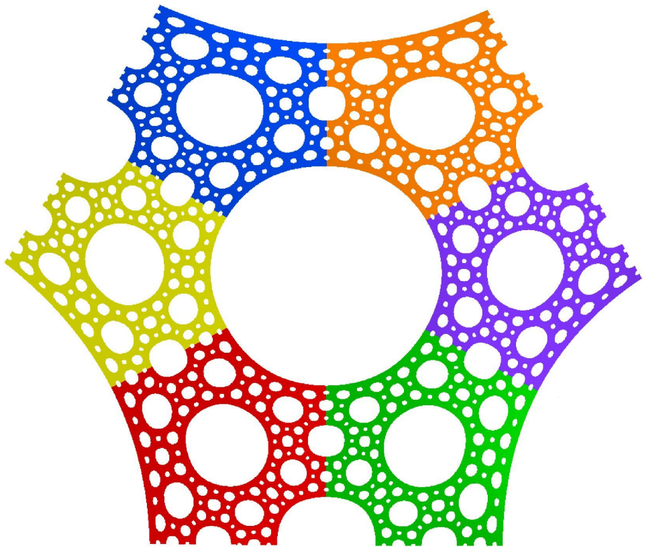


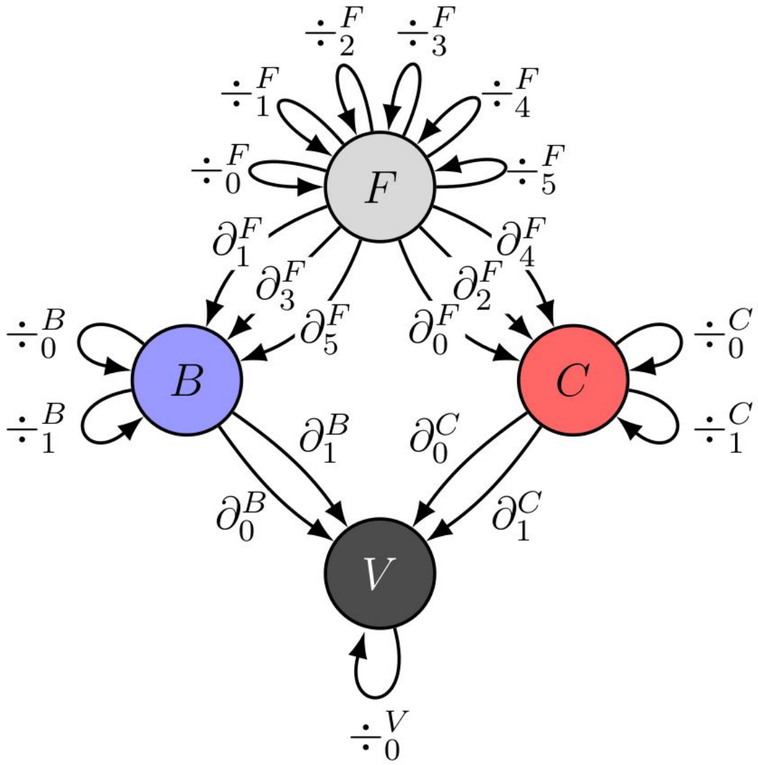
[GGS21]

Université Bourgogne Europe | LIB Laboratoire Informatique de Bourgogne

Modèle fractal

- BC-IFS
  - Décomposition cellulaire
  - Contraintes d'incidence et d'adjacence



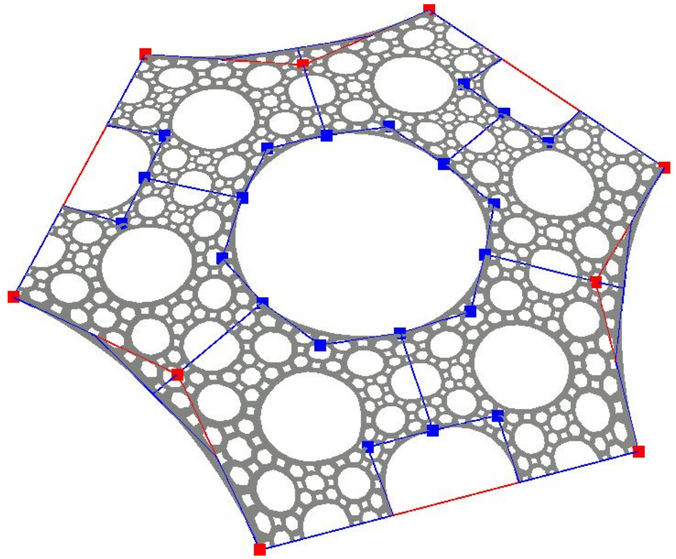


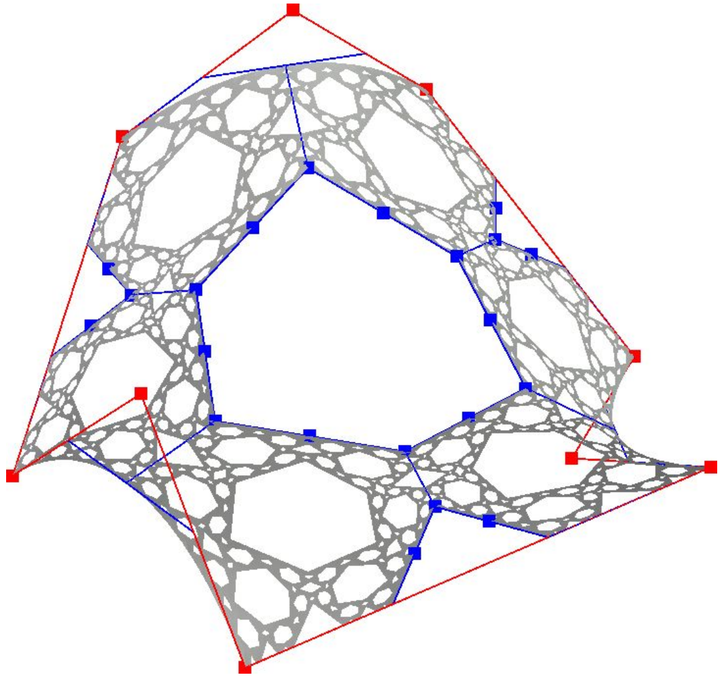
$$\partial_0^F \div_0^C = \div_2^F \partial_0^F$$
$$\partial_0^F \div_1^C = \div_3^F \partial_0^F$$
$$\partial_1^F \div_0^B = \div_3^F \partial_1^F$$
$$\partial_1^F \div_1^B = \div_4^F \partial_5^F$$
$$\partial_2^F \div_0^C = \div_4^F \partial_0^F$$
$$\partial_2^F \div_1^C = \div_5^F \partial_0^F$$
$$\partial_3^F \div_0^B = \div_5^F \partial_1^F$$
$$\partial_3^F \div_1^B = \div_0^F \partial_5^F$$
$$\partial_4^F \div_0^C = \div_0^F \partial_0^F$$
$$\partial_4^F \div_1^C = \div_1^F \partial_0^F$$
$$\partial_5^F \div_0^B = \div_1^F \partial_1^F$$
$$\partial_5^F \div_1^B = \div_2^F \partial_5^F$$

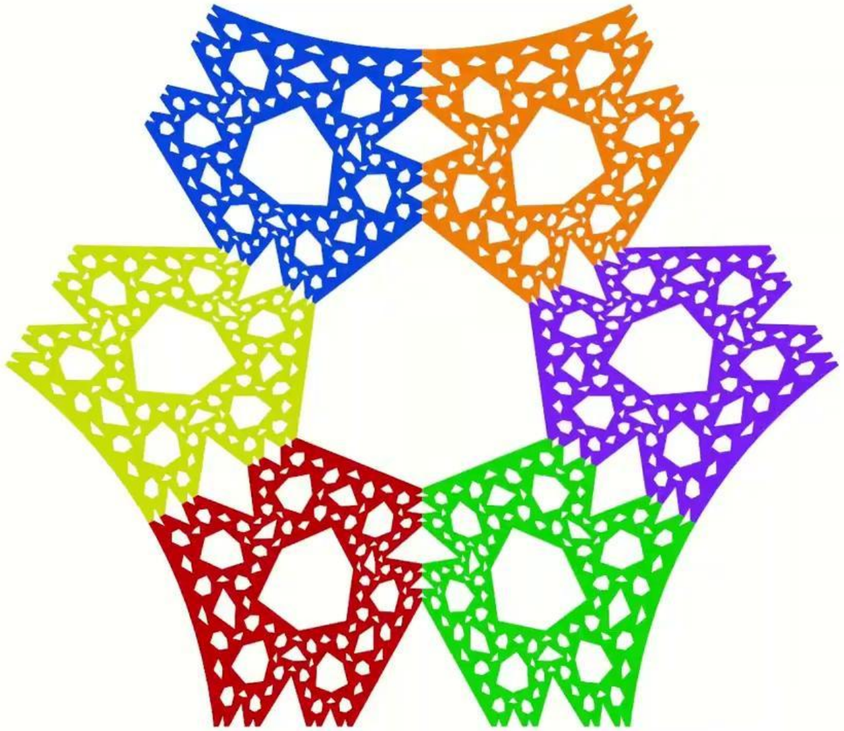
$$\div_0^F \partial_2^F = \div_1^F \partial_4^F$$
$$\div_1^F \partial_2^F = \div_2^F \partial_4^F$$
$$\div_2^F \partial_2^F = \div_3^F \partial_4^F$$
$$\div_3^F \partial_2^F = \div_4^F \partial_4^F$$
$$\div_4^F \partial_2^F = \div_5^F \partial_4^F$$
$$\div_5^F \partial_2^F = \div_0^F \partial_4^F$$

Modèle fractal

- BC-IFS
  - Décomposition cellulaire
  - Contraintes d’incidence et d’adjacence
  - Réalisation géométrique





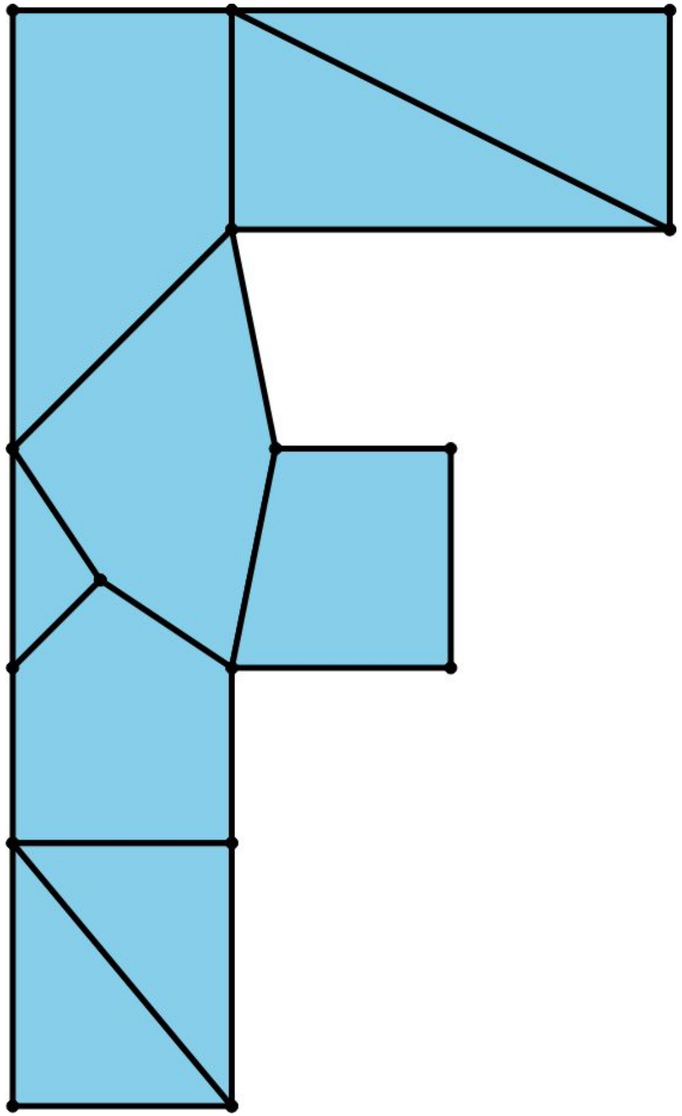


Problématique

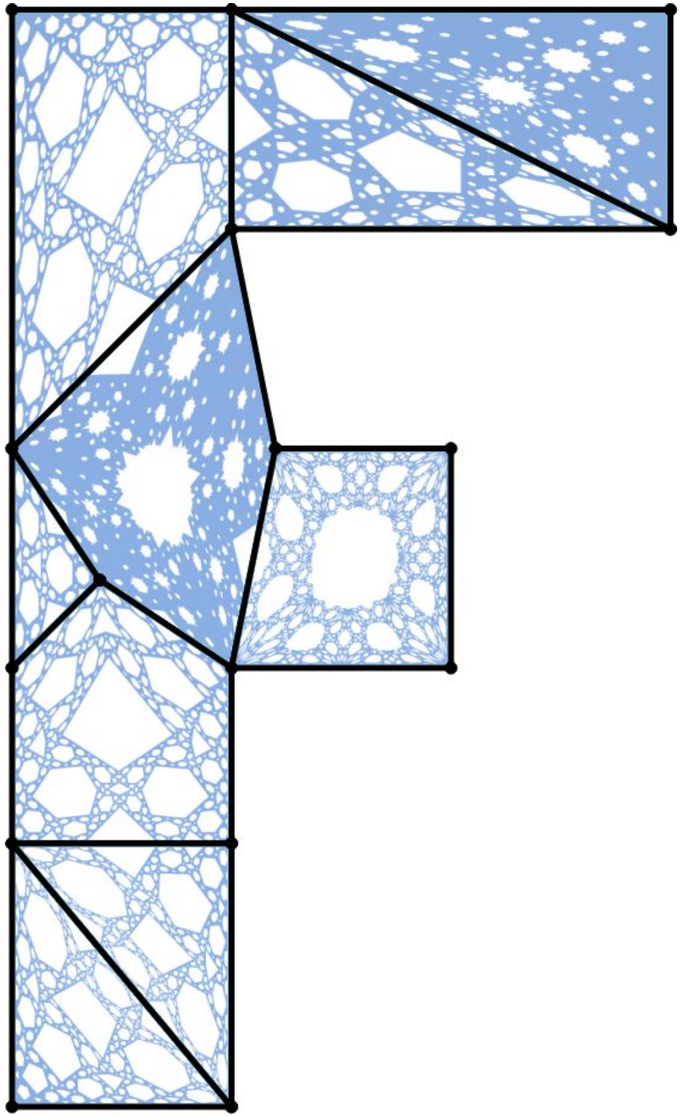
Problématique



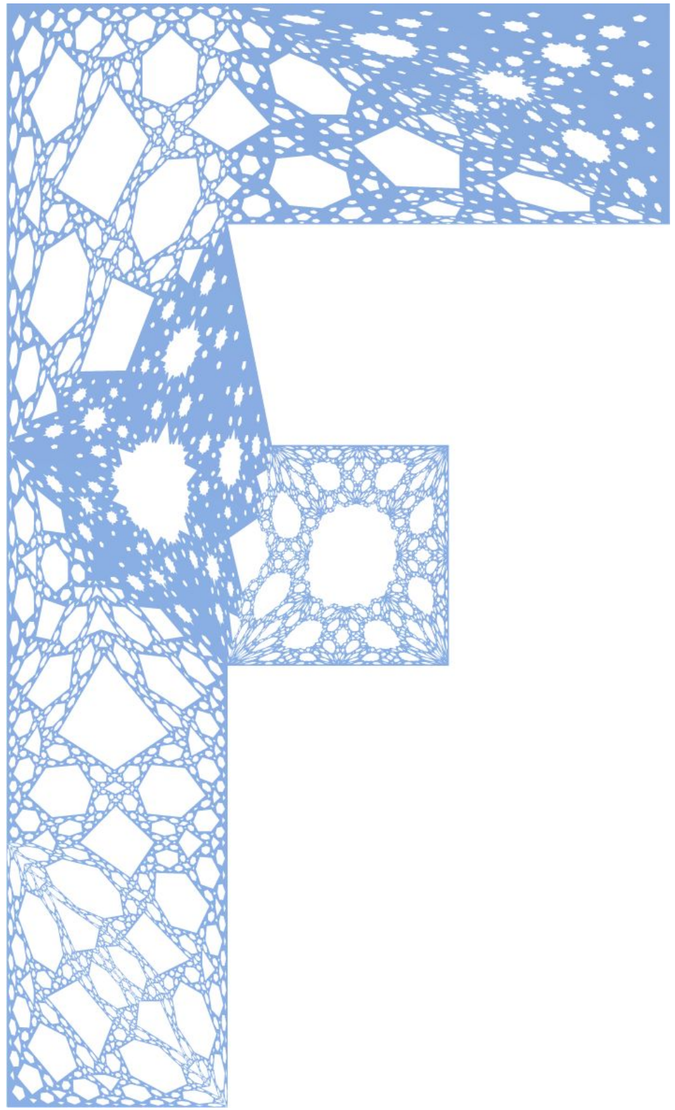
Problématique



Problématique

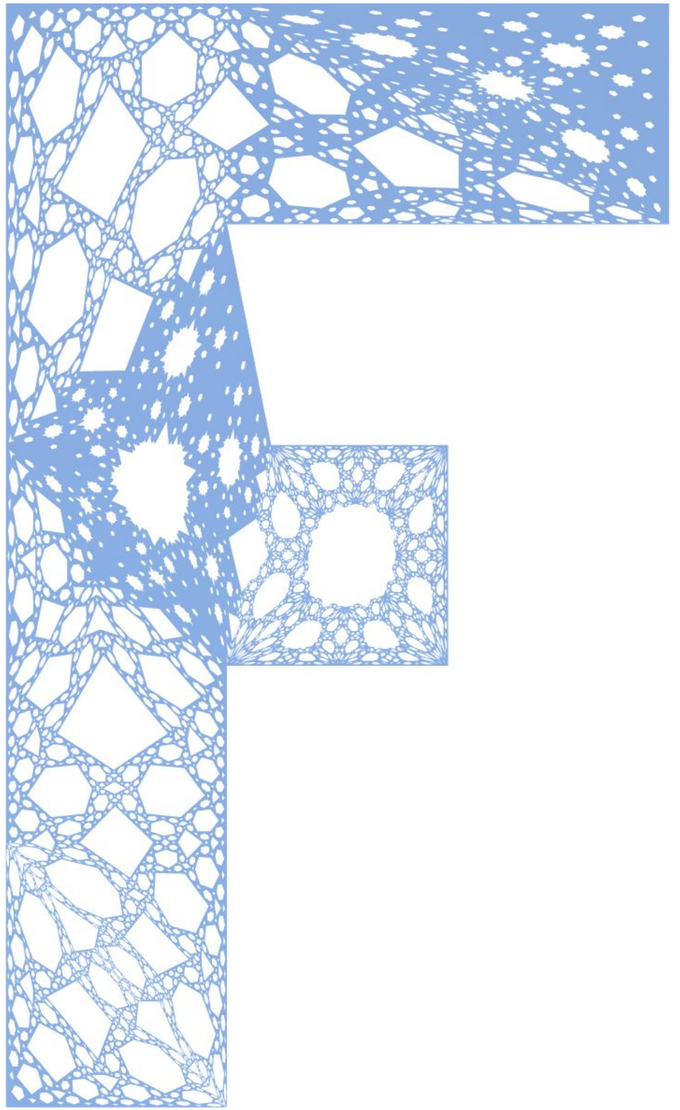


Problématique



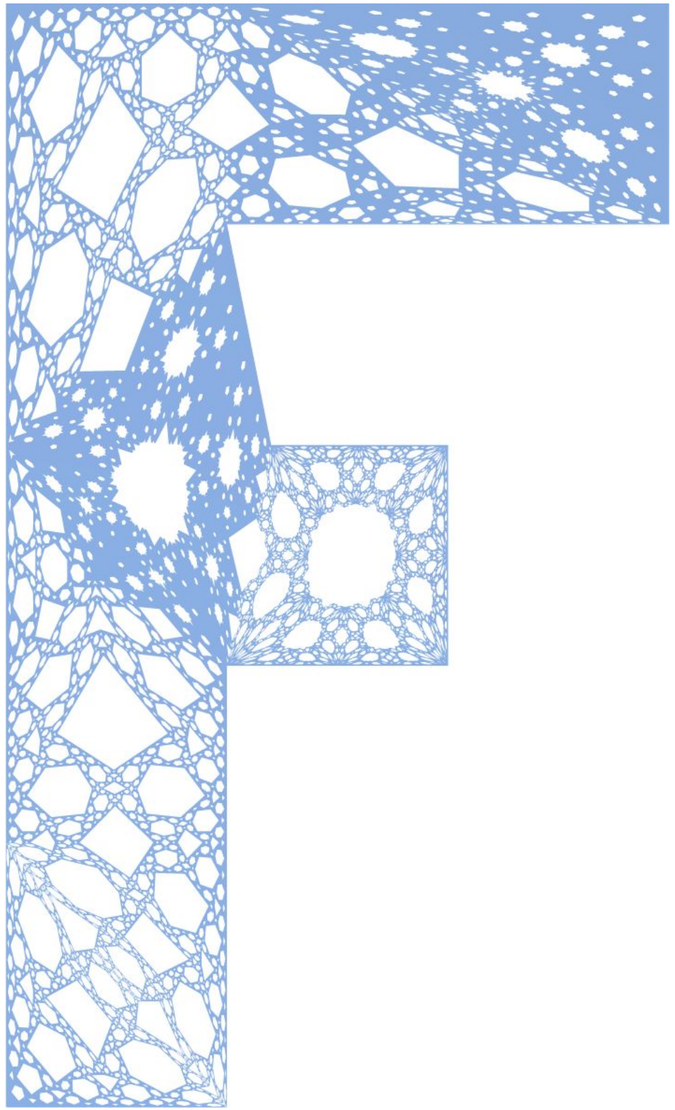
Démarche

- Utilisation du modèle BC-IFS
  - Gestion des raccords topologiques
  - Déformation avec des points de contrôle



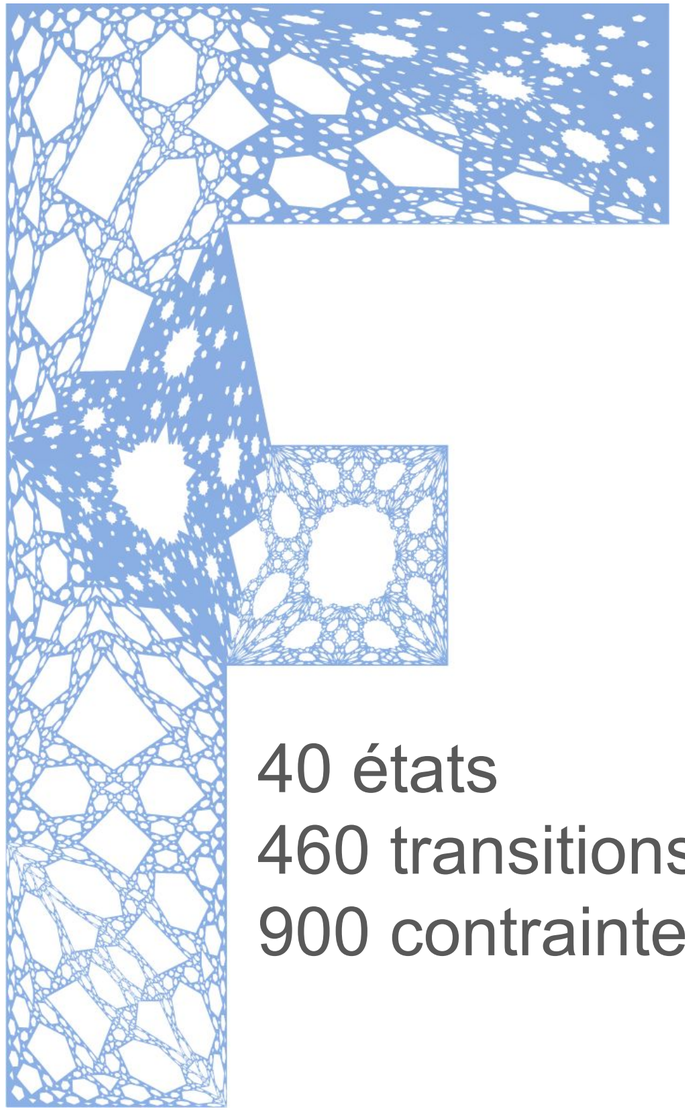
Démarche

- Utilisation du modèle BC-IFS
  - Gestion des raccords topologiques
  - Déformation avec des points de contrôle
- Modélisation fastidieuse
  - Imaginer le processus de subdivision
  - Le traduire avec un BC-IFS (automate + contraintes)



Démarche

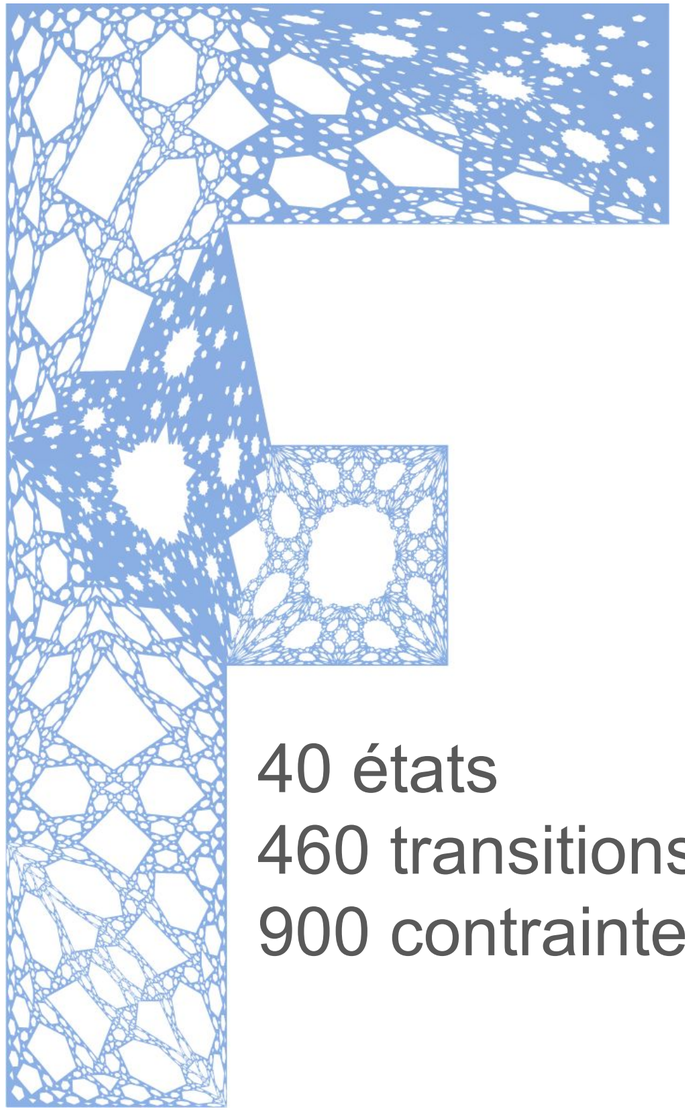
- Utilisation du modèle BC-IFS
  - Gestion des raccords topologiques
  - Déformation avec des points de contrôle
- Modélisation fastidieuse
  - Imaginer le processus de subdivision
  - Le traduire avec un BC-IFS (automate + contraintes)



40 états
460 transitions
900 contraintes

Démarche

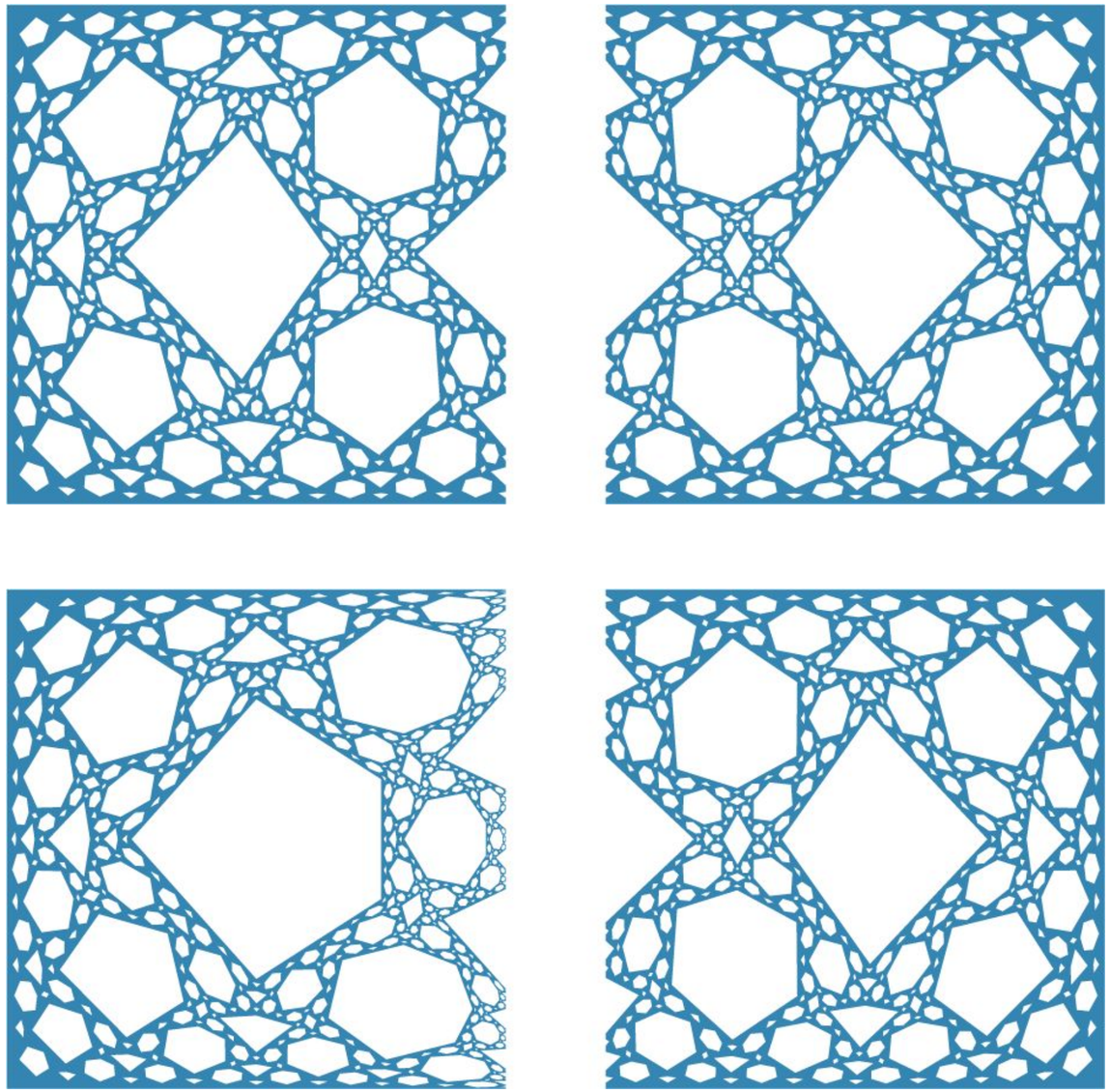
- Utilisation du modèle BC-IFS
  - Gestion des raccords topologiques
  - Déformation avec des points de contrôle
- Modélisation fastidieuse
  - Imaginer le processus de subdivision
  - Le traduire avec un BC-IFS (automate + contraintes)
- Nécessité de faciliter la conception
  - Deux processus de subdivision génériques
  - Algorithmes de traduction dans le modèle BC-IFS



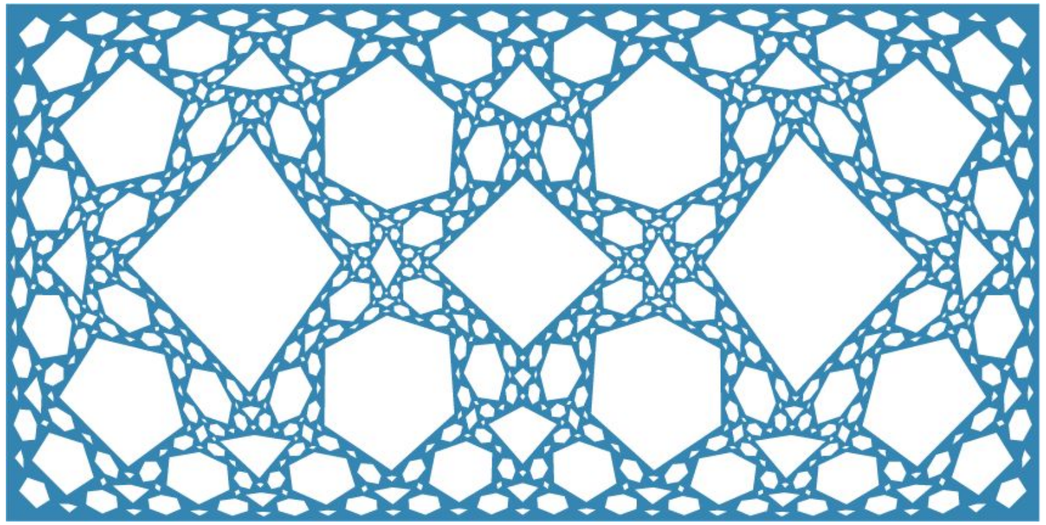
40 états
460 transitions
900 contraintes

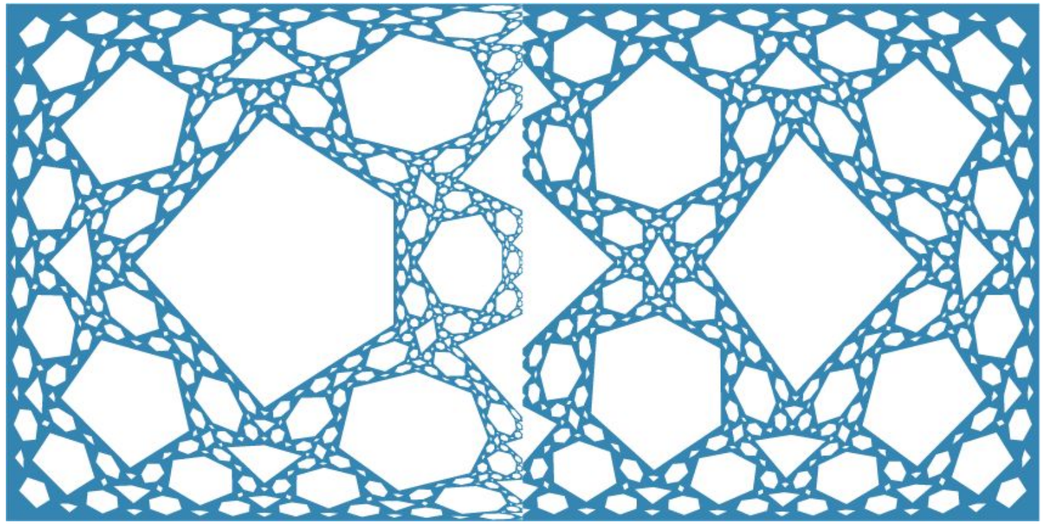
Conception automatique à partir des bords

Pourquoi à partir des bords ?



Pourquoi à partir des bords ?





Les types de bords

- Bords de type Cantor
  - Subdivisions non connexes de type Cantor

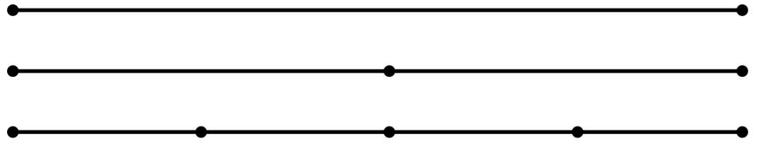


C_2

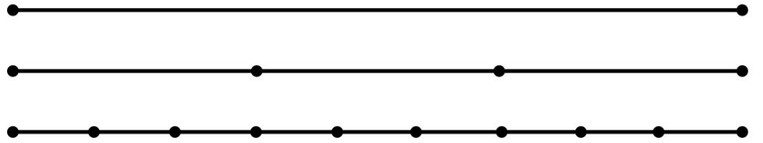


C_3

- Bords de type Courbe
  - Subdivisions connexes de type Courbe



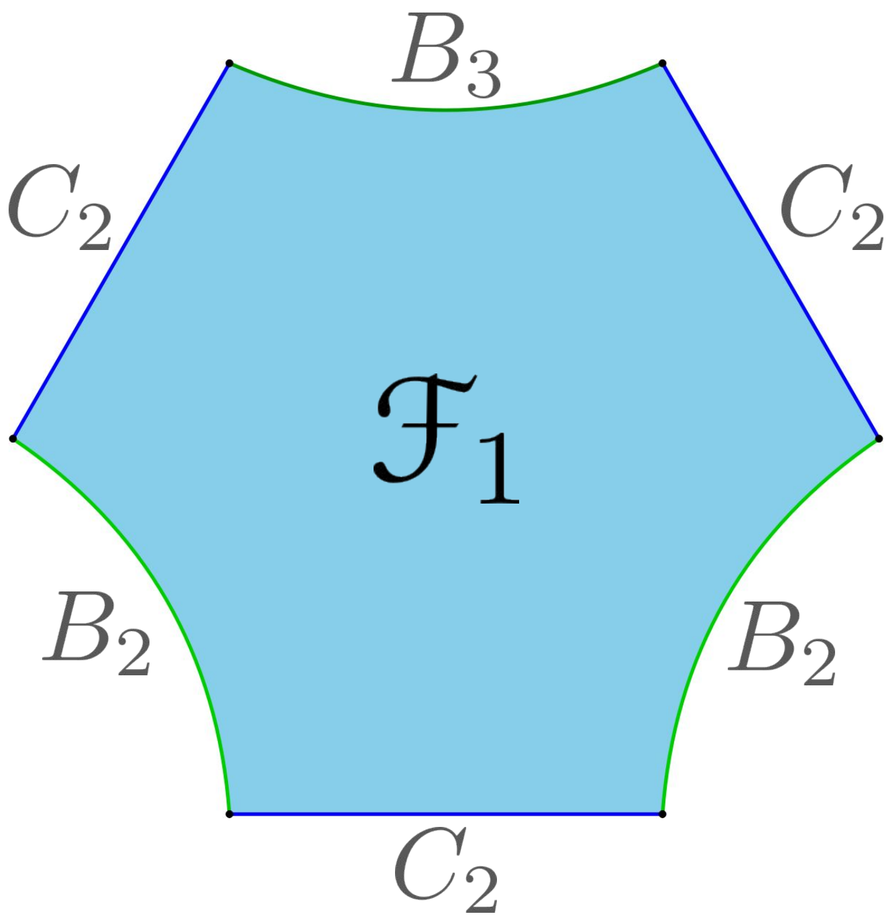
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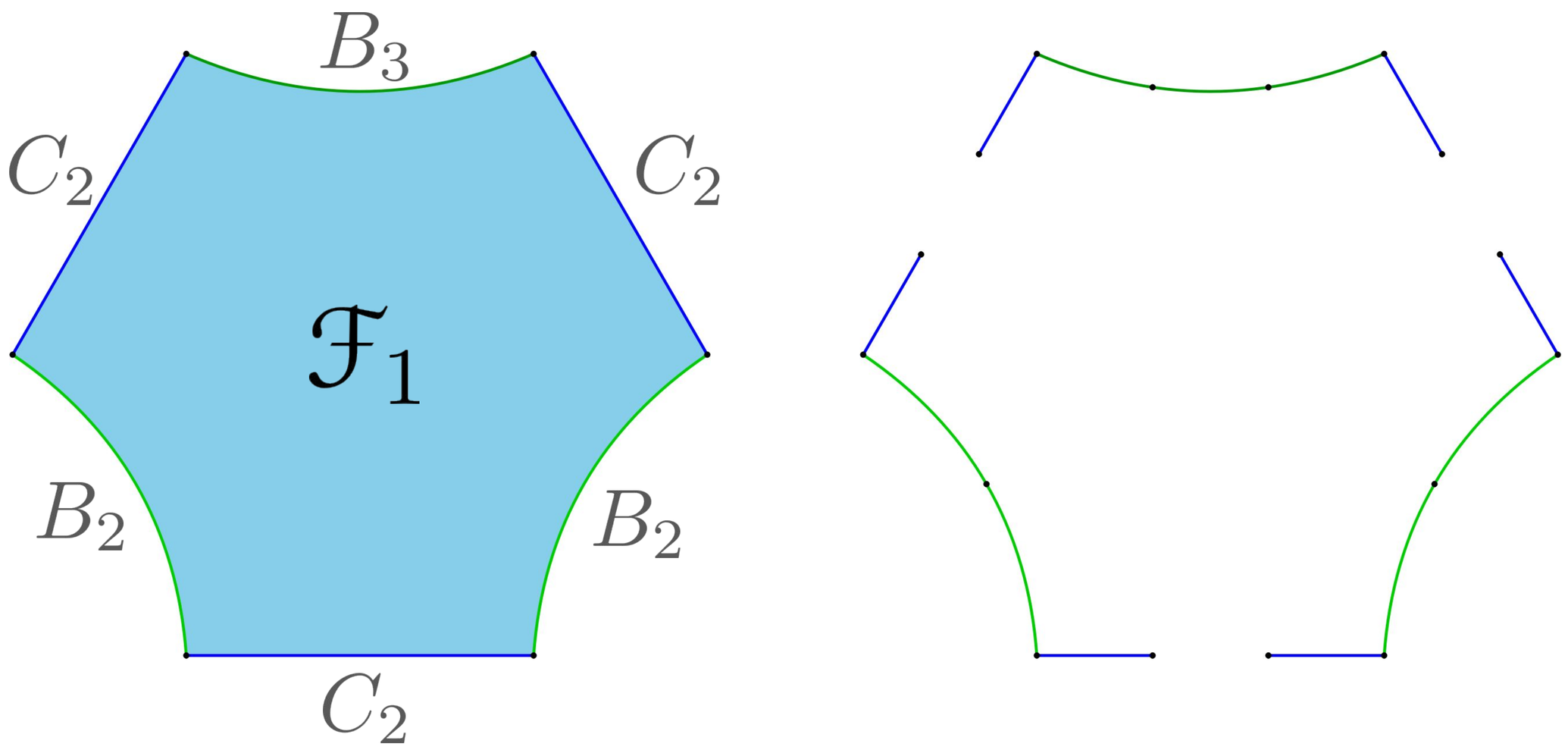
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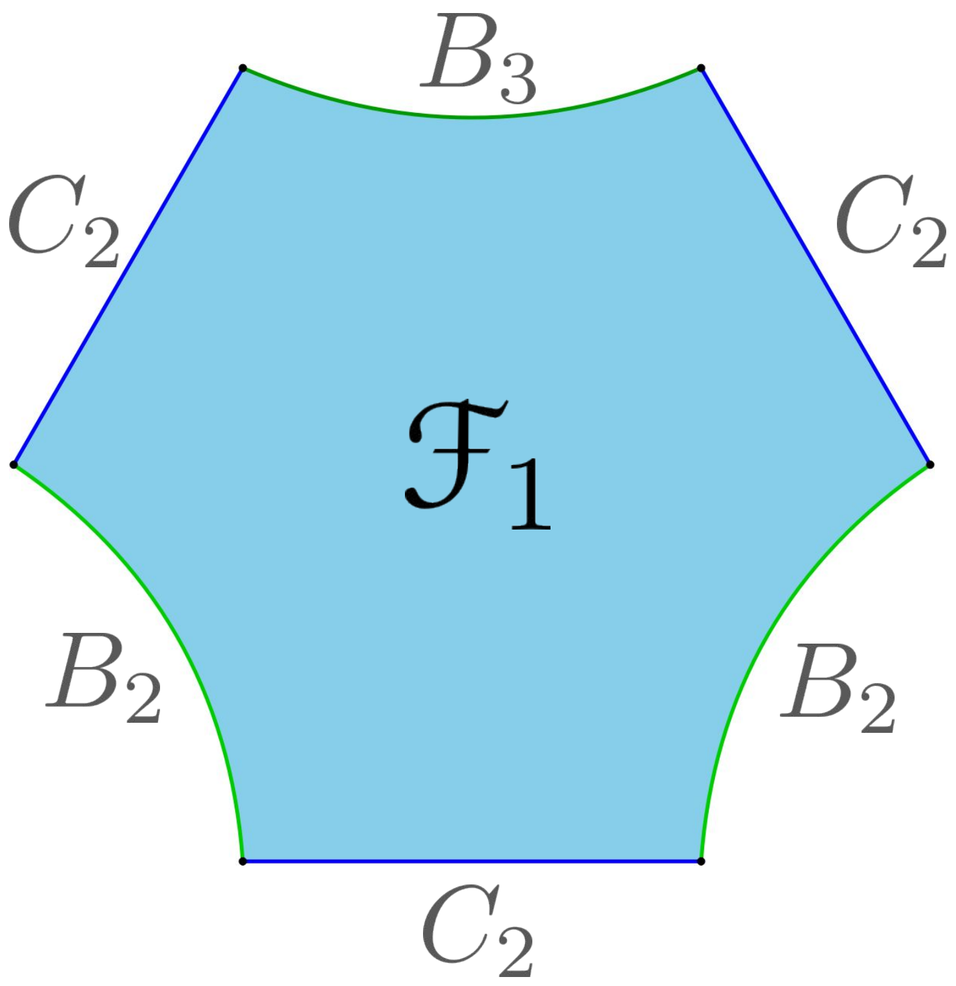
Comment définir les règles de subdivision ?

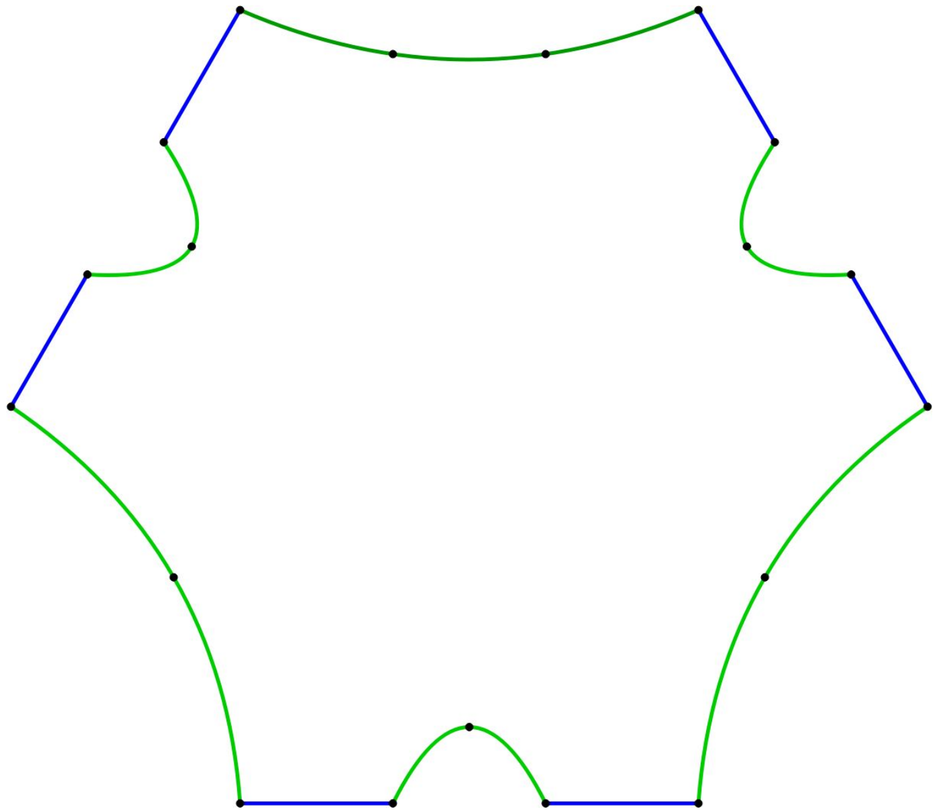


Processus de subdivision

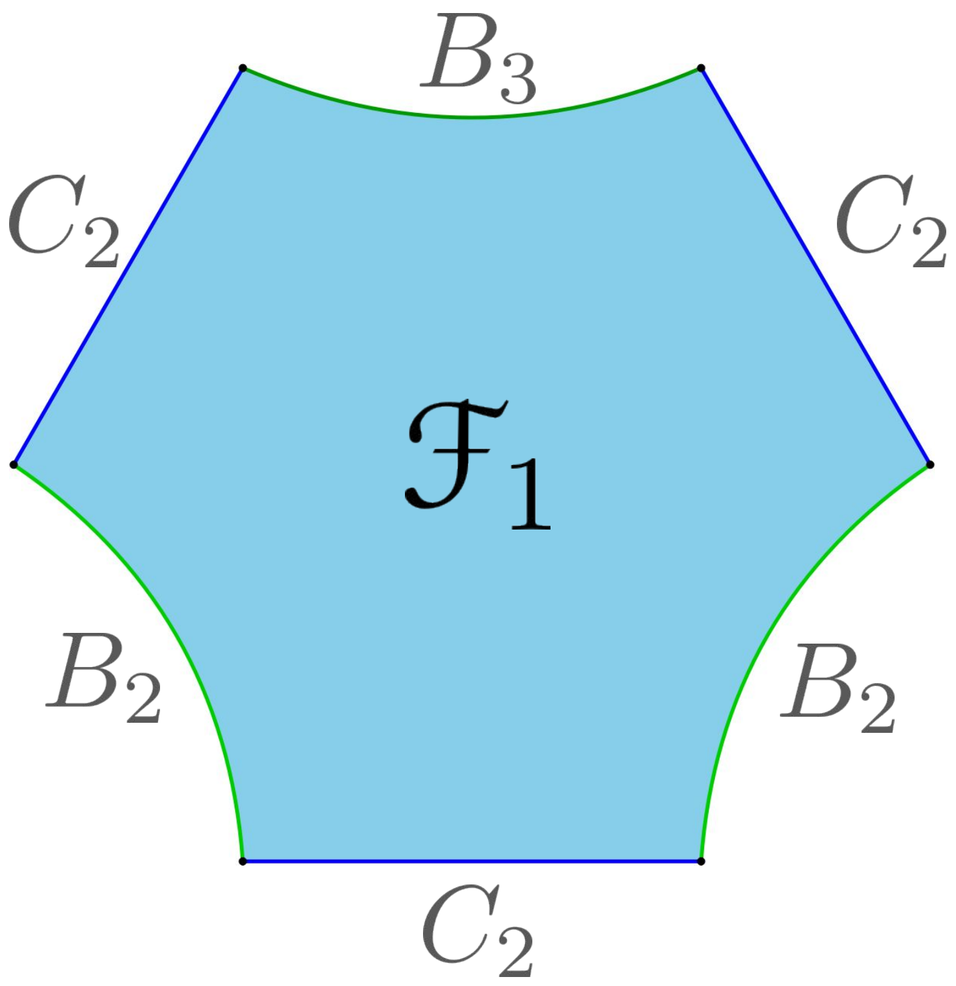


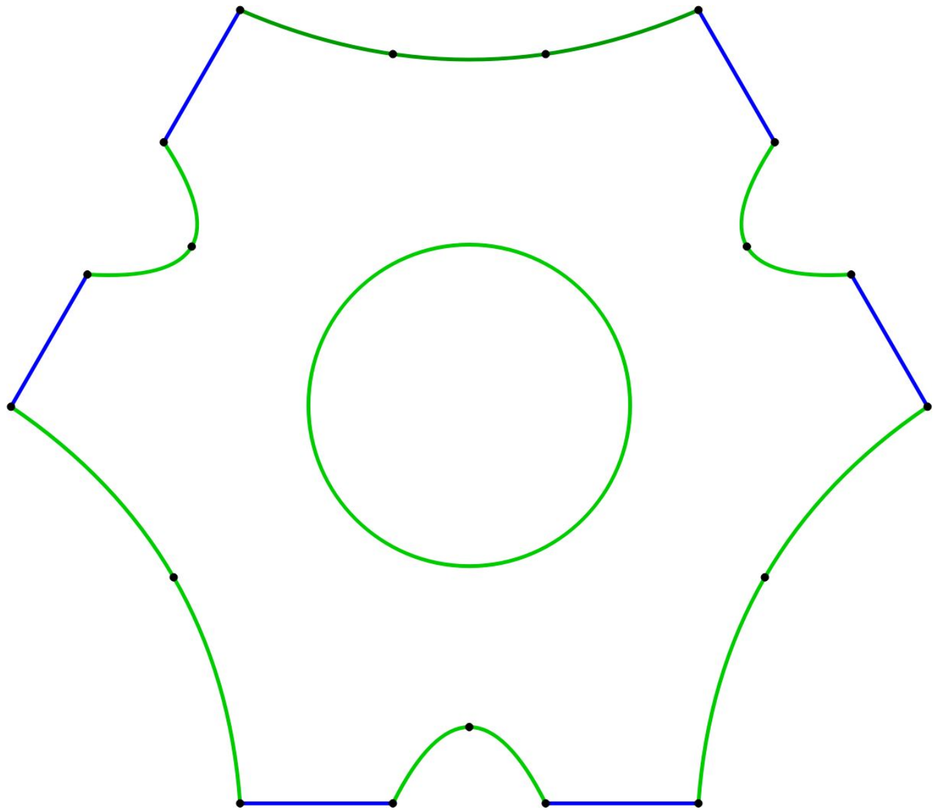
Processus de subdivision



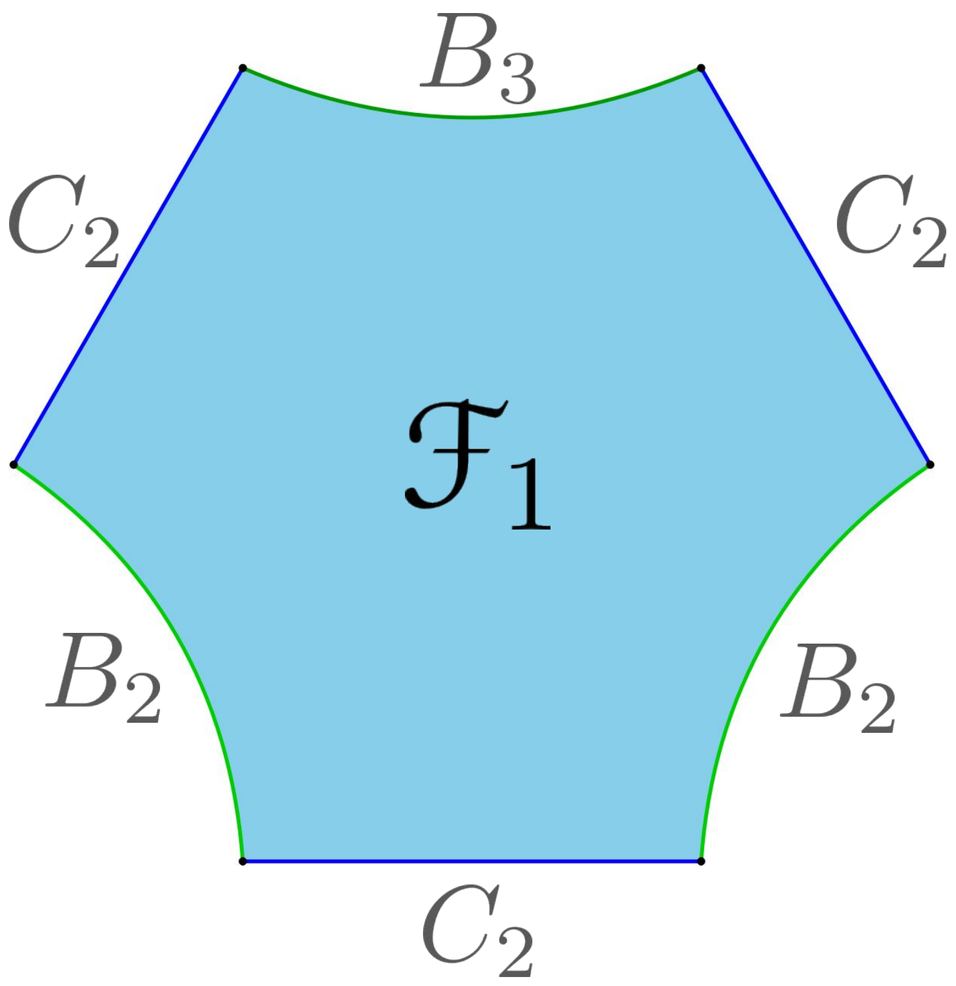


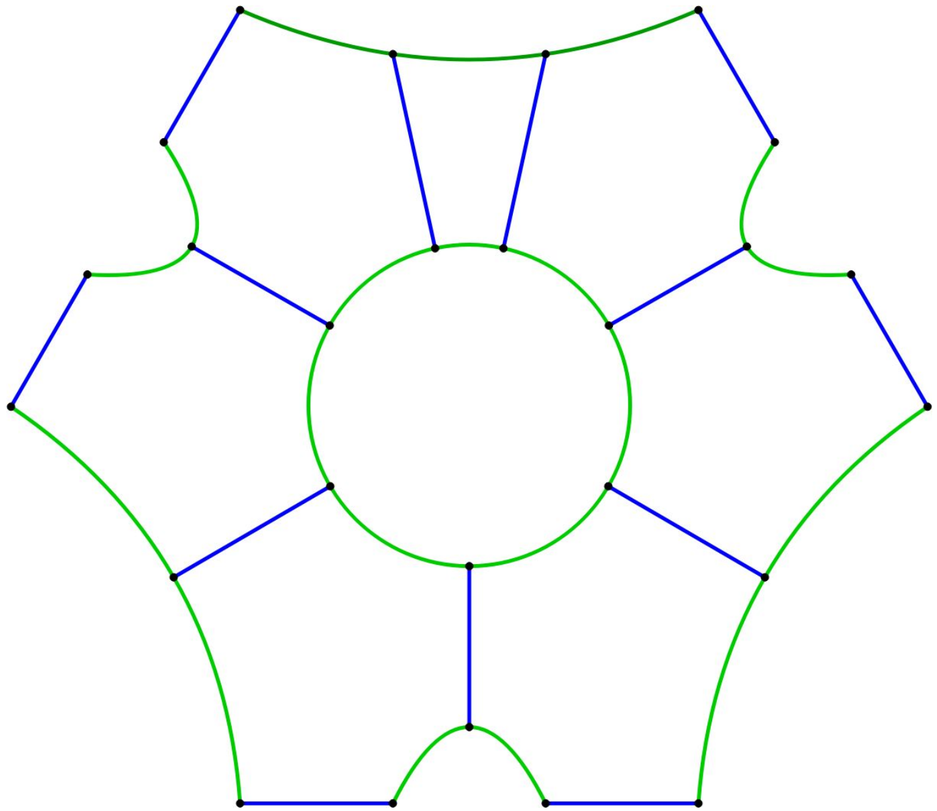
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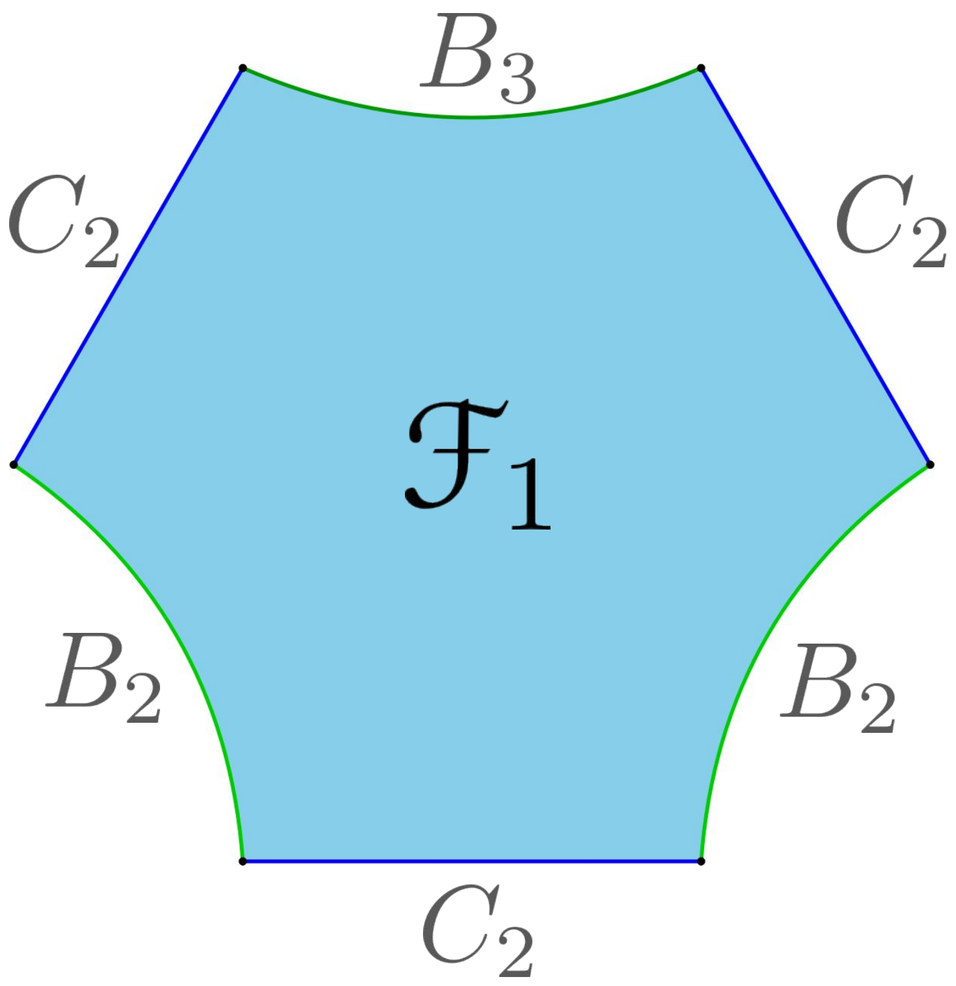


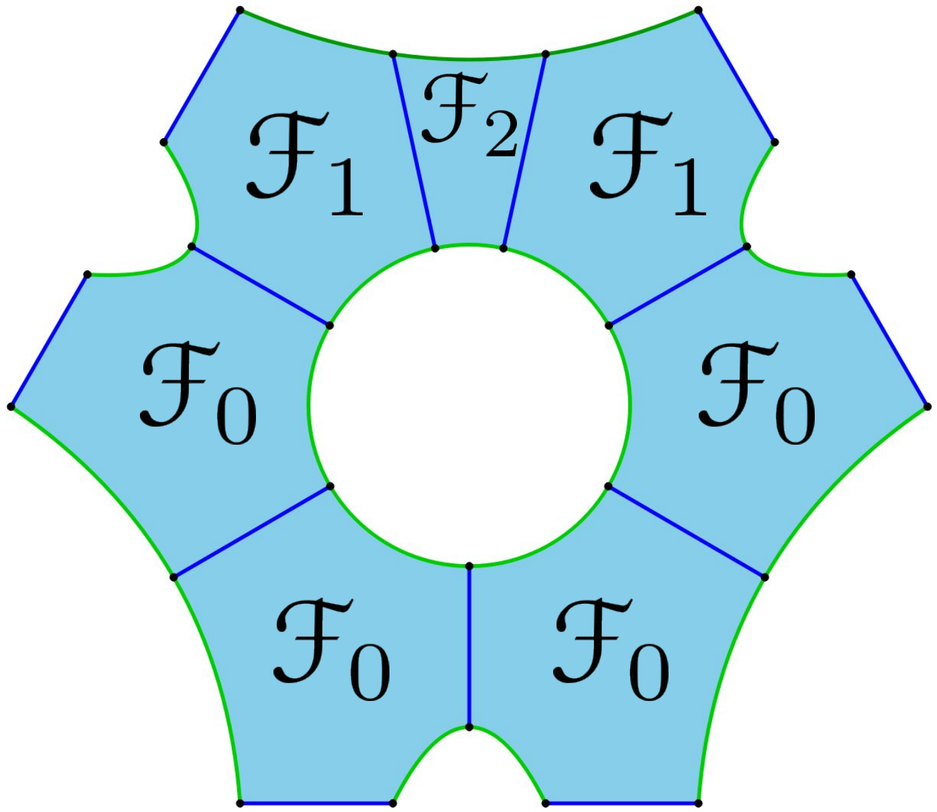
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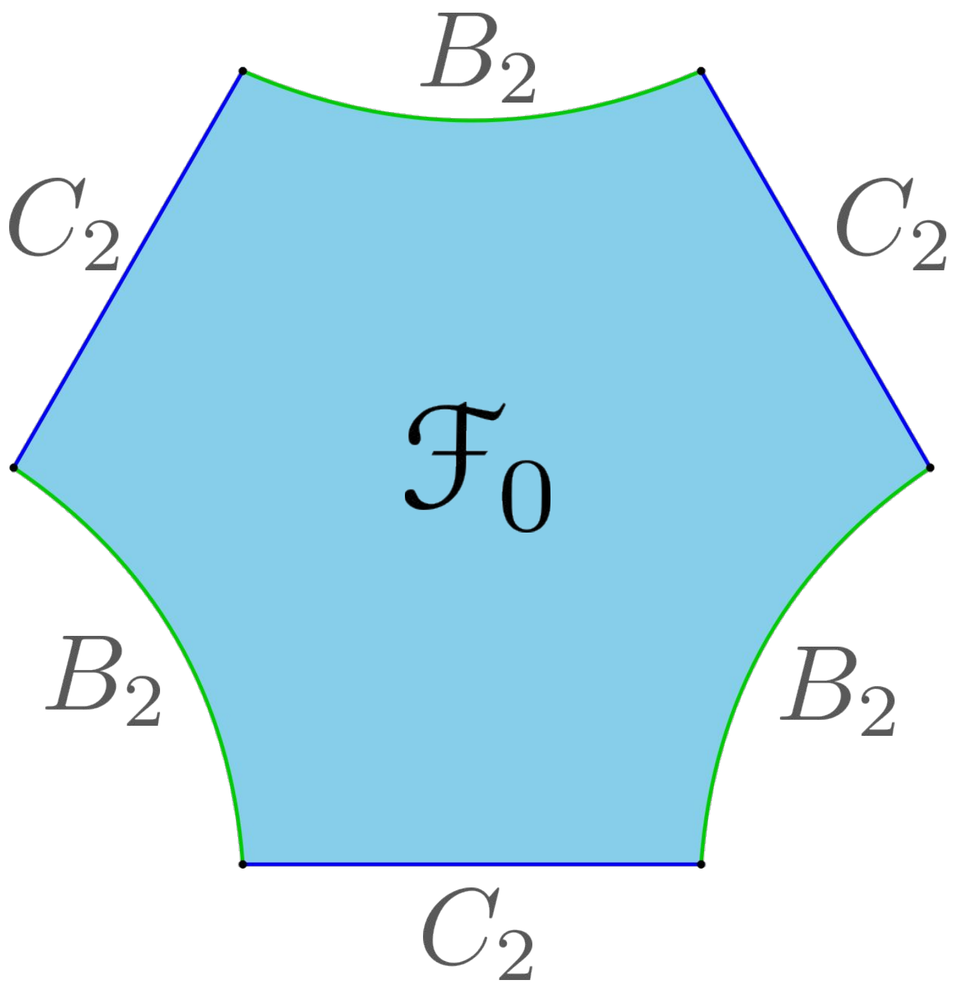


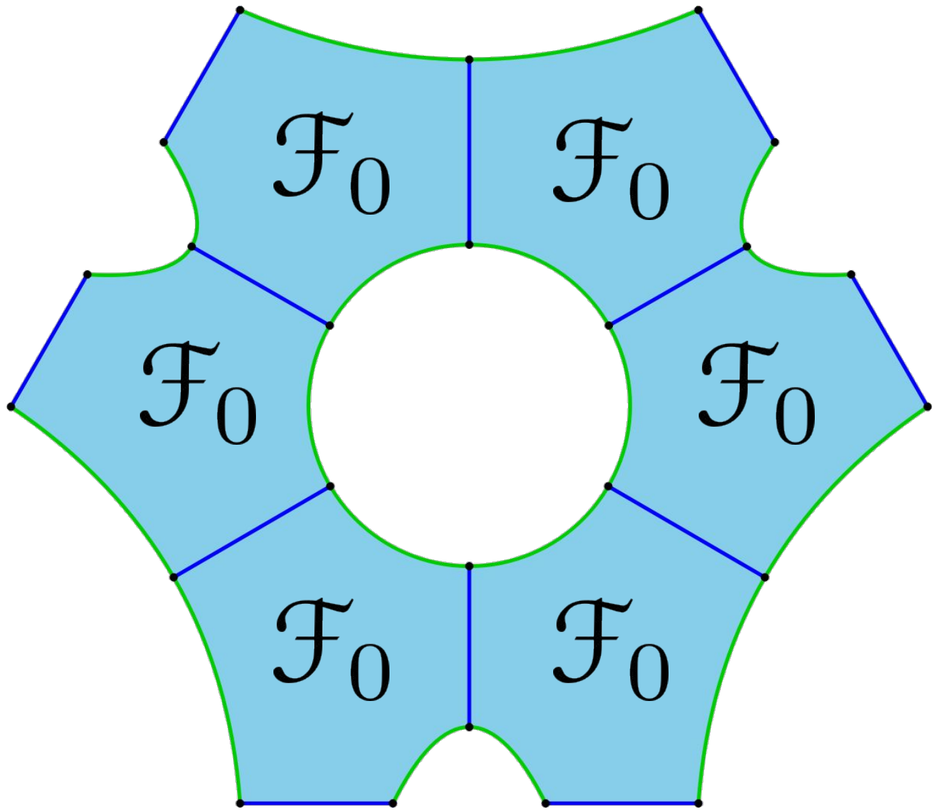
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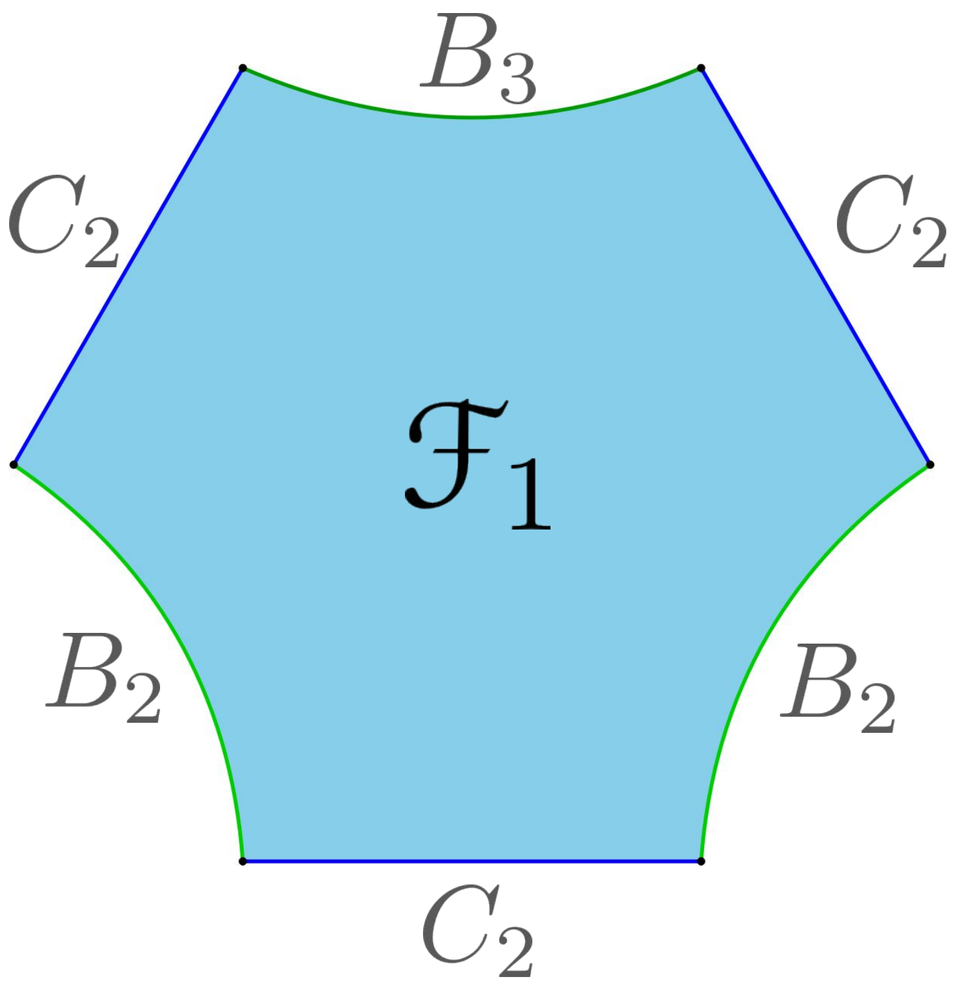


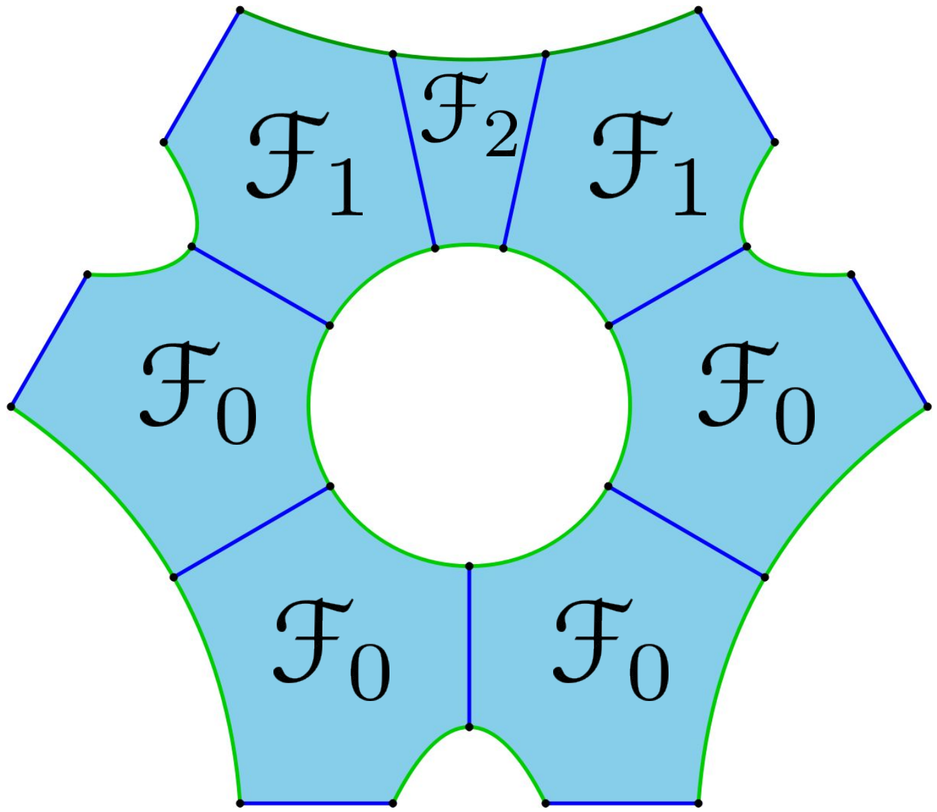
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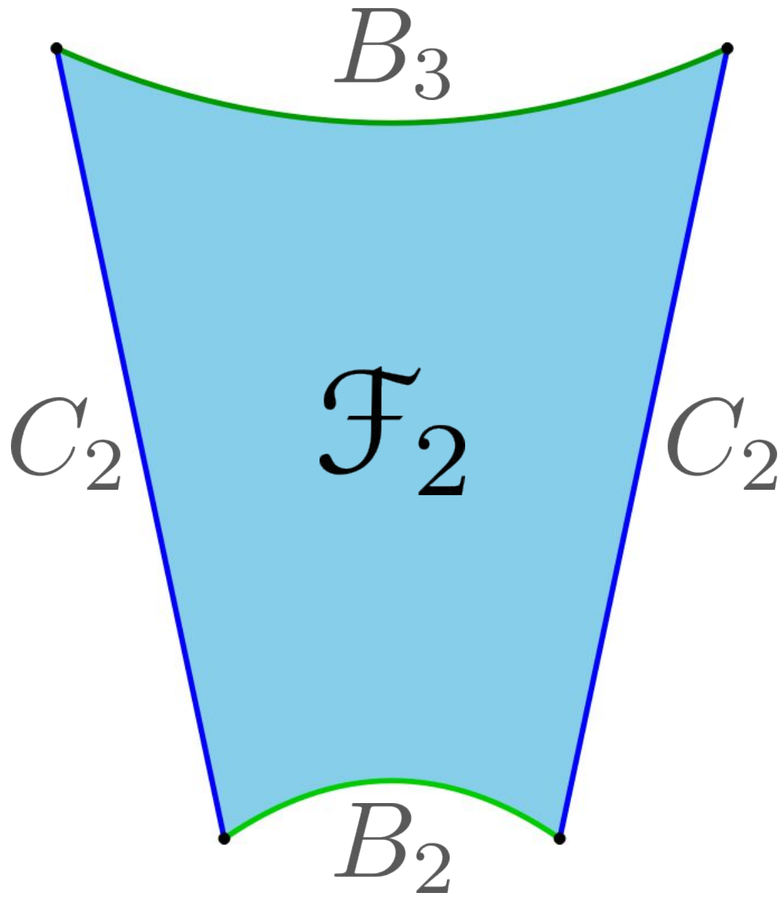


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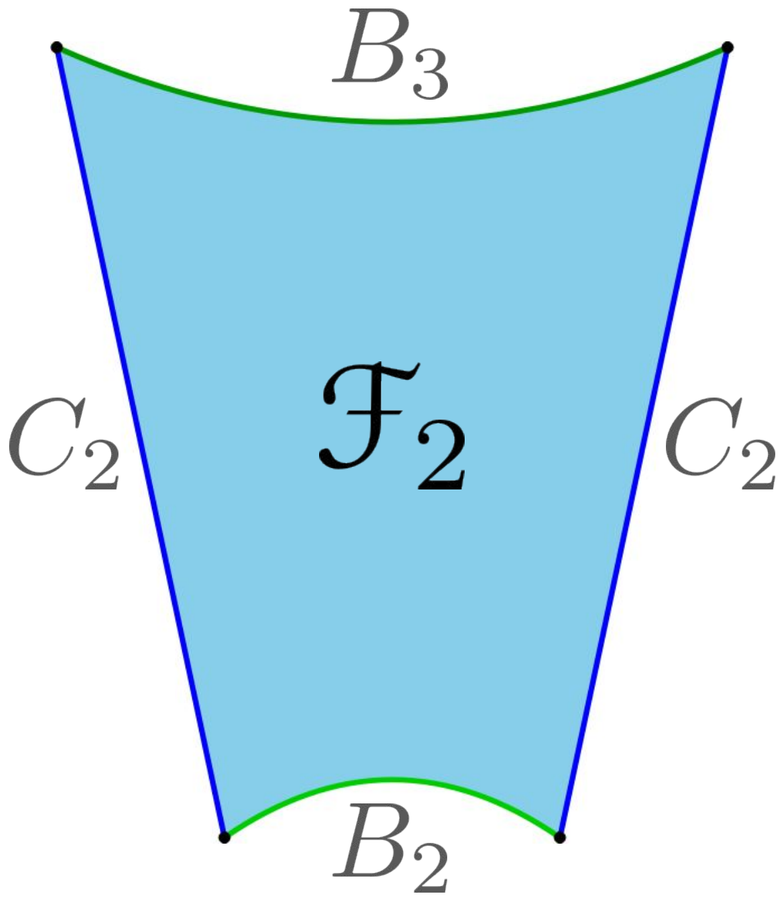


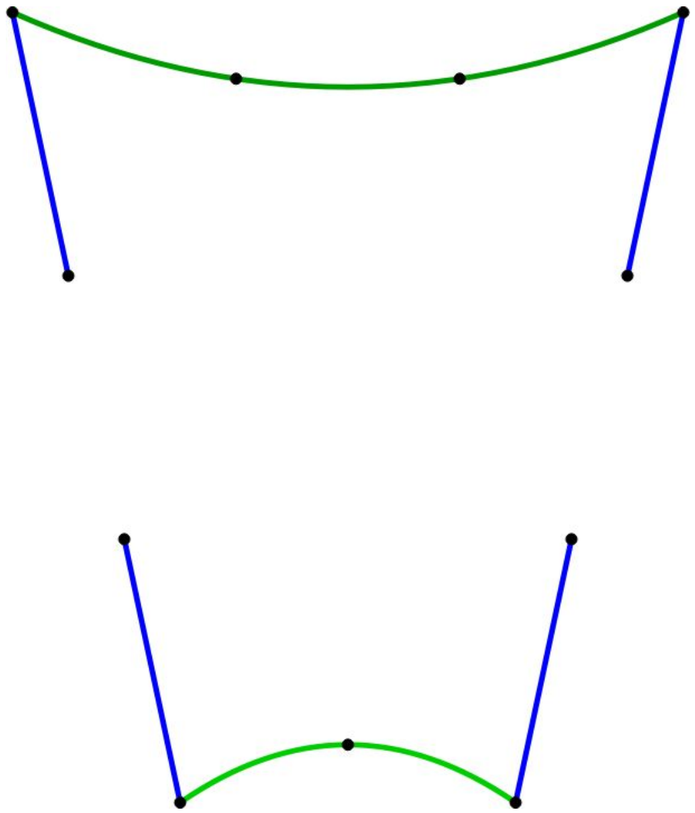


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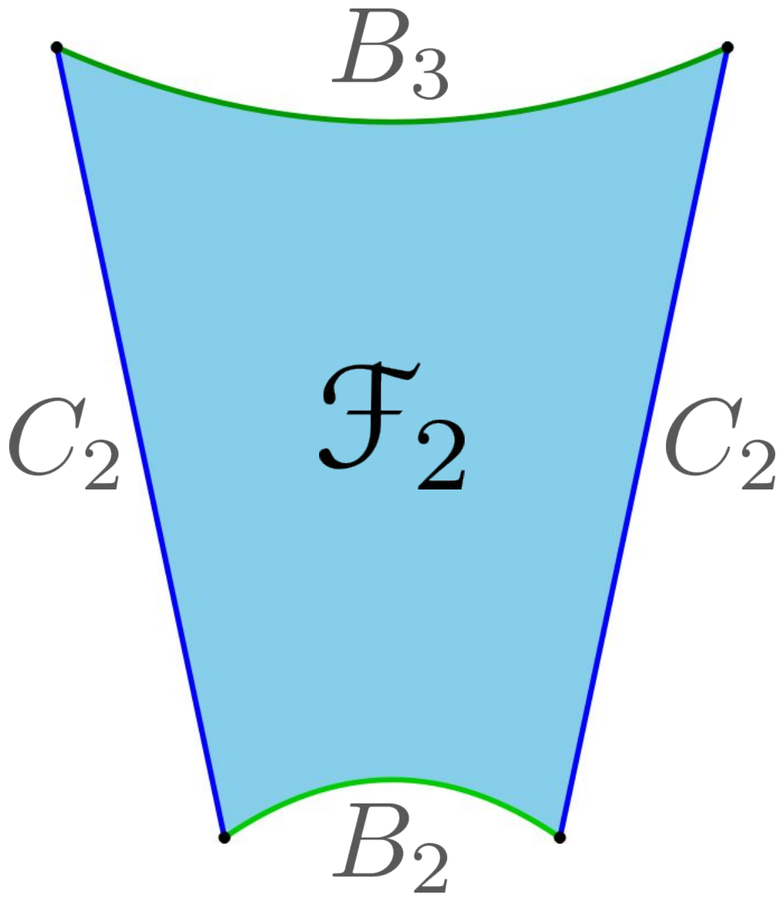


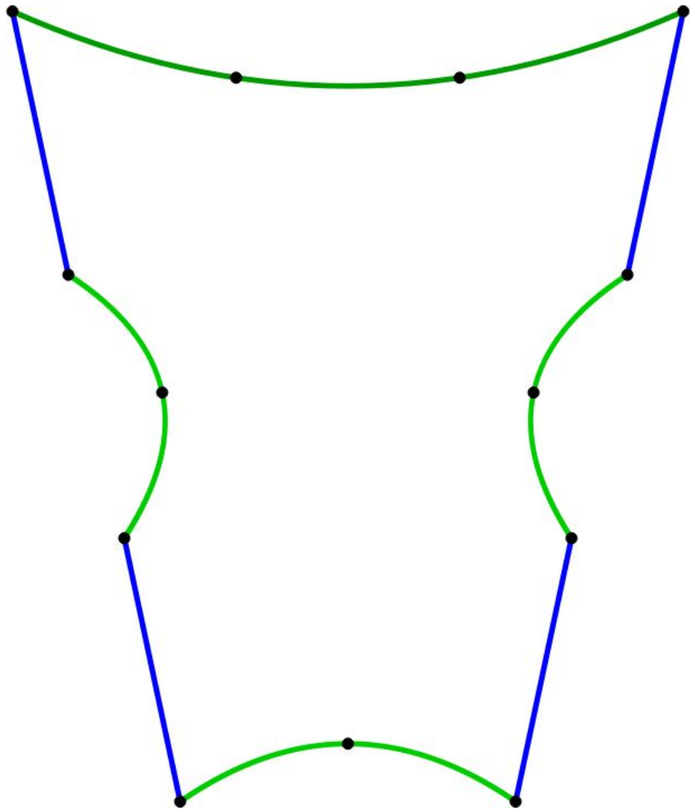
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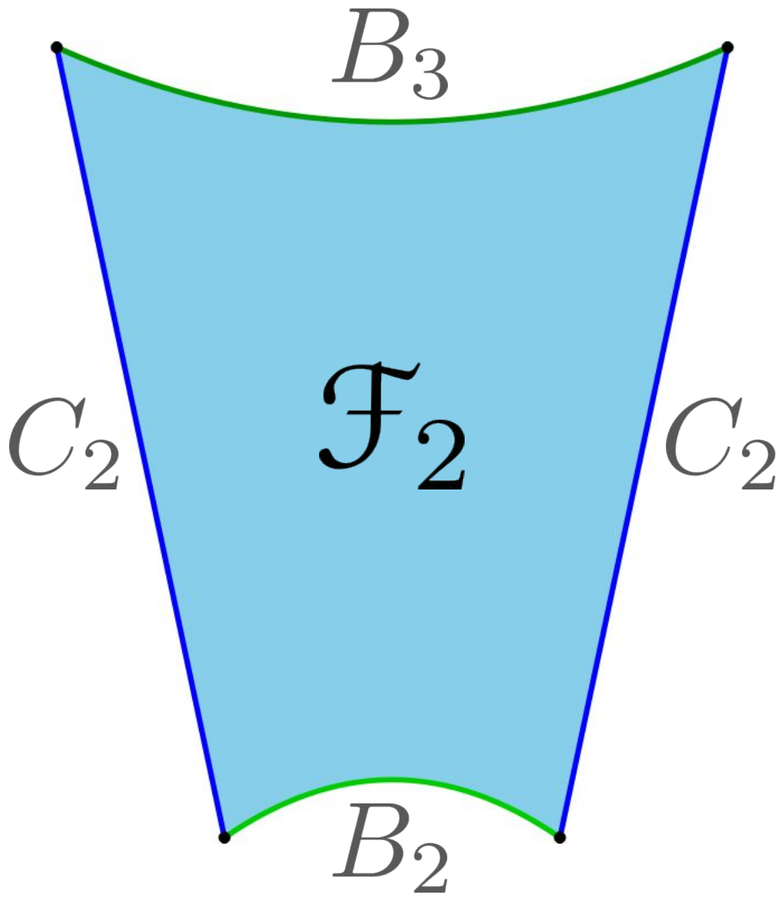


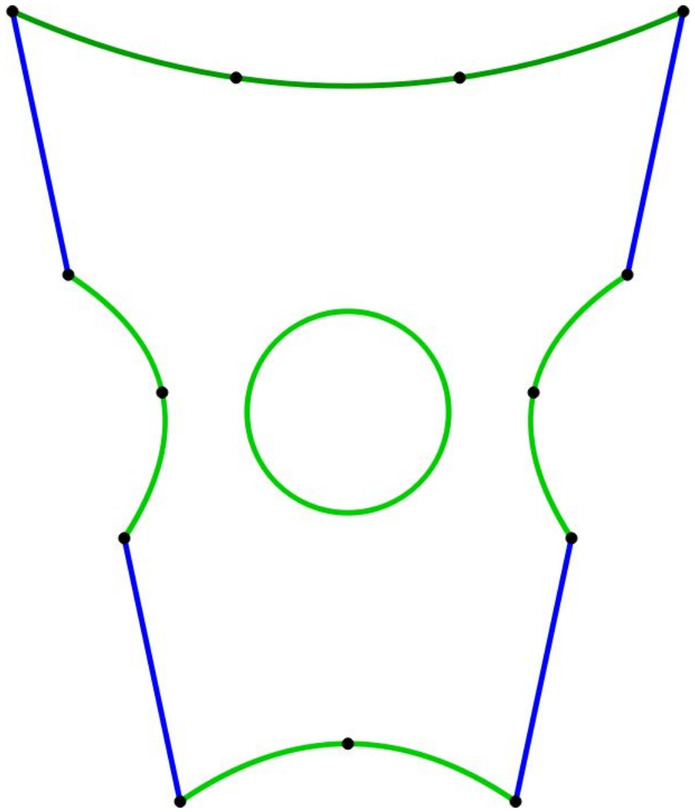
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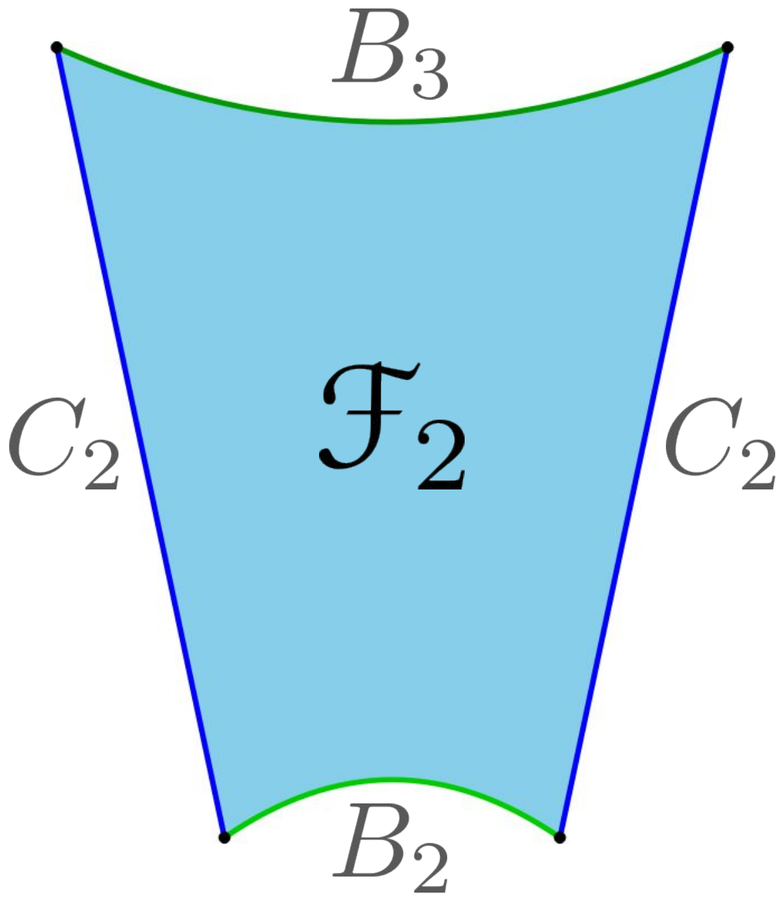


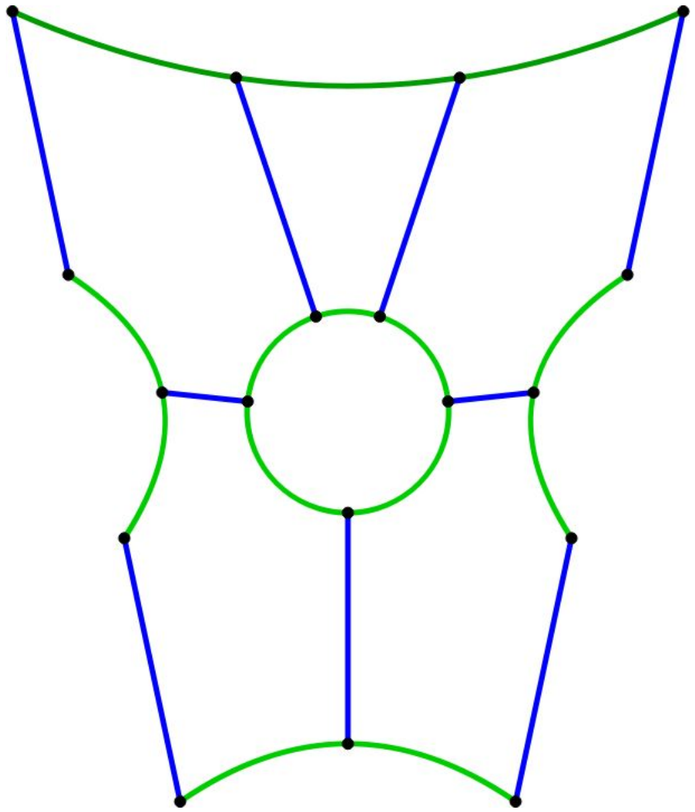
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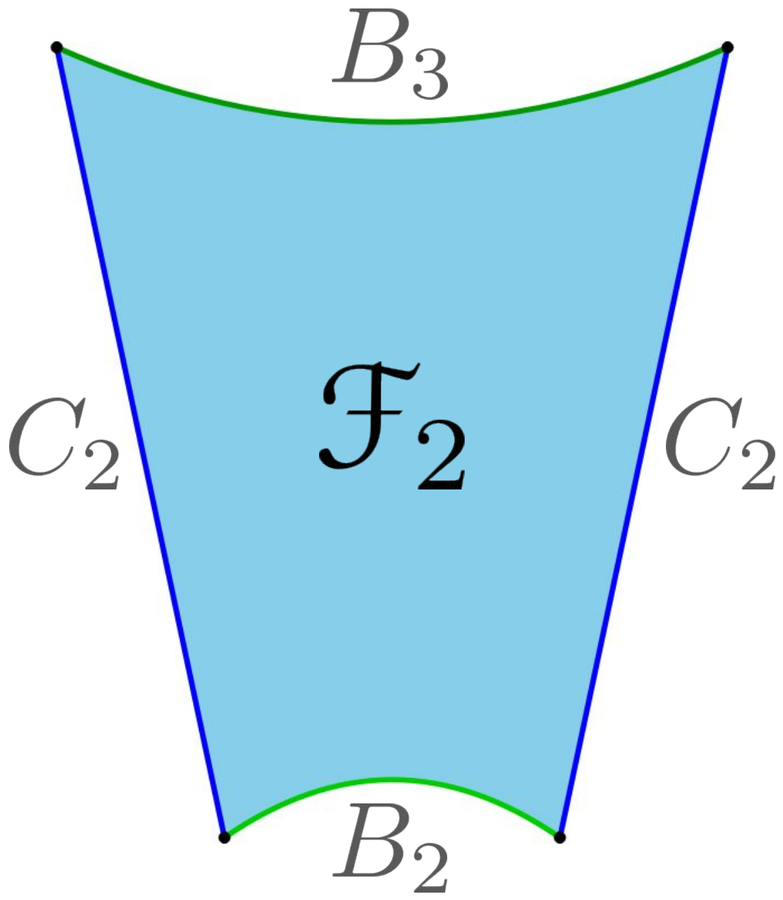


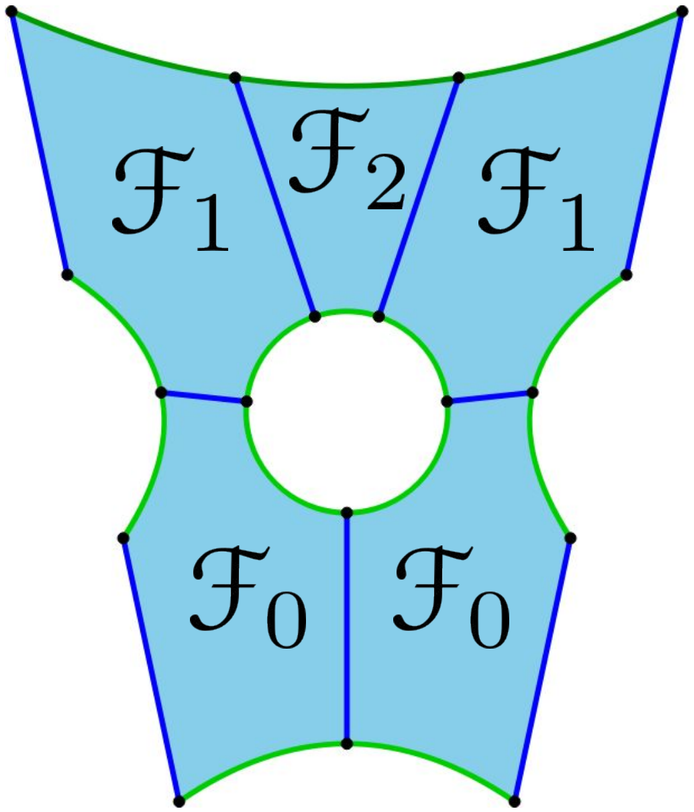
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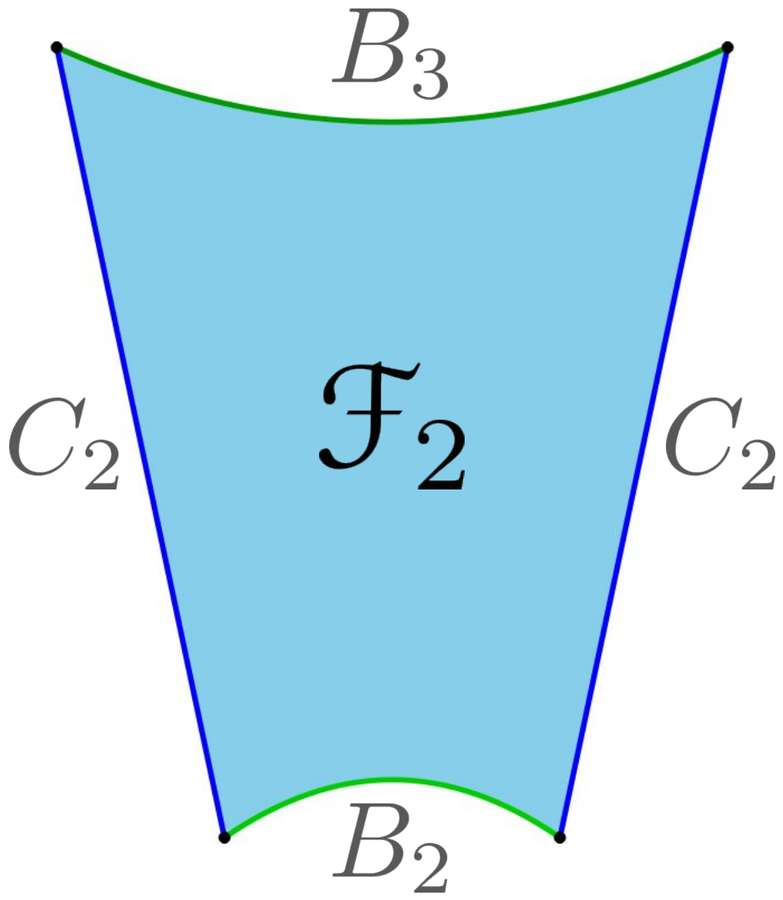


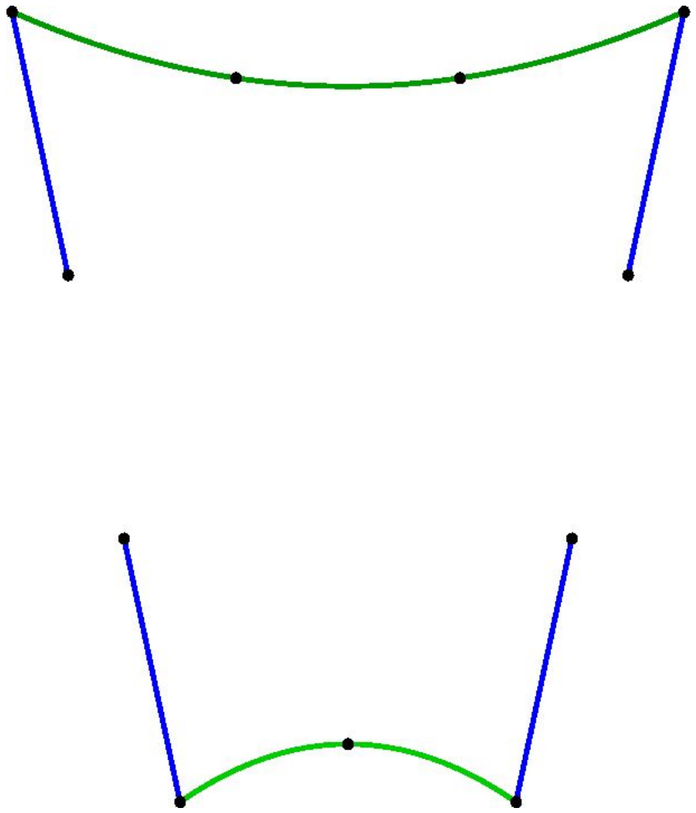
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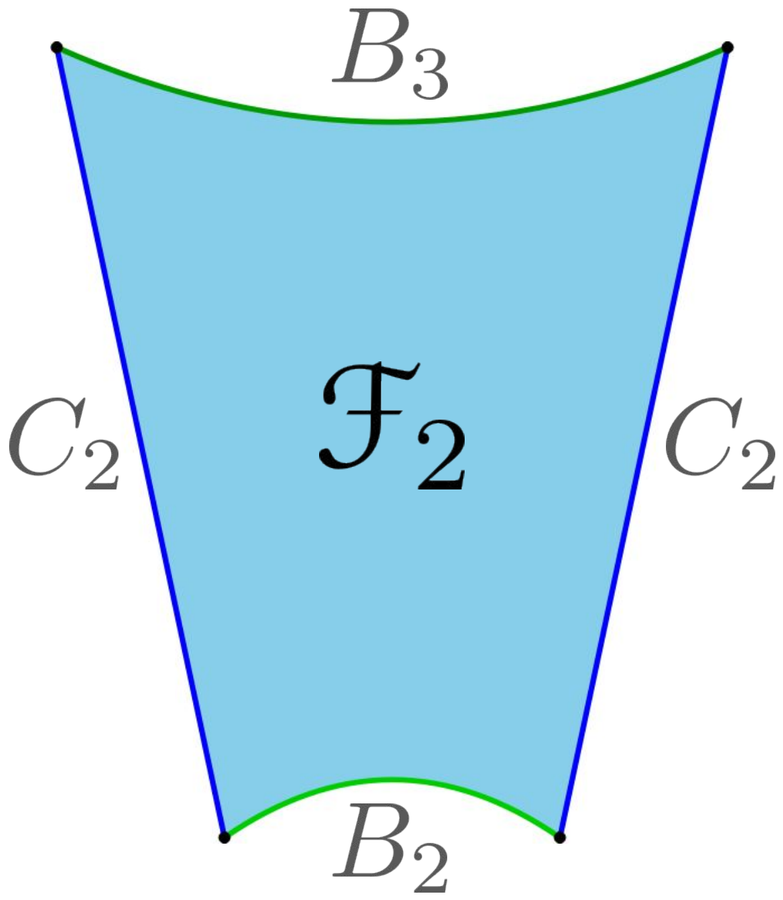


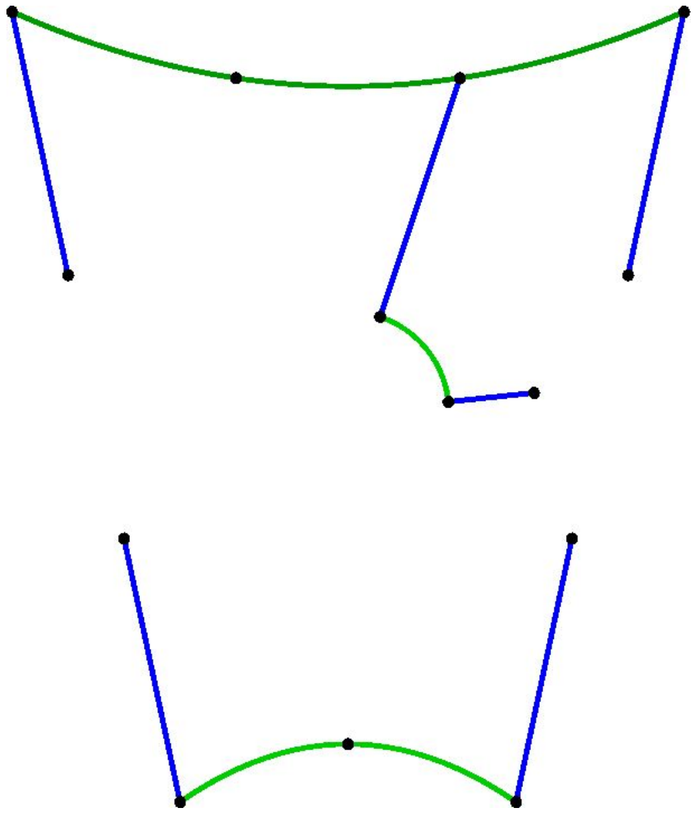
Algorithme de traduction en BC-IFS



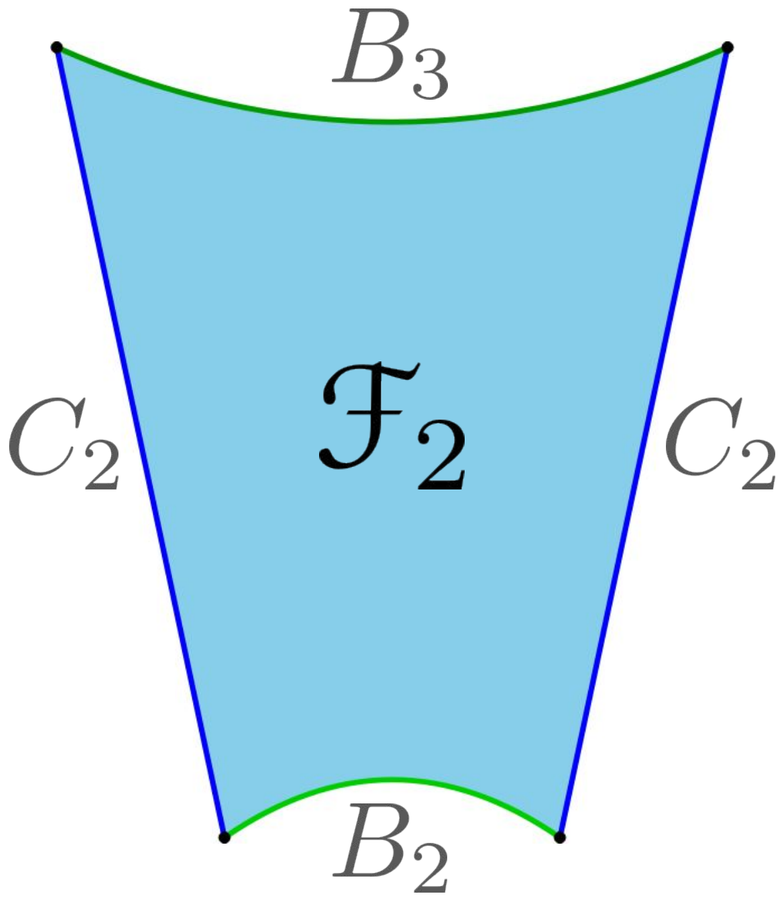


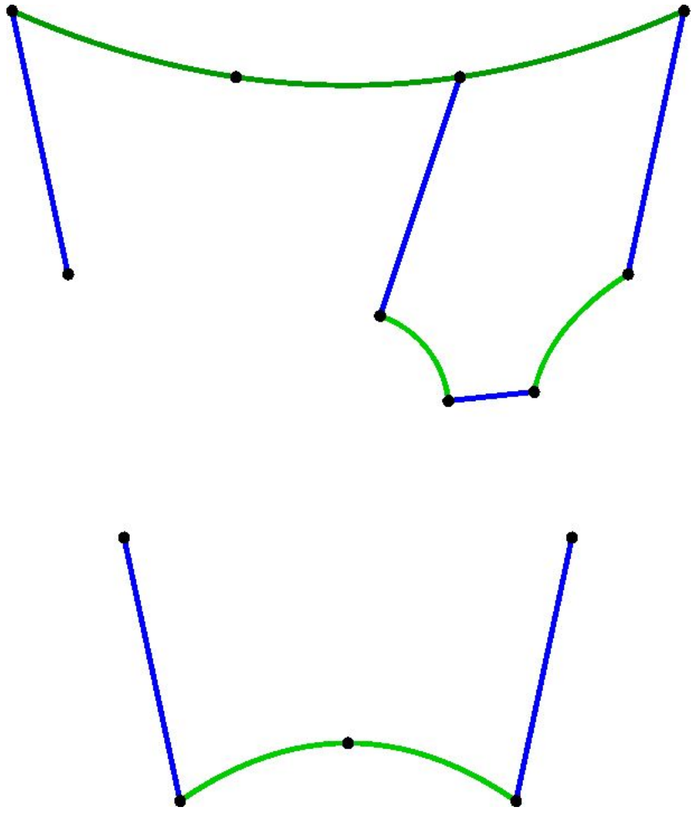
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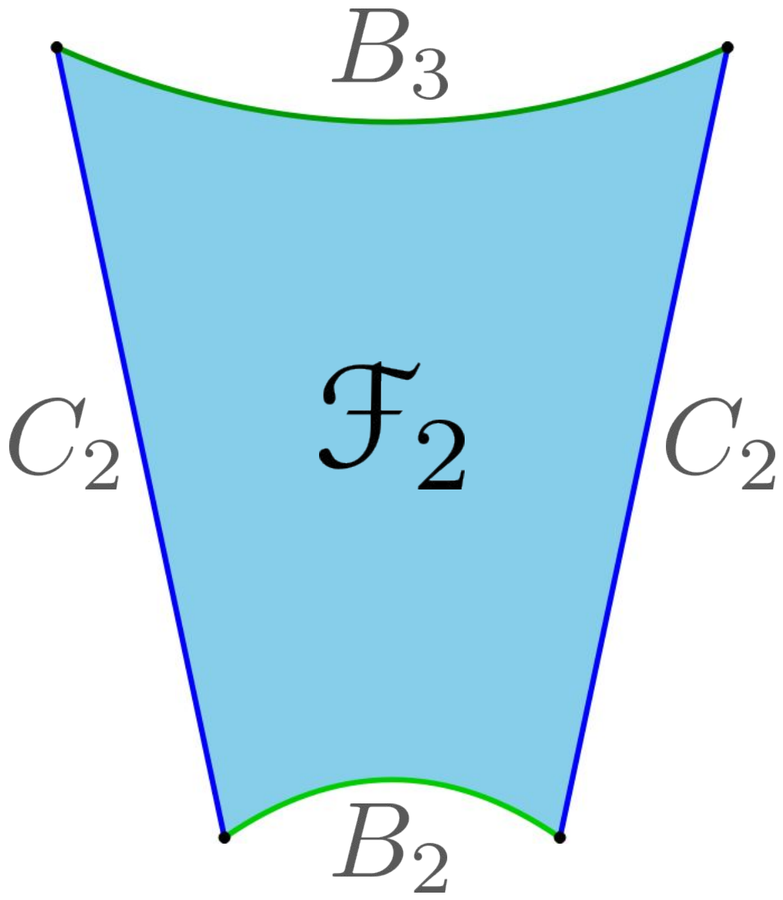


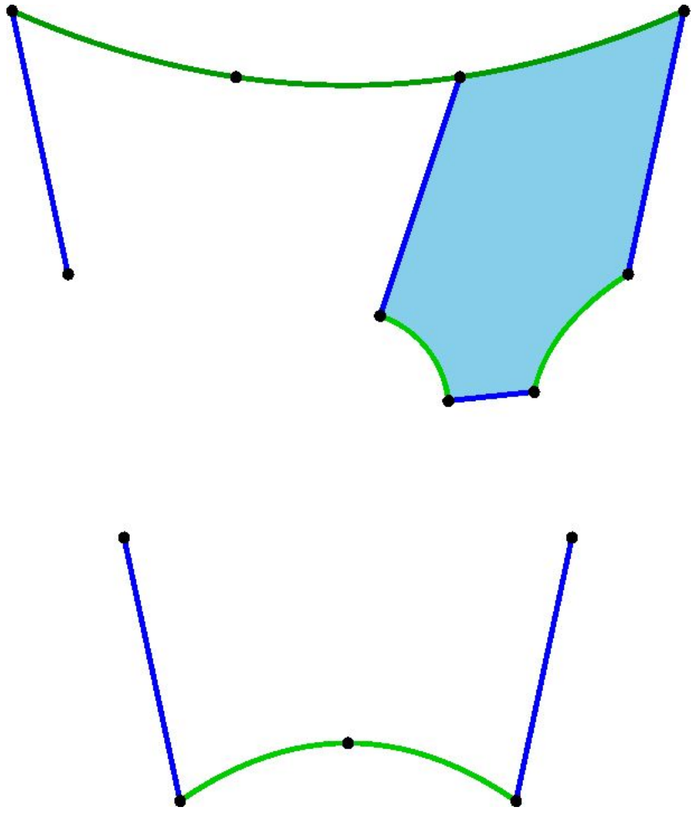
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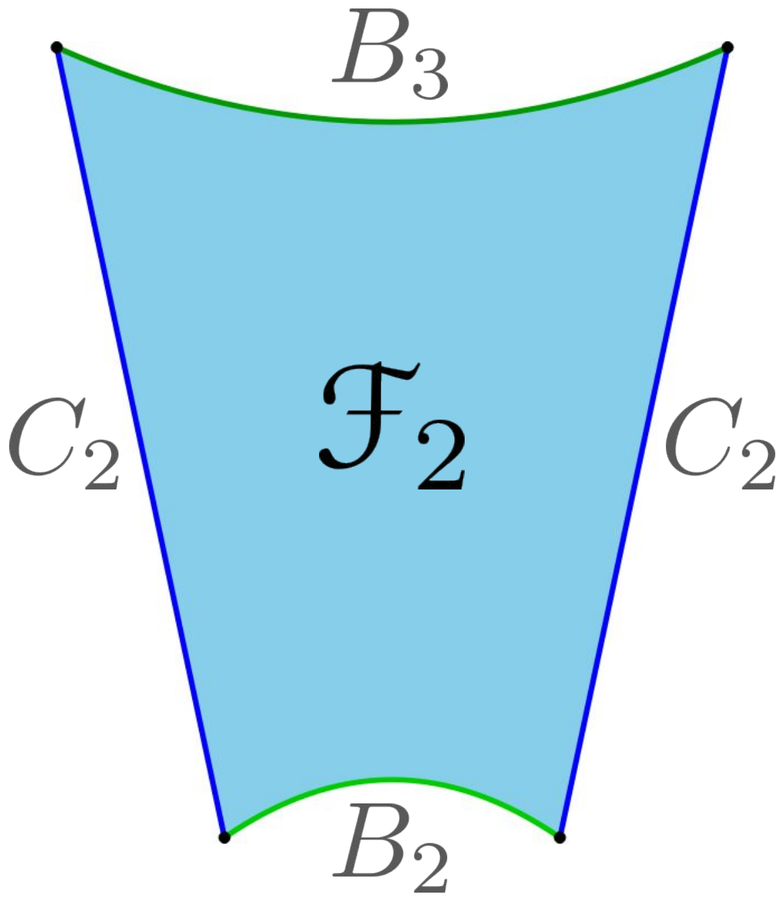


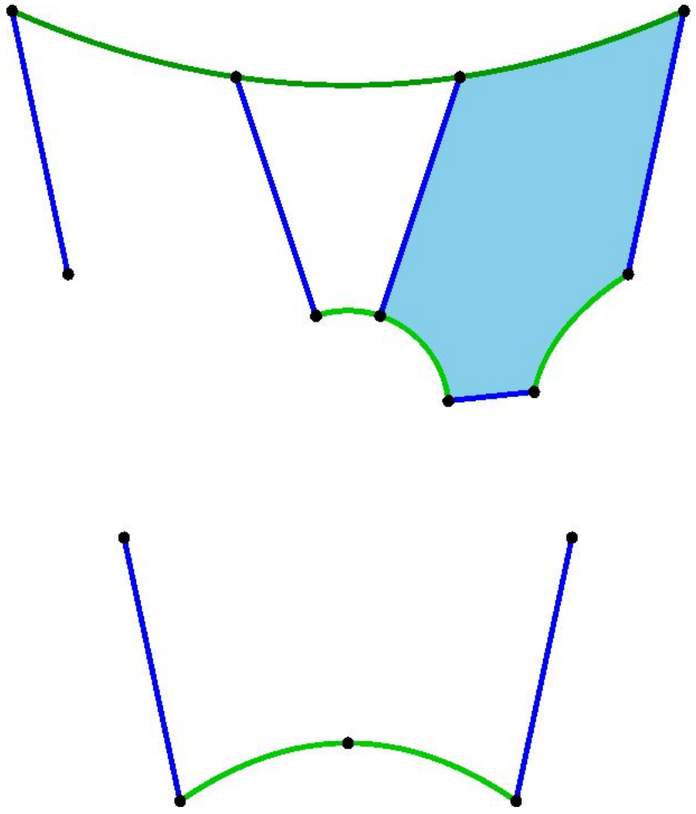
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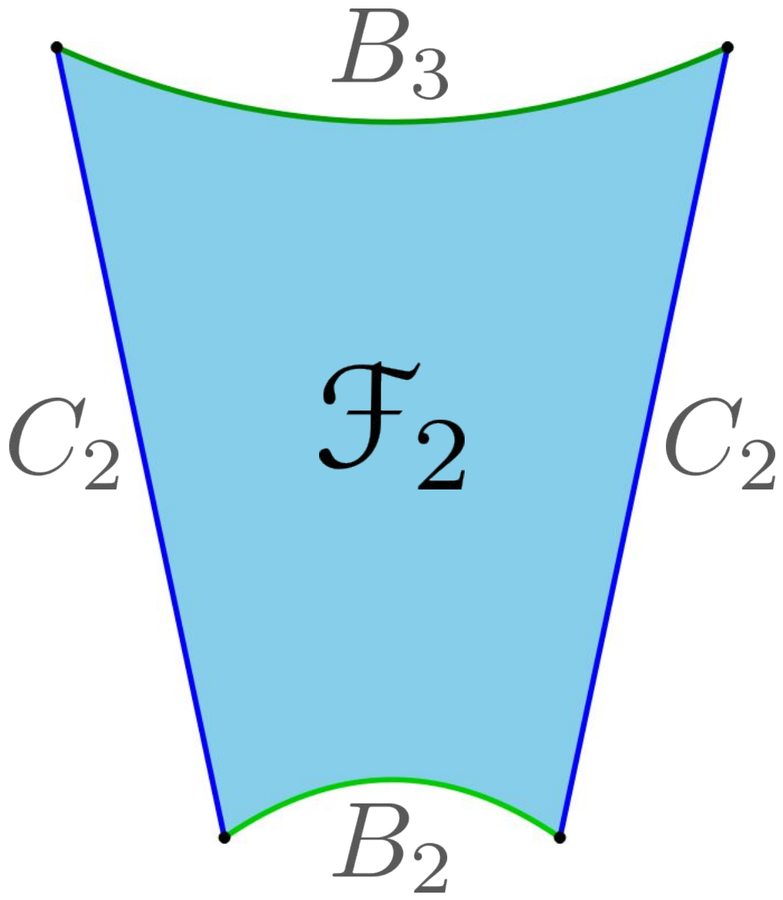


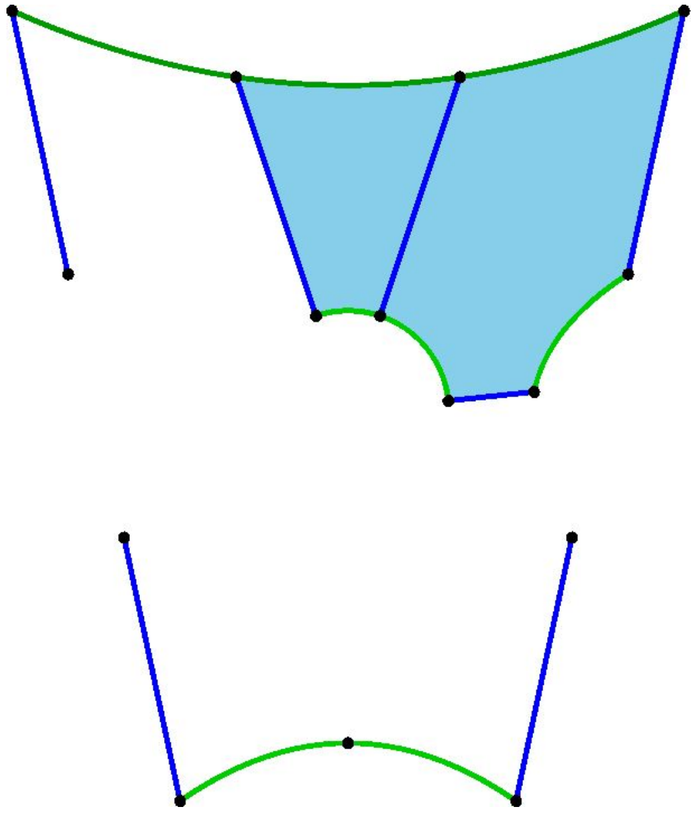
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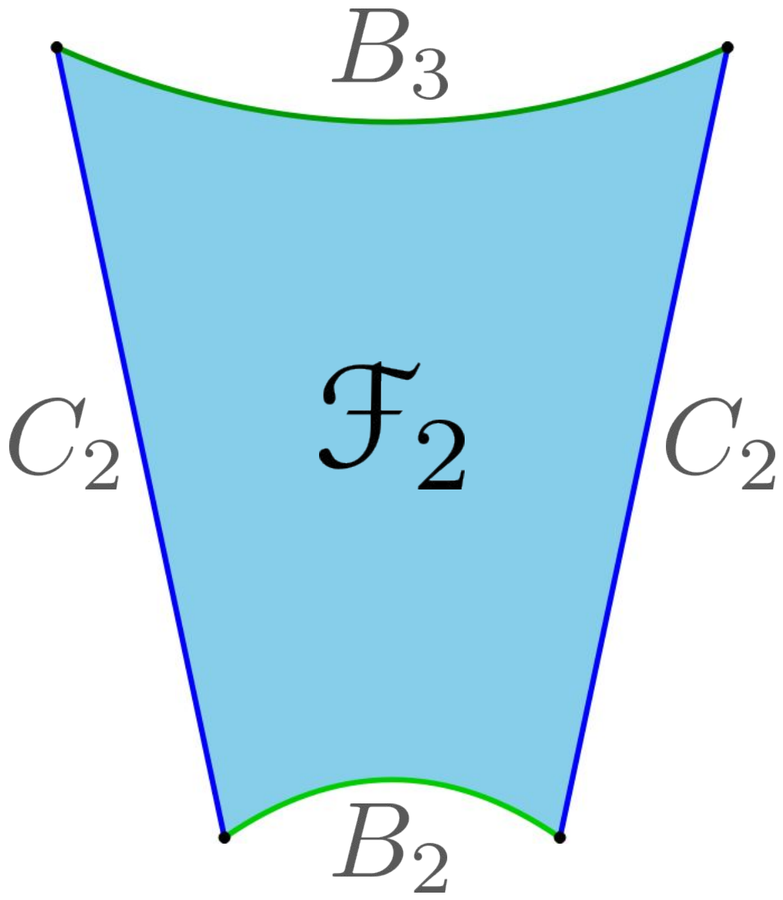


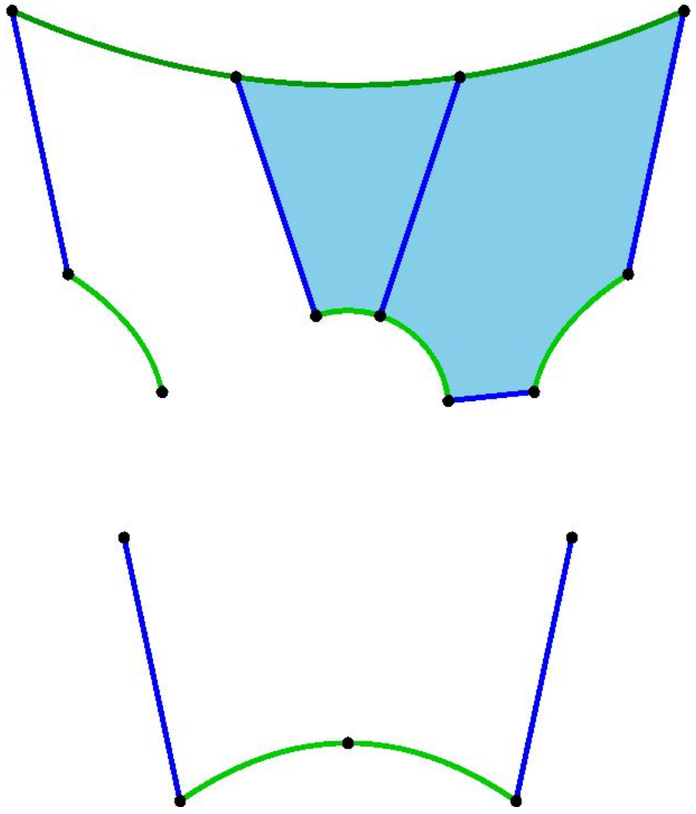
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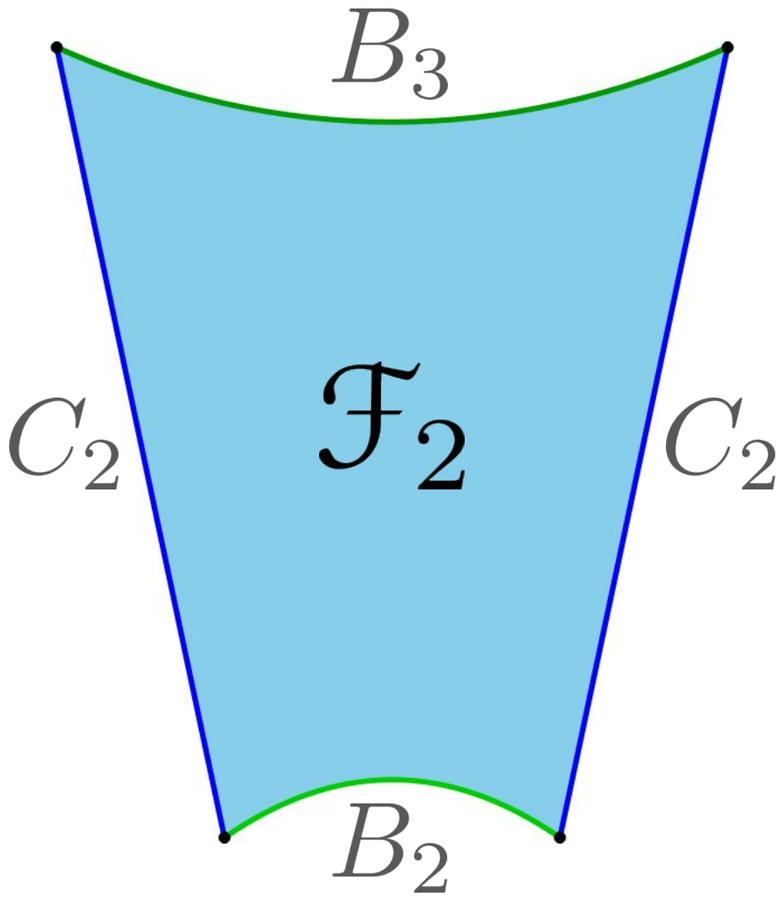


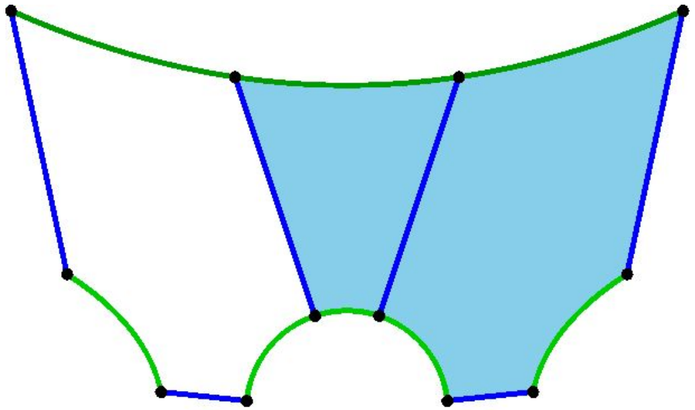
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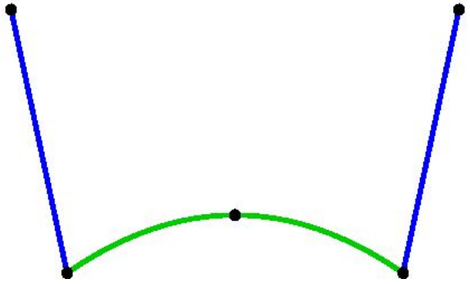




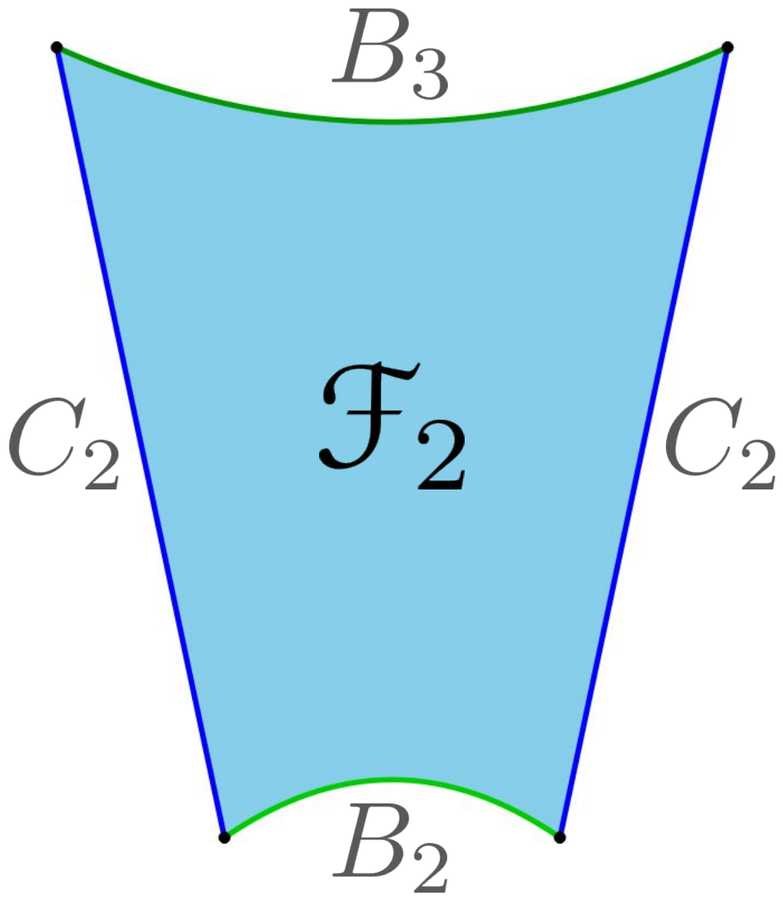
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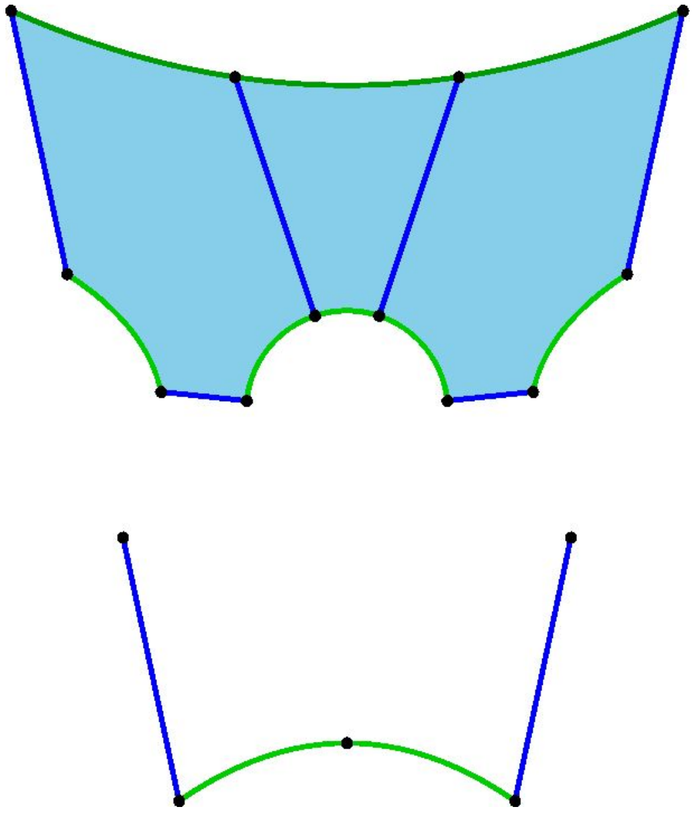




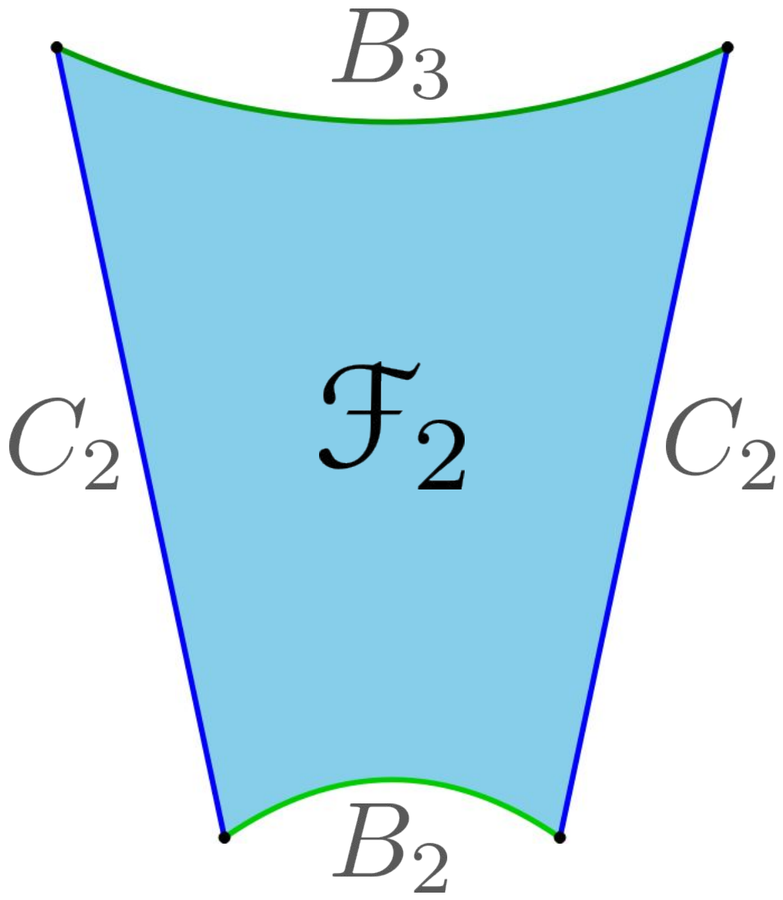


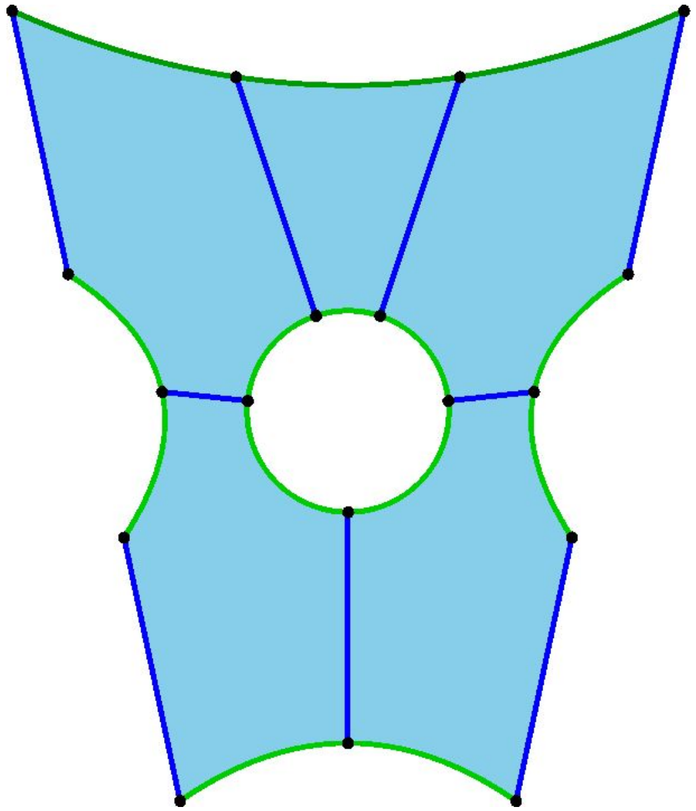
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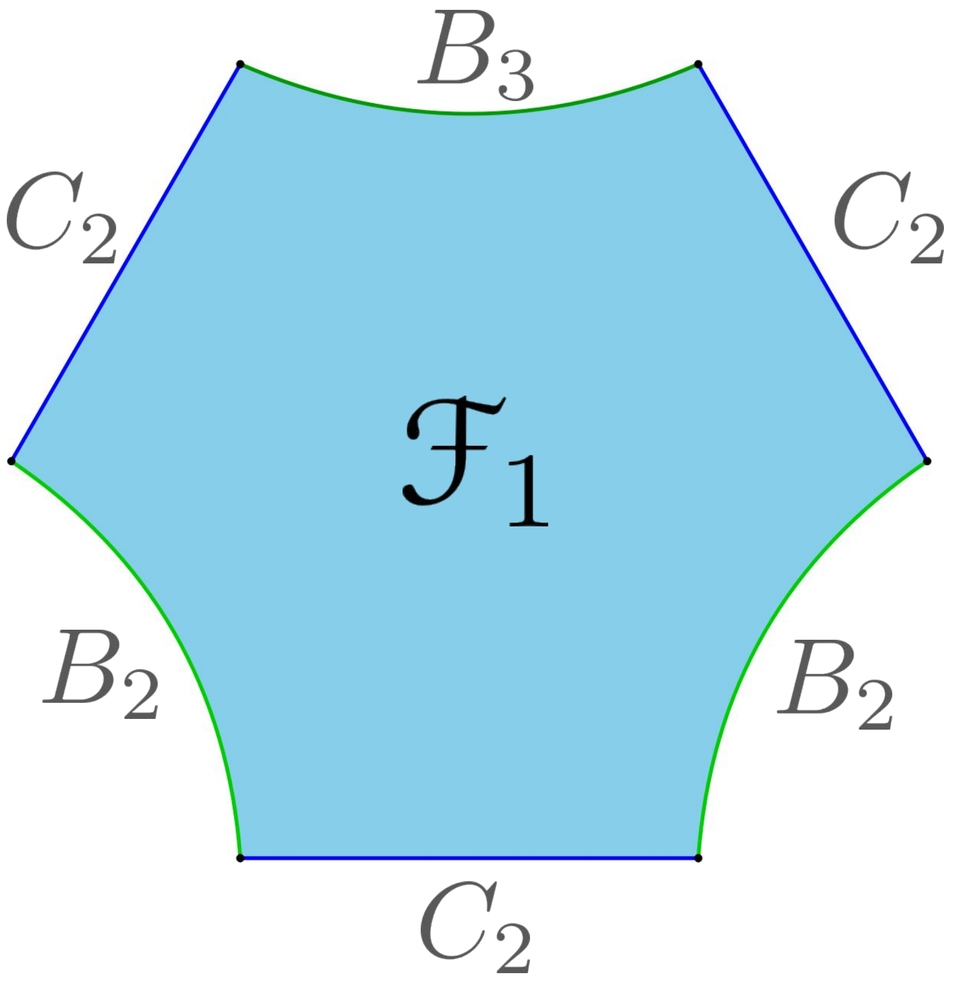


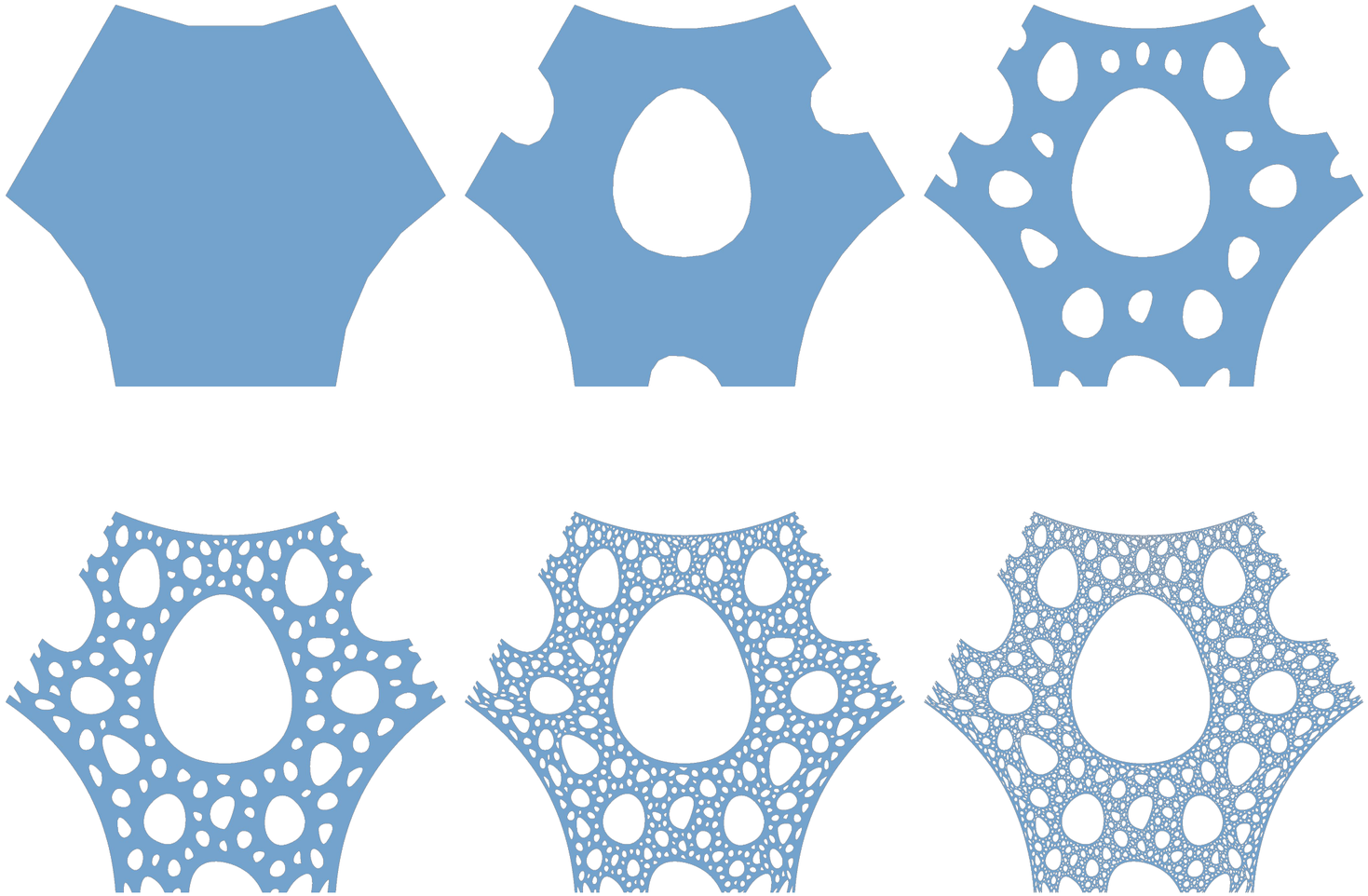
Algorithme de traduction en BC-IFS





Processus de subdivision



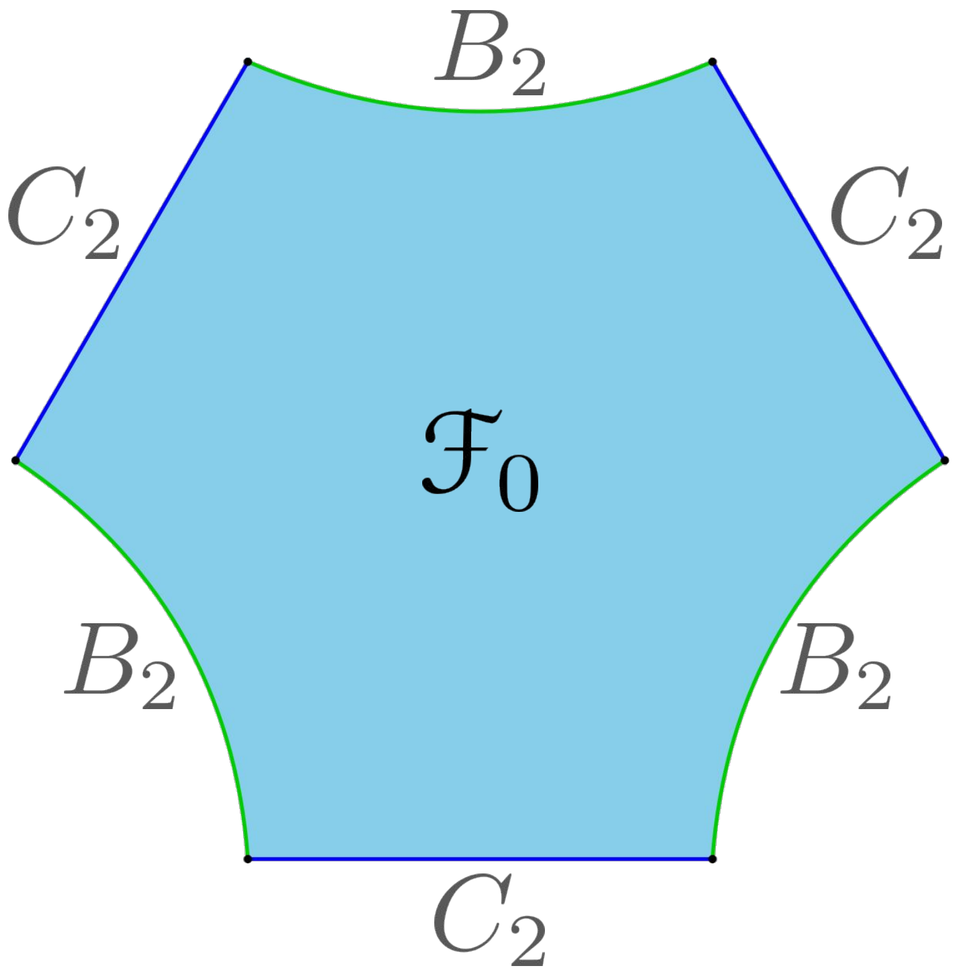


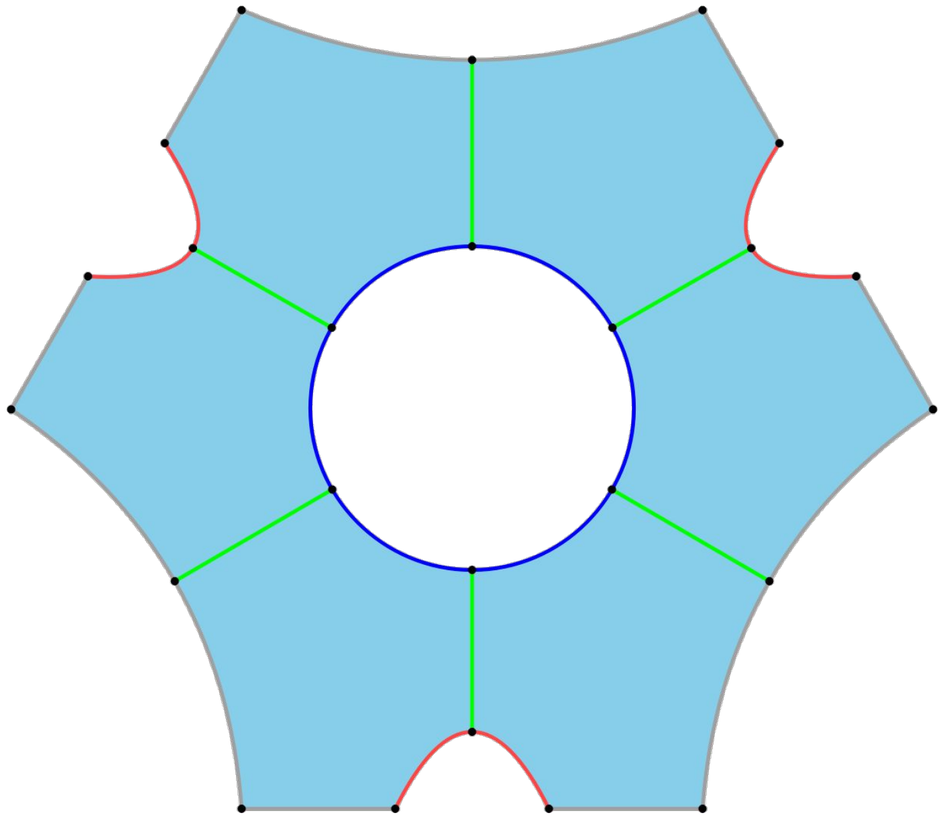
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Contrôle local de la lacunarité

Paramètre $\mathcal{E}$

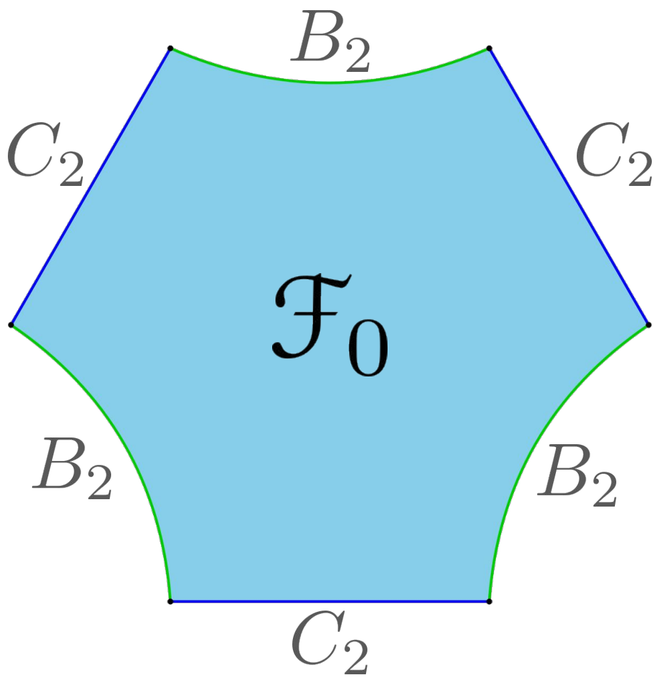
- $\mathcal{E}_a$ (vert)
- $\mathcal{E}_l$ (bleu)
- $\mathcal{E}_c$ (rouge)

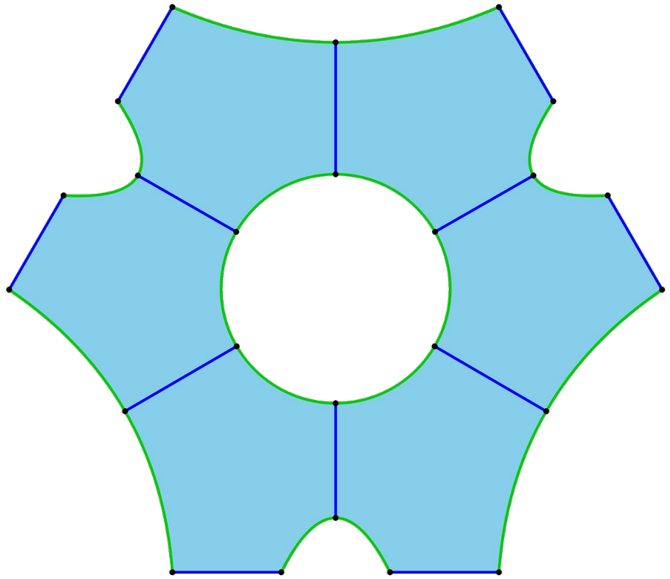


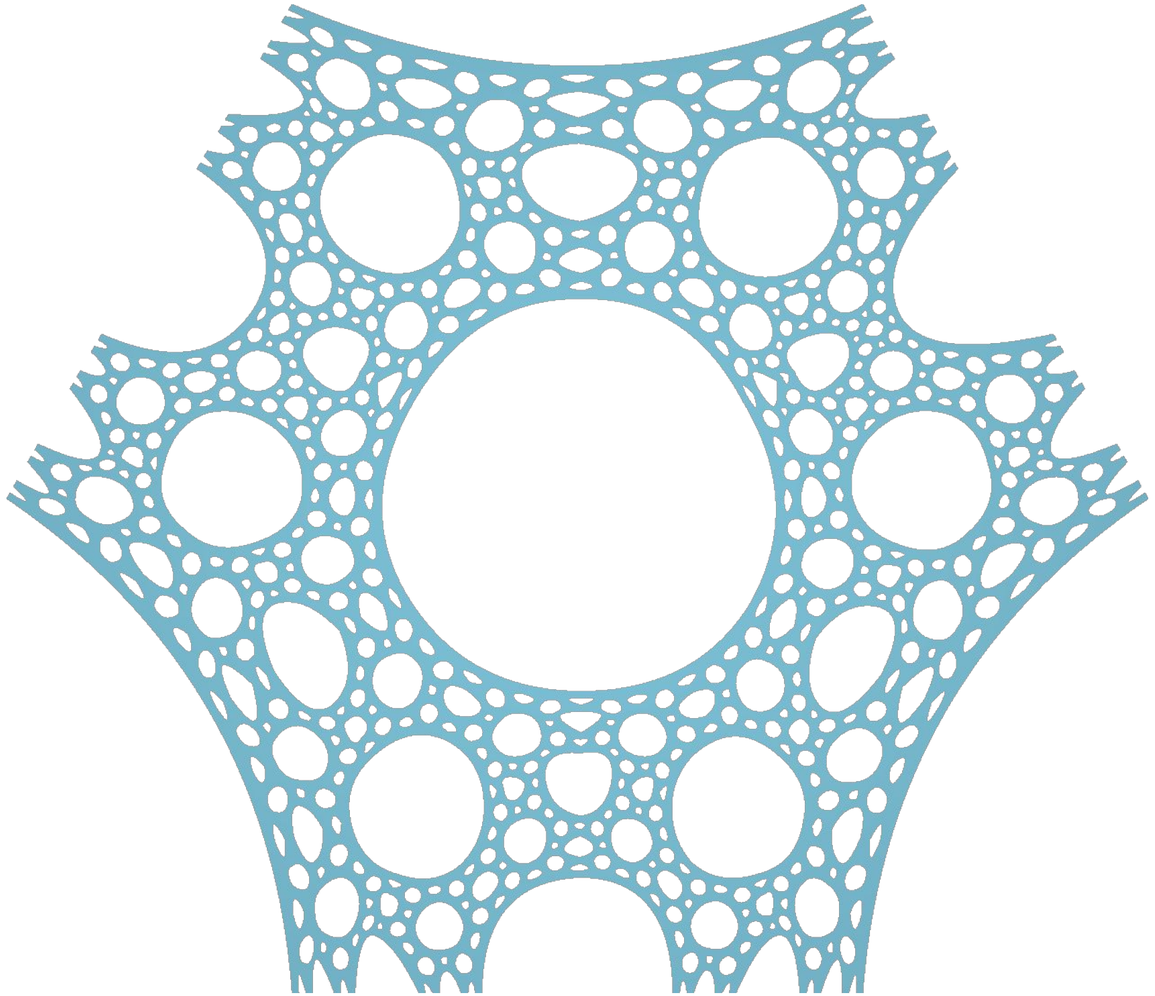


Paramètre $\mathcal{E}$

- $\mathcal{E}_a = C_2$
- $\mathcal{E}_l = B_2$
- $\mathcal{E}_c = B_2$

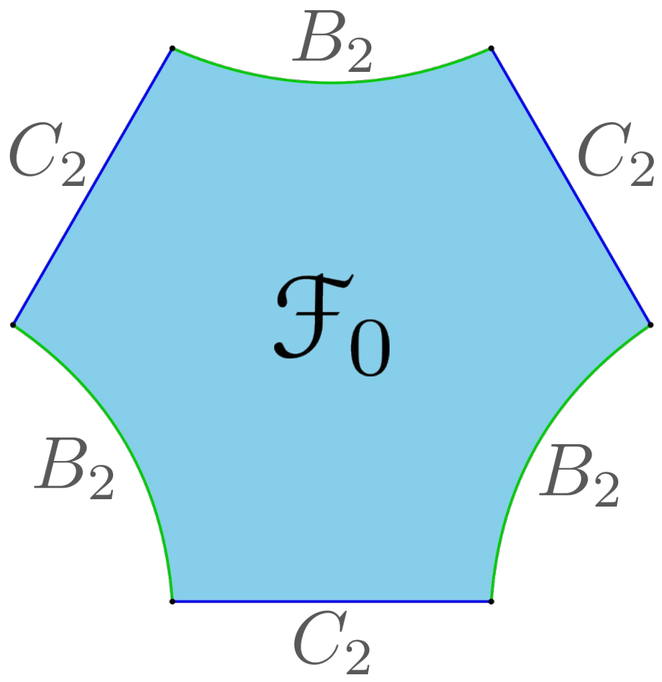


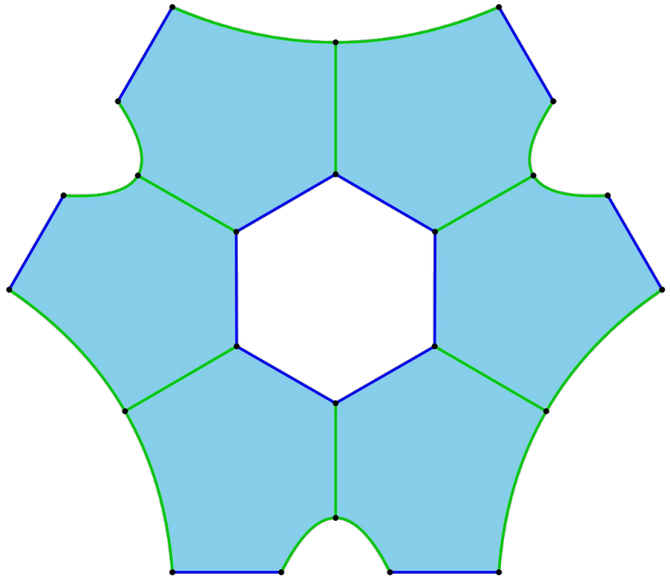


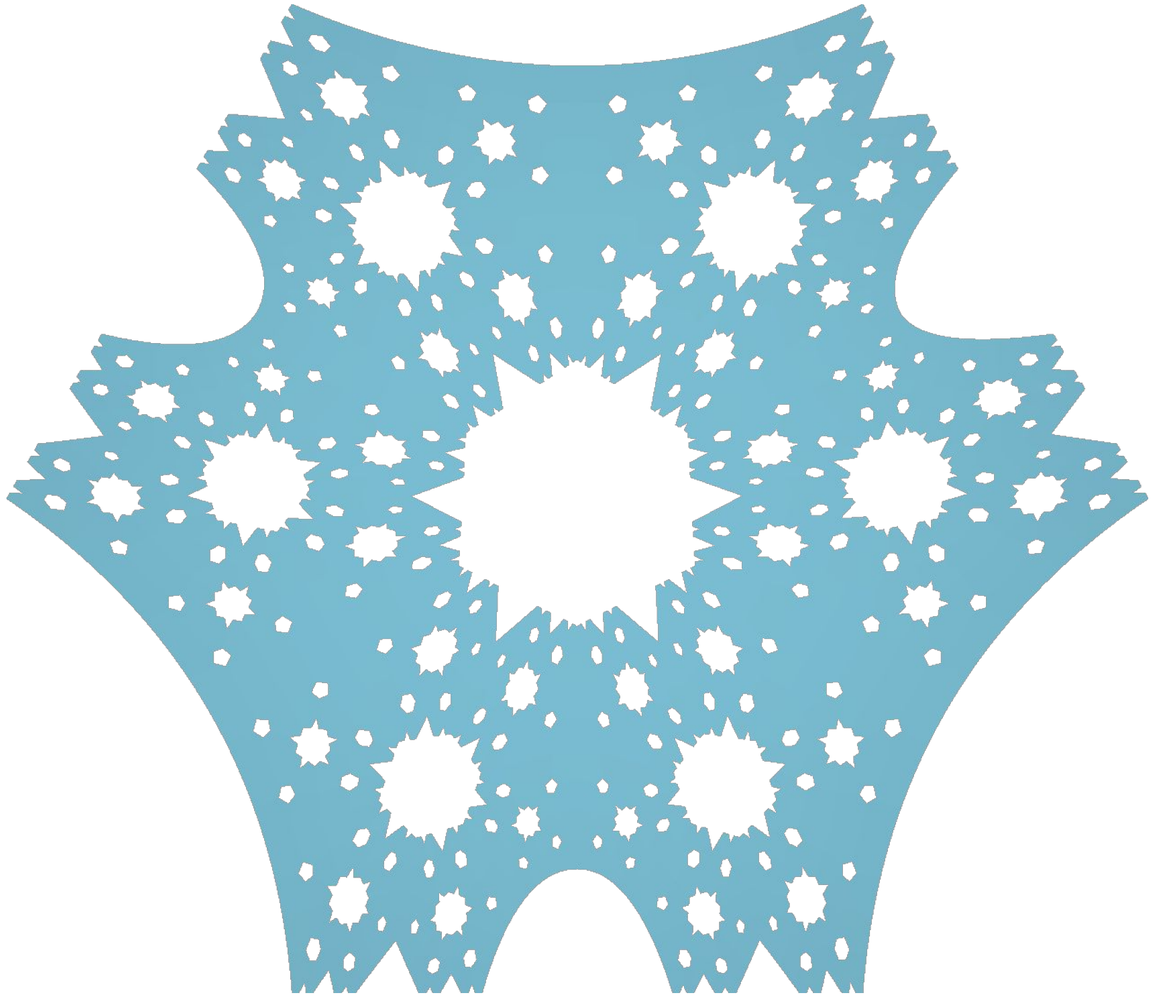


Paramètre $\mathcal{E}$

- $\mathcal{E}_a = B_2$
- $\mathcal{E}_l = C_2$
- $\mathcal{E}_c = B_2$

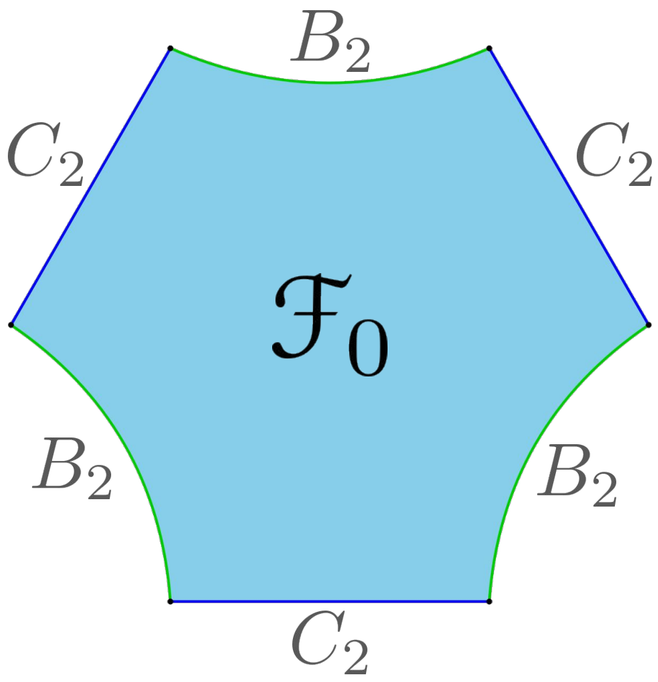


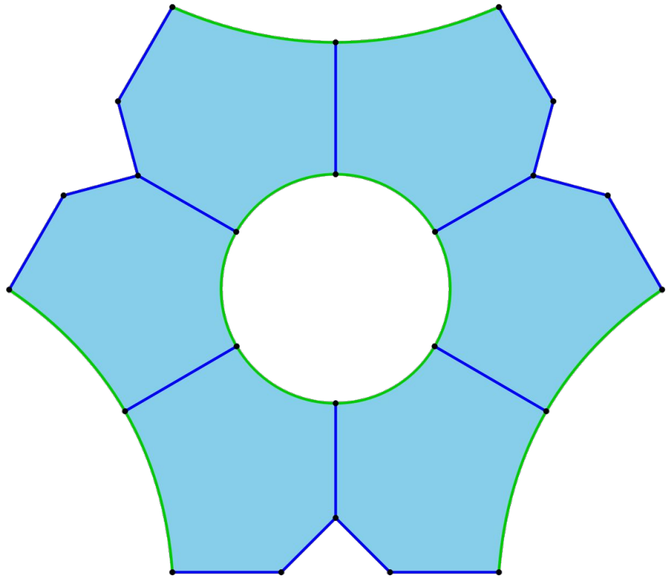


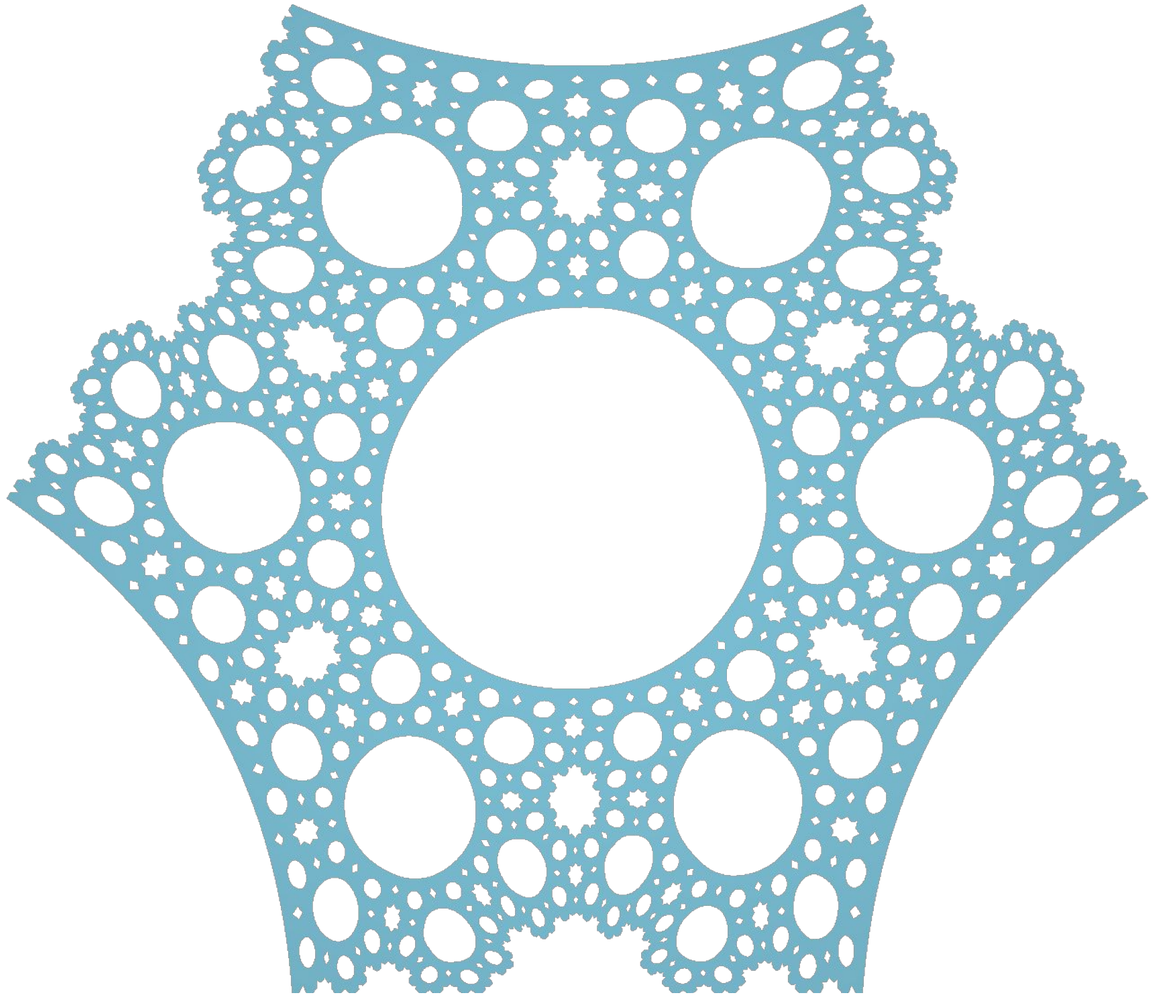


Paramètre $\mathcal{E}$

- $\mathcal{E}_a = C_2$
- $\mathcal{E}_l = B_2$
- $\mathcal{E}_c = C_2$

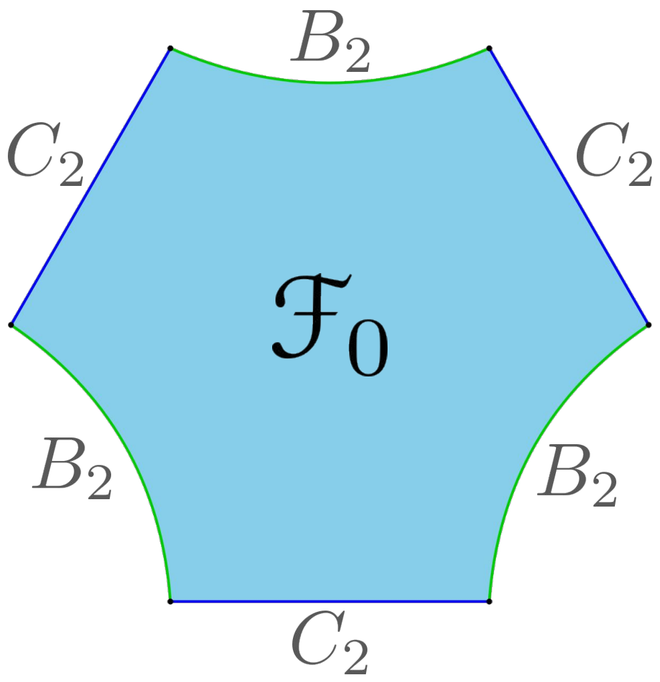


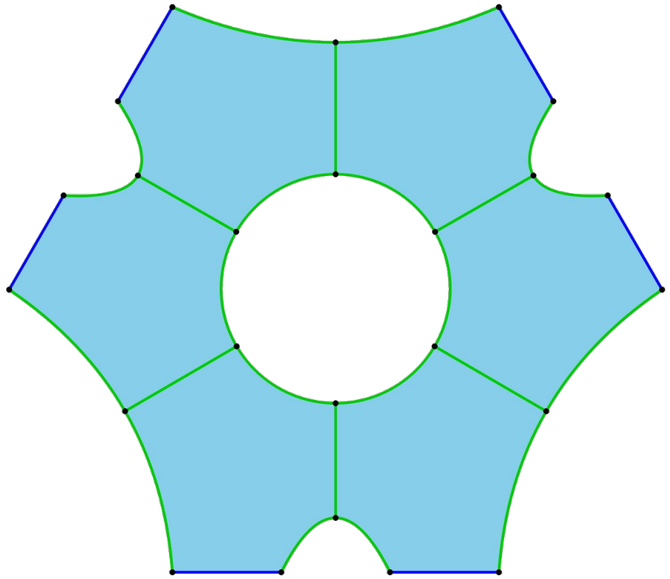


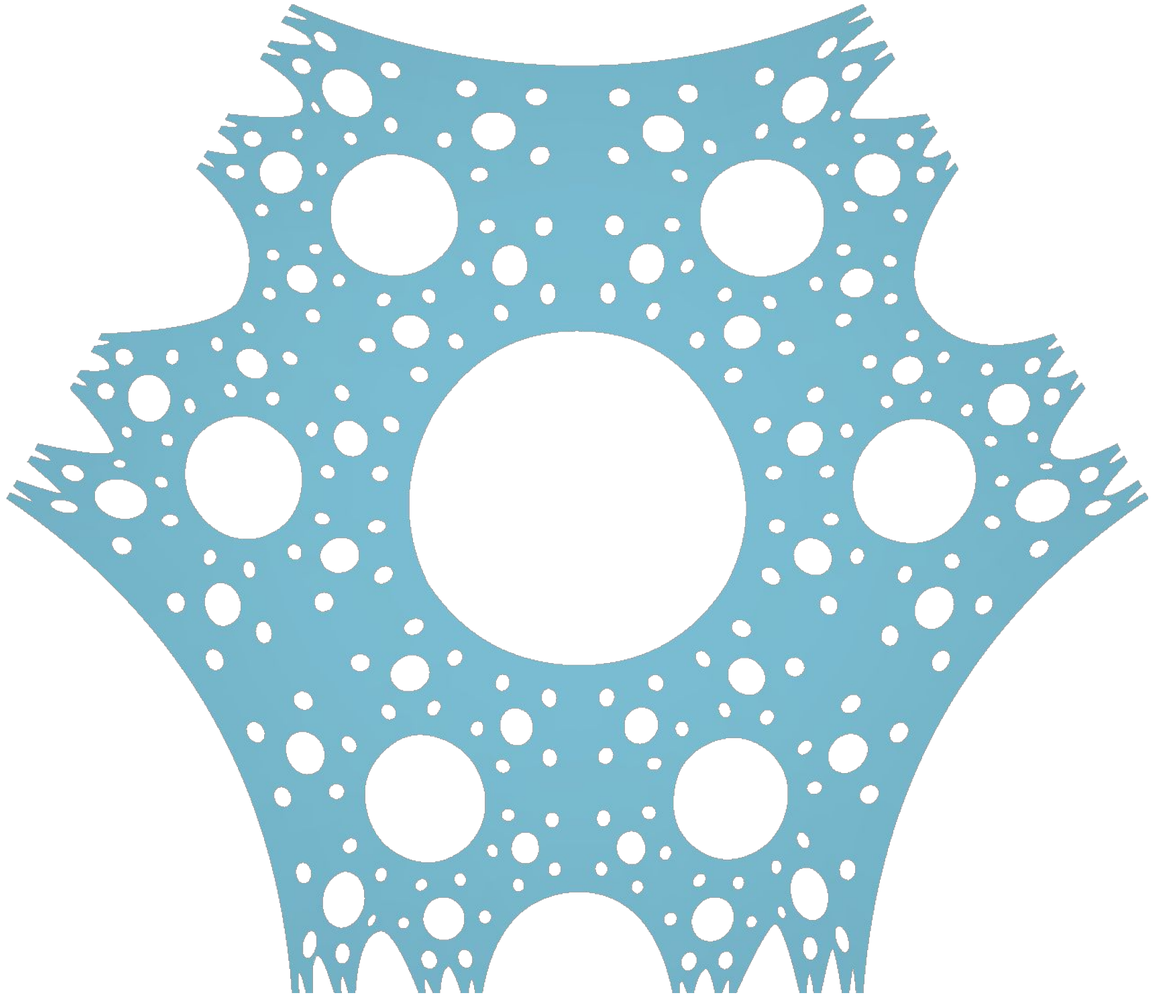


Paramètre $\mathcal{E}$

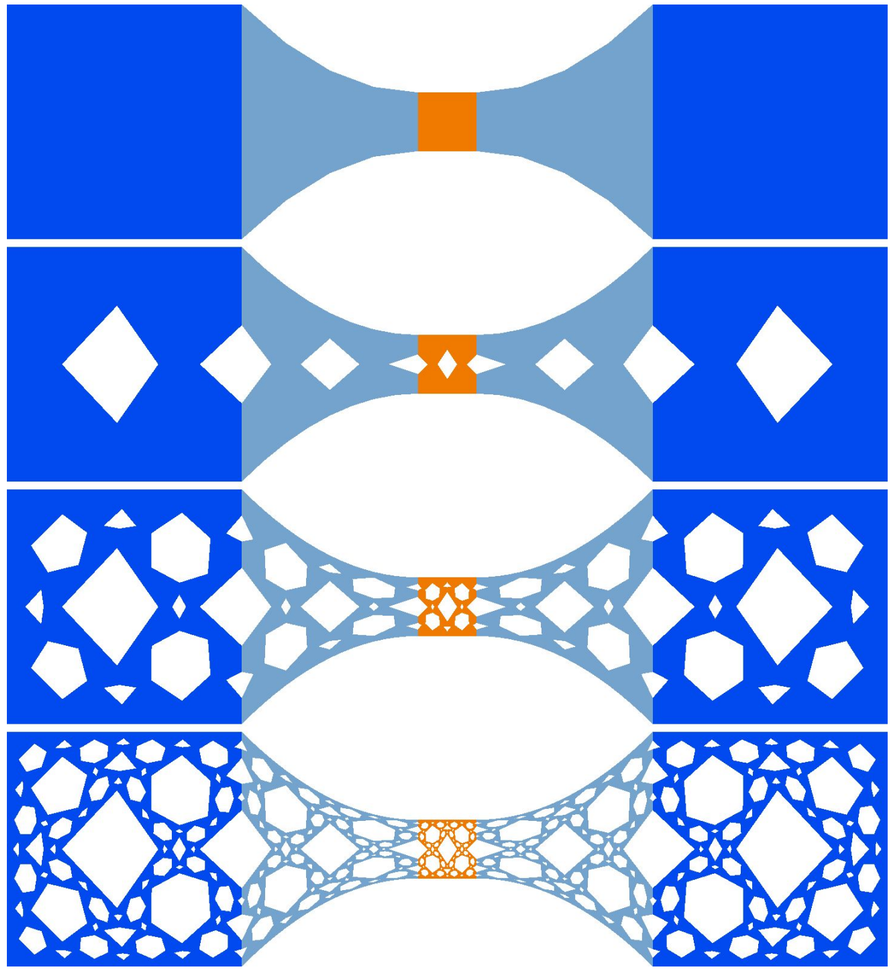
- $\mathcal{E}_a = B_2$
- $\mathcal{E}_l = B_2$
- $\mathcal{E}_c = B_2$

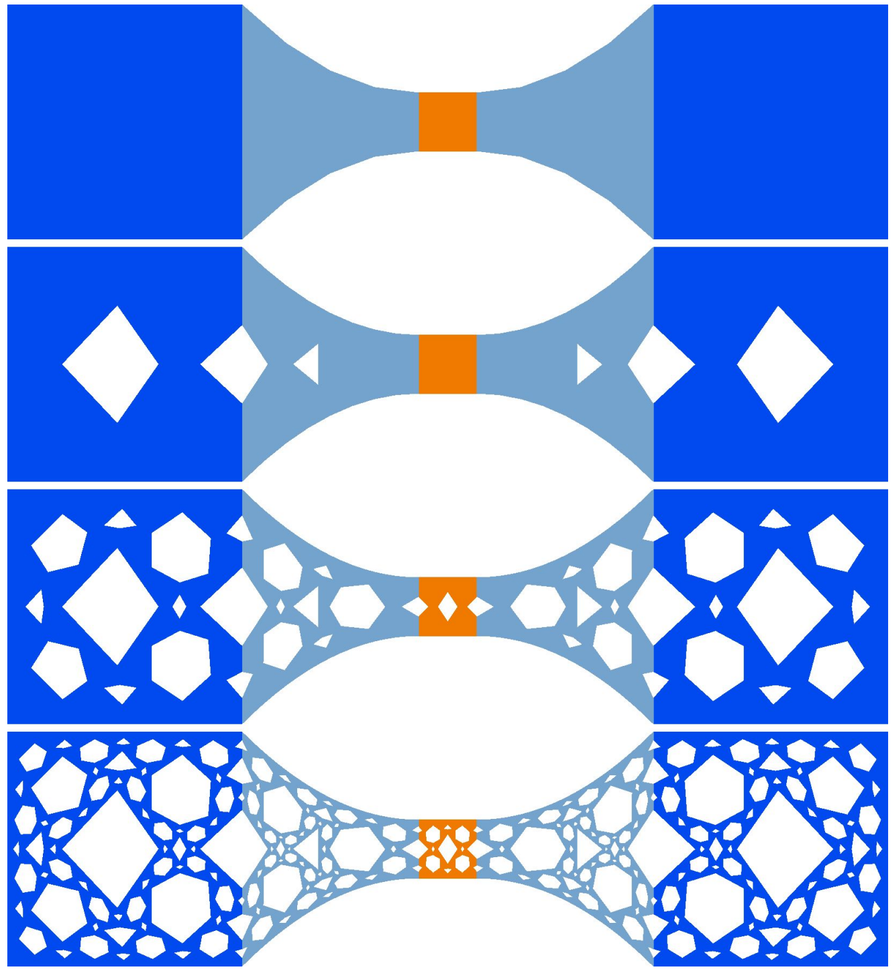




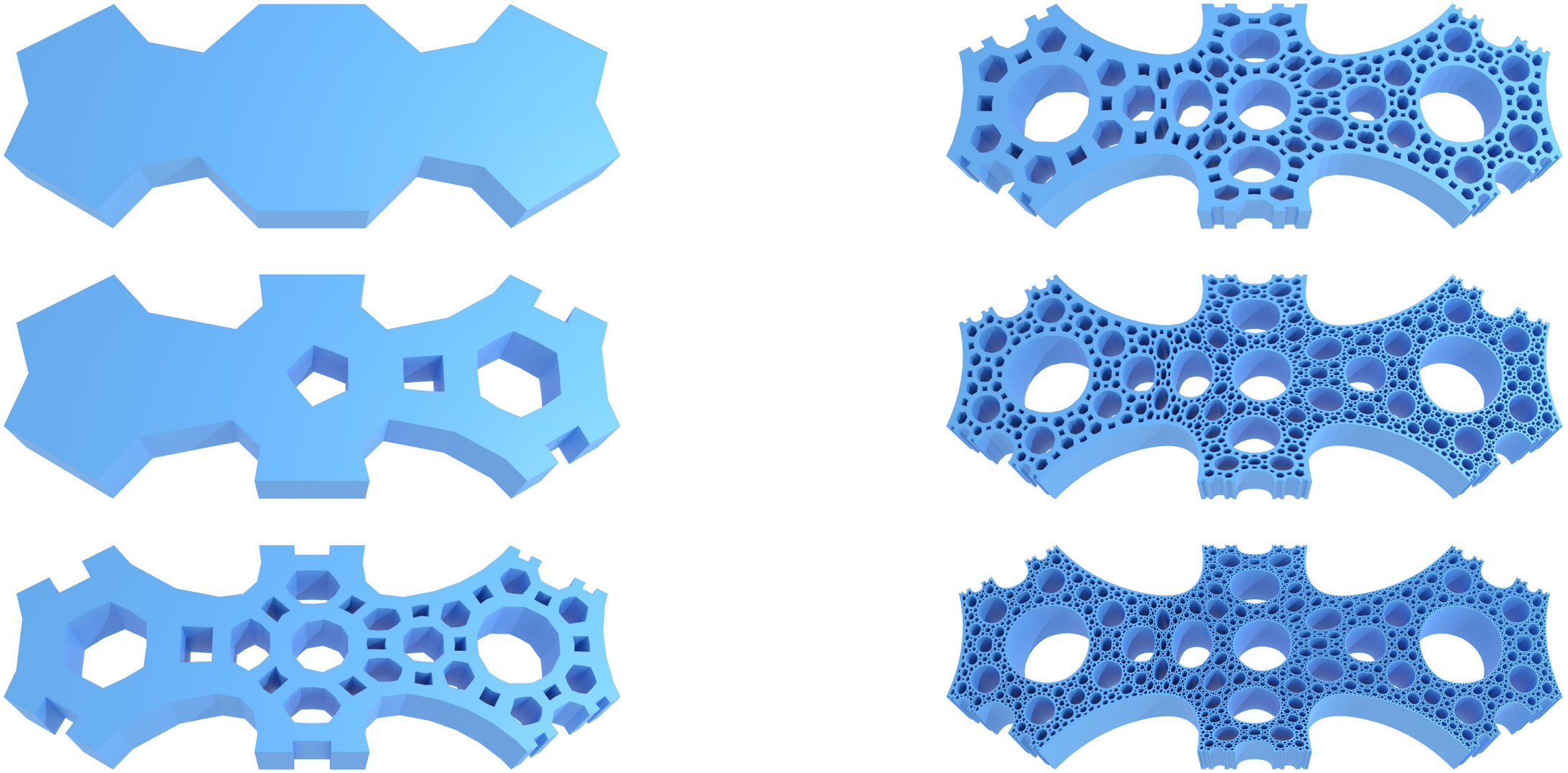


Délai dans la subdivision



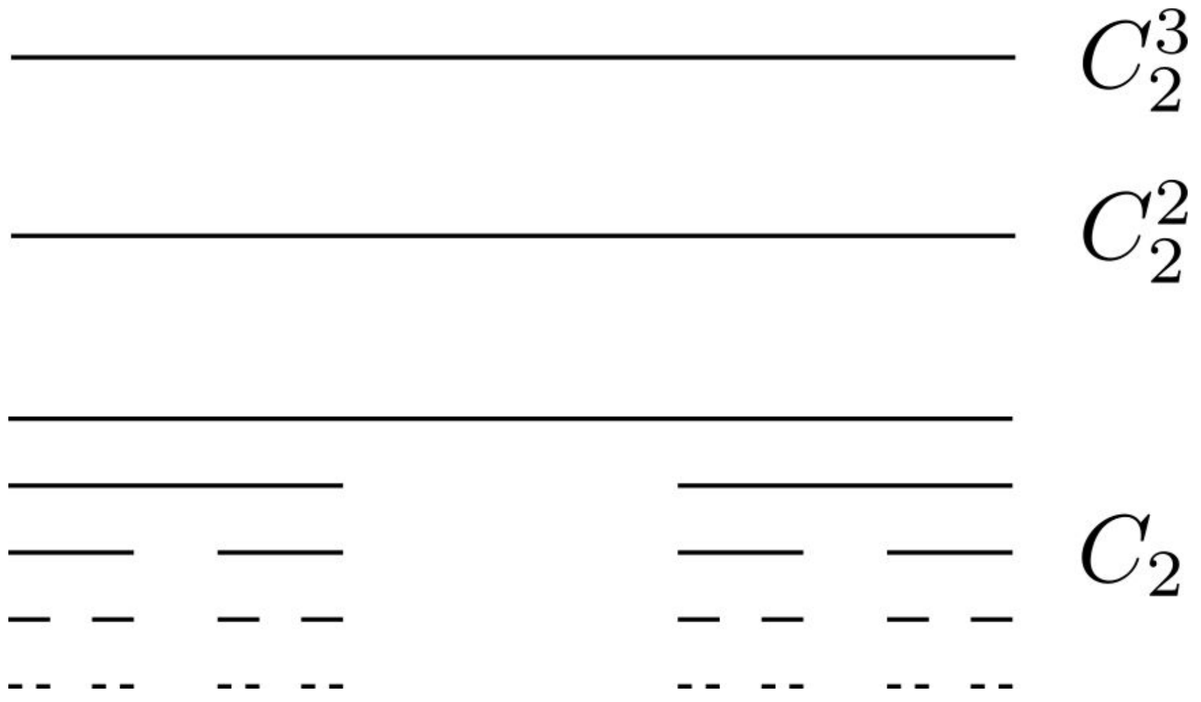


Délai dans la subdivision



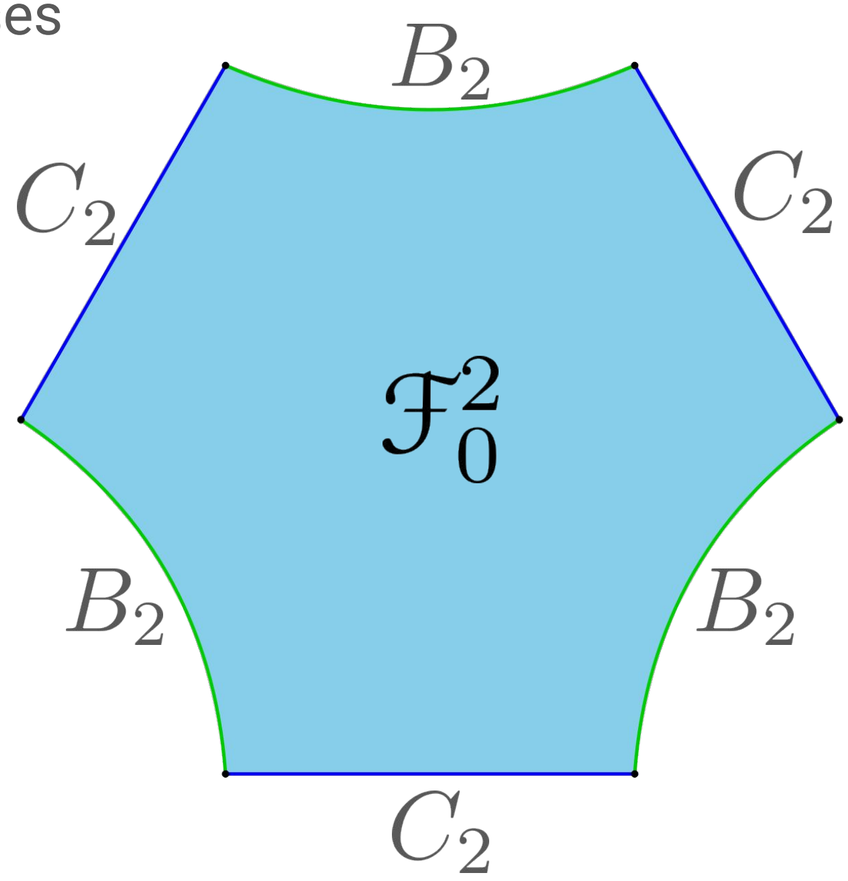
Délai dans la subdivision

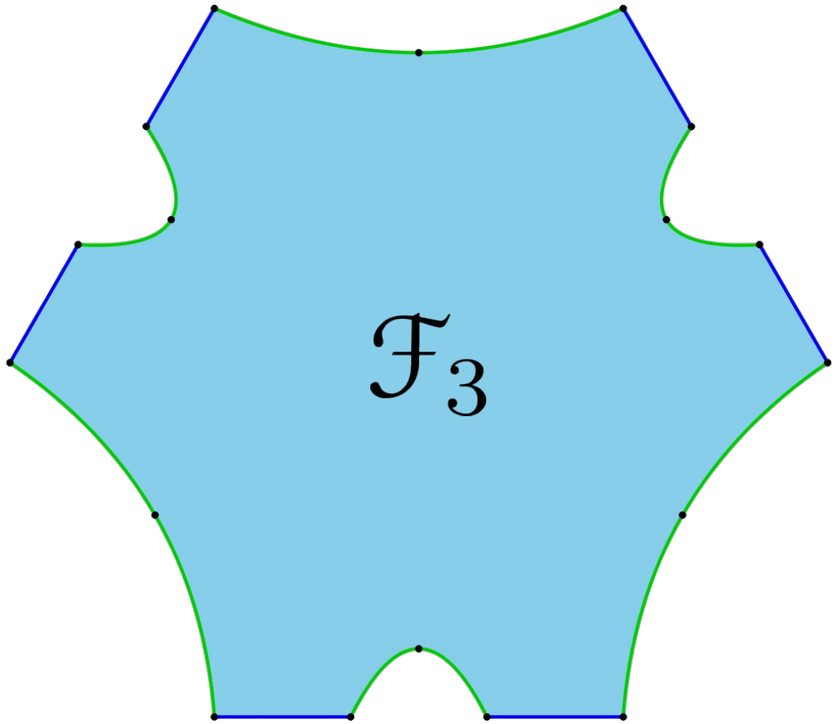
- Arêtes
  - Le délai diminue



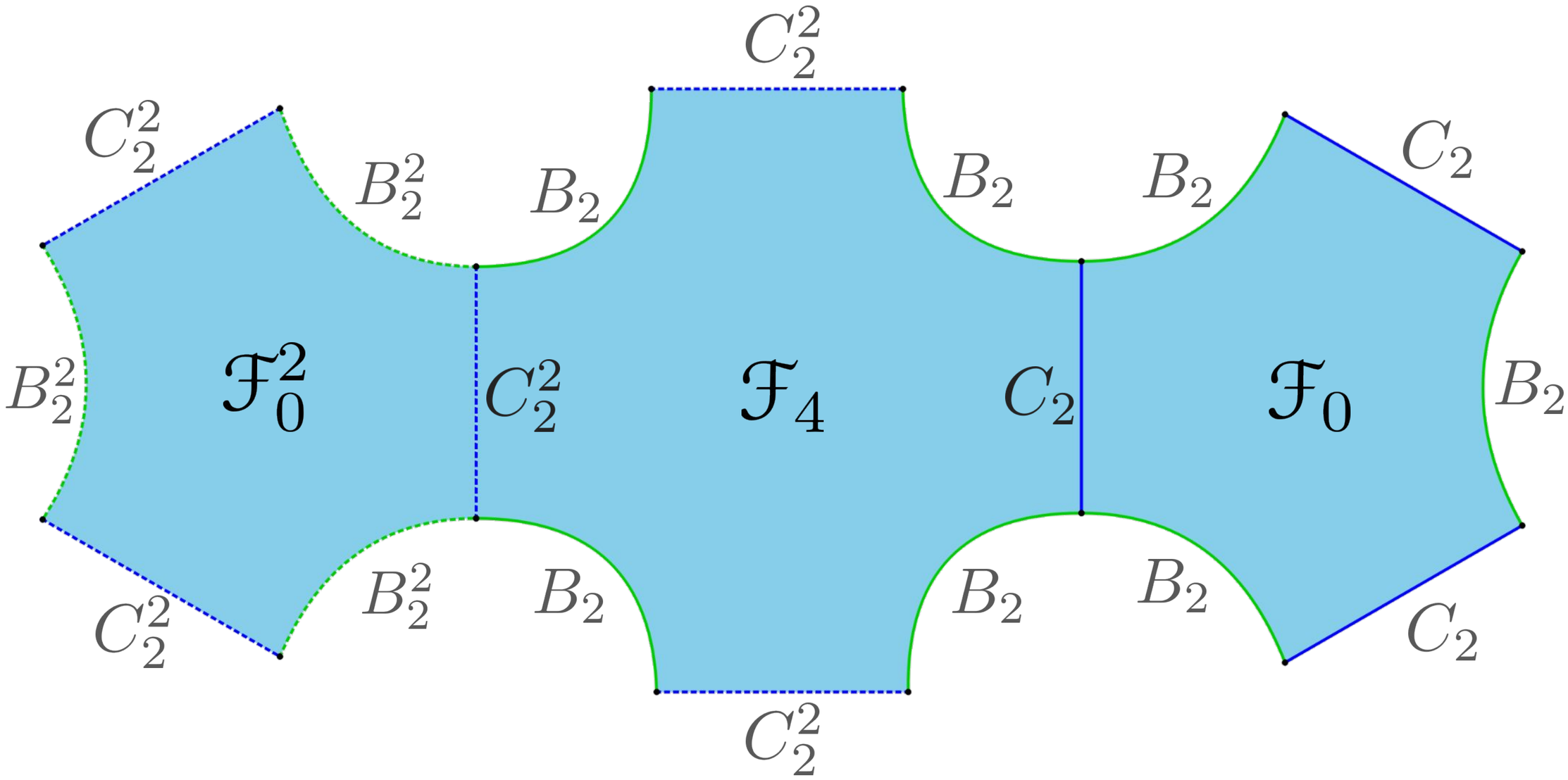
Délai dans la subdivision

- Faces
  - Pas de lacune centrale
  - Les bords sont subdivisés

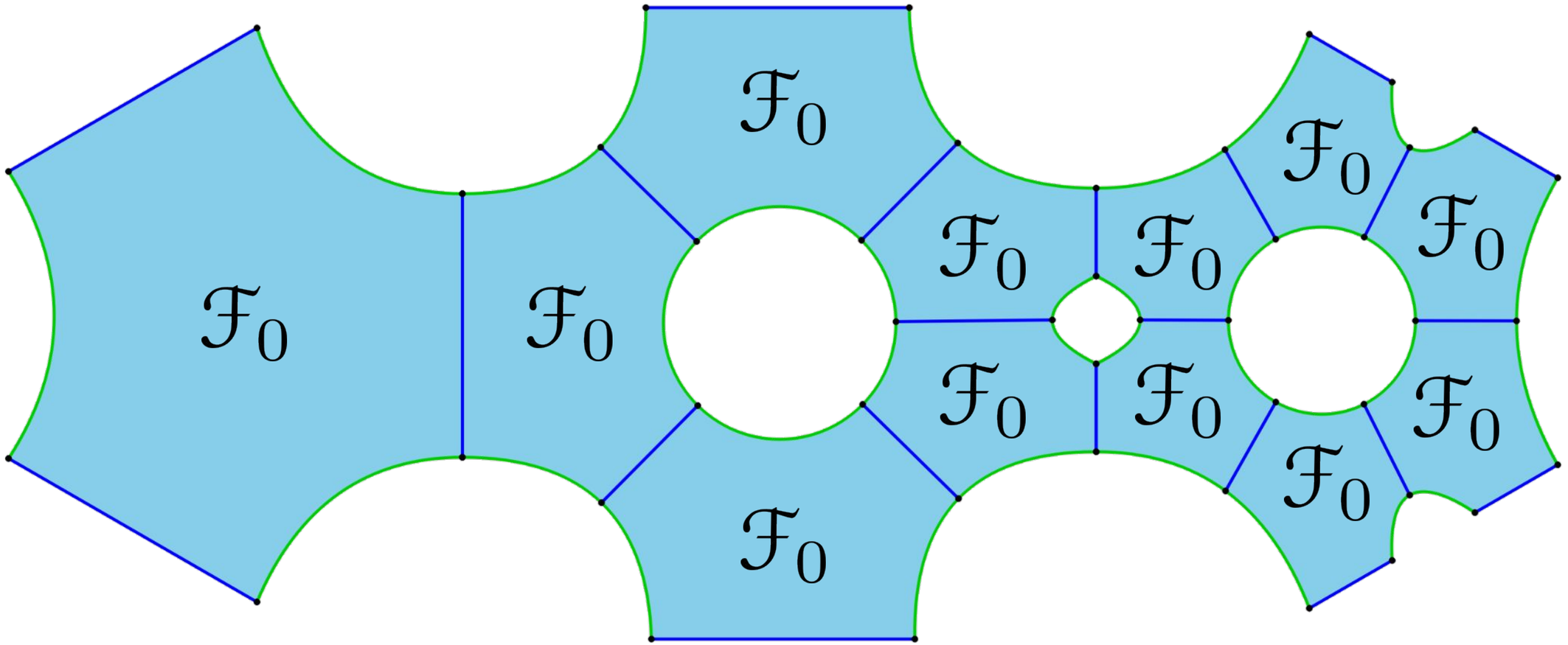




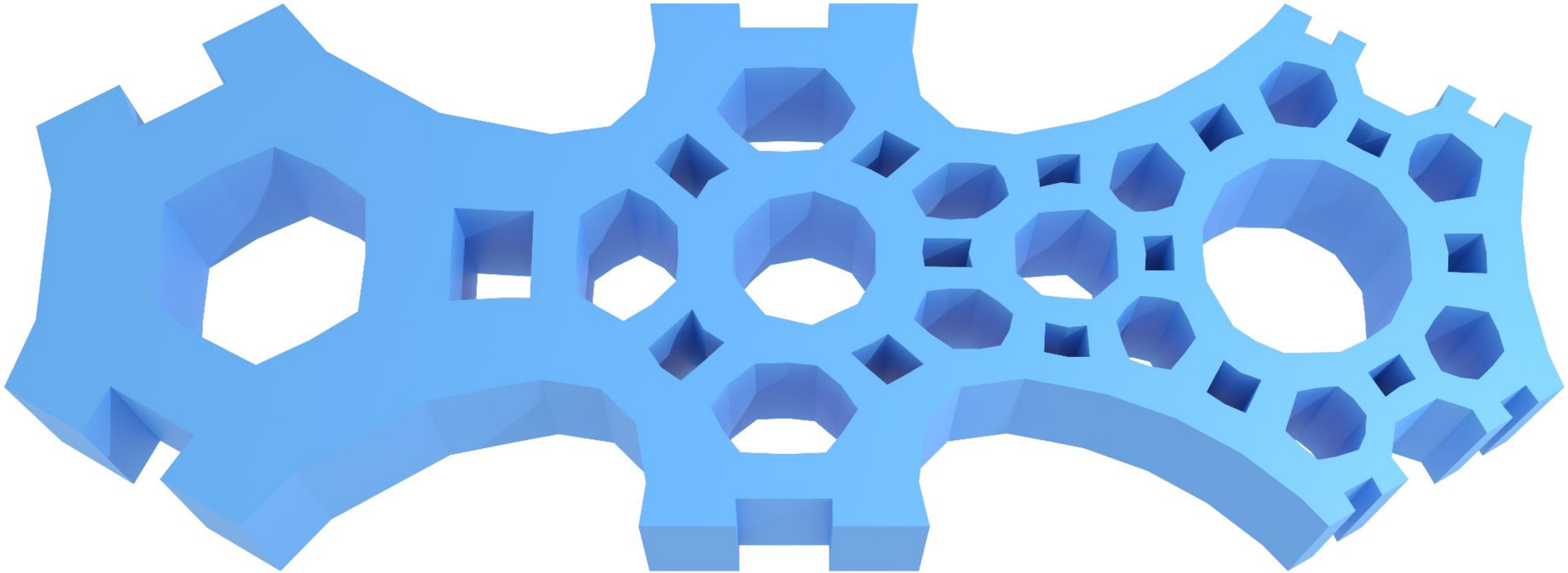
Délai dans la subdivision



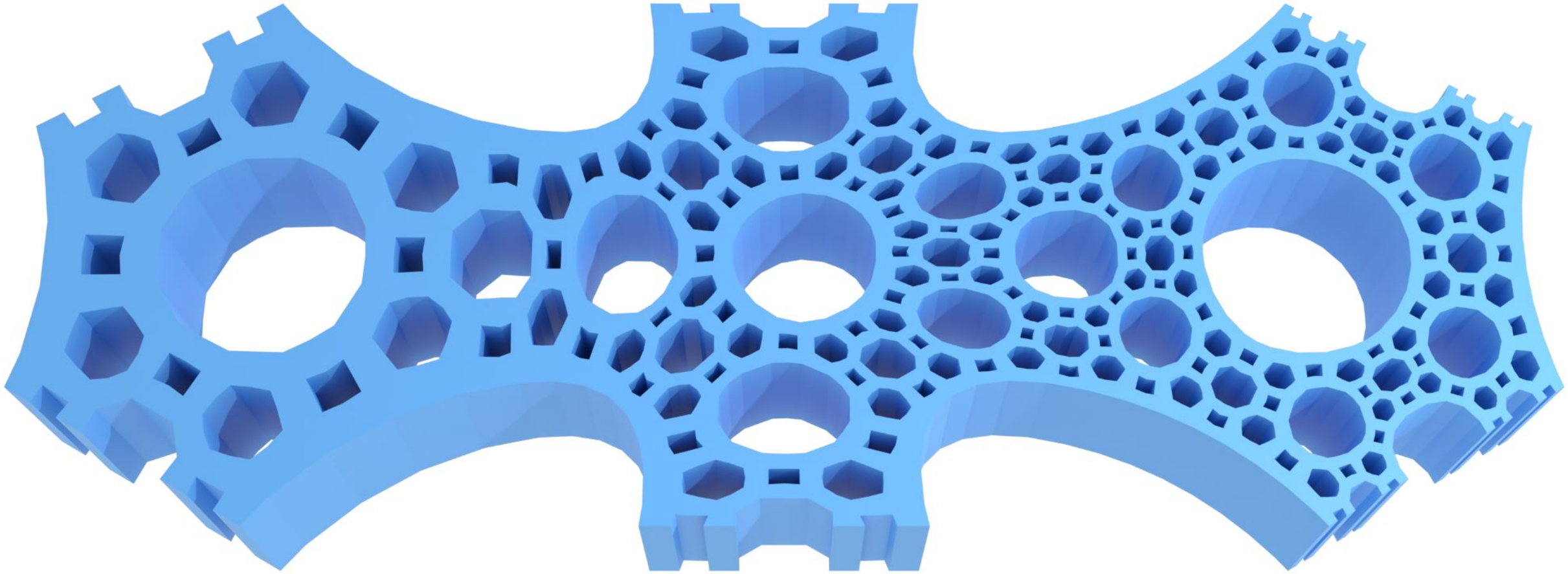
Délai dans la subdivision



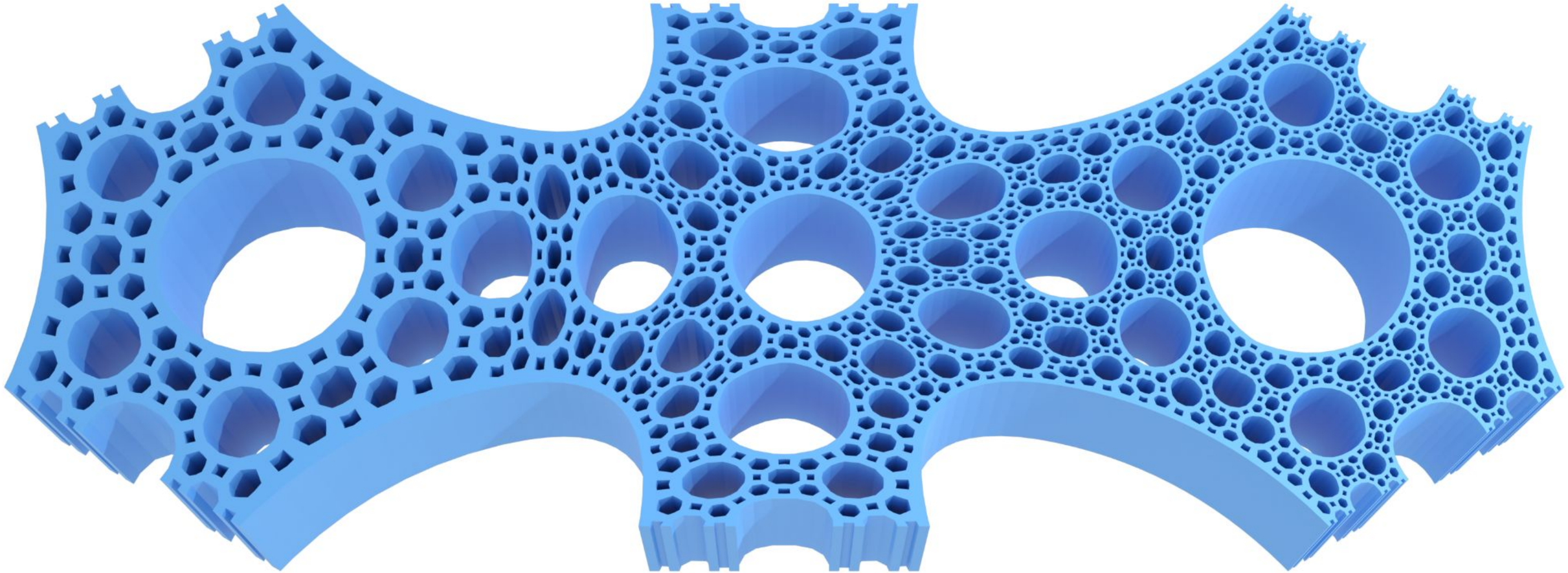
Délai dans la subdivision



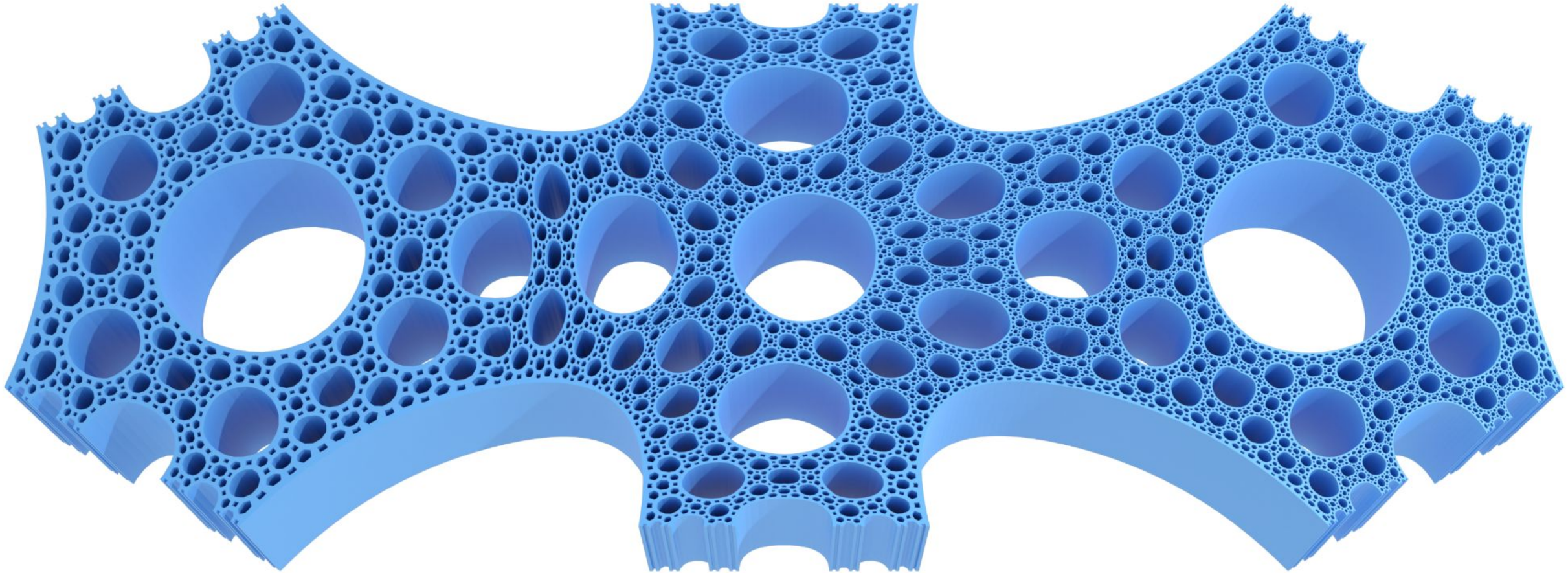
Délai dans la subdivision



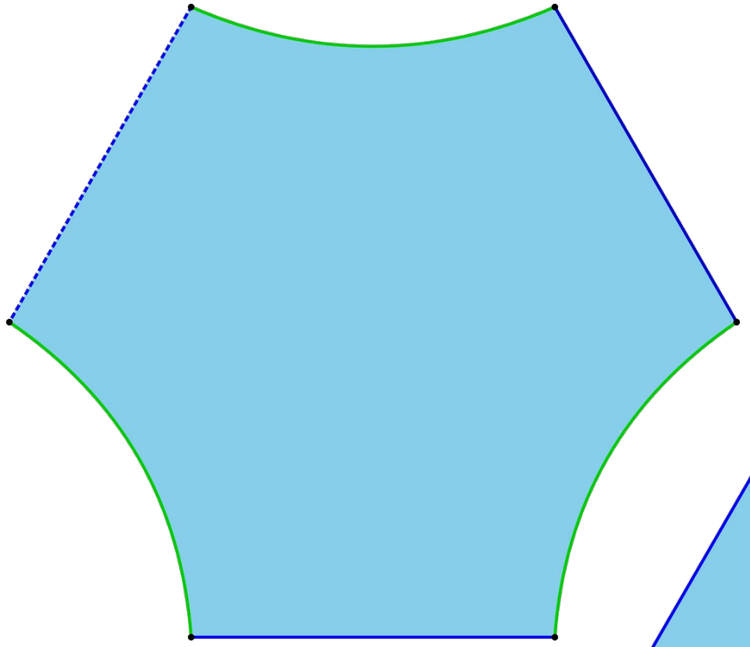
Délai dans la subdivision

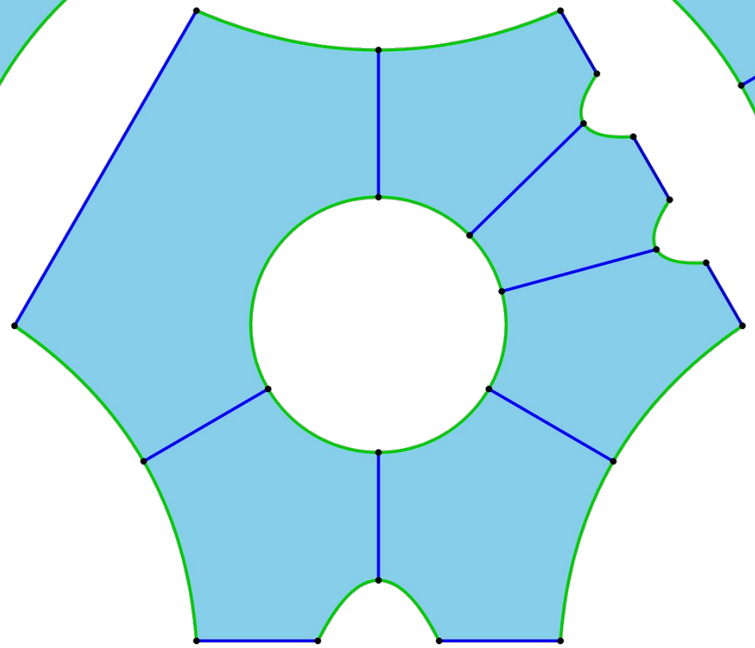


Délai dans la subdivision

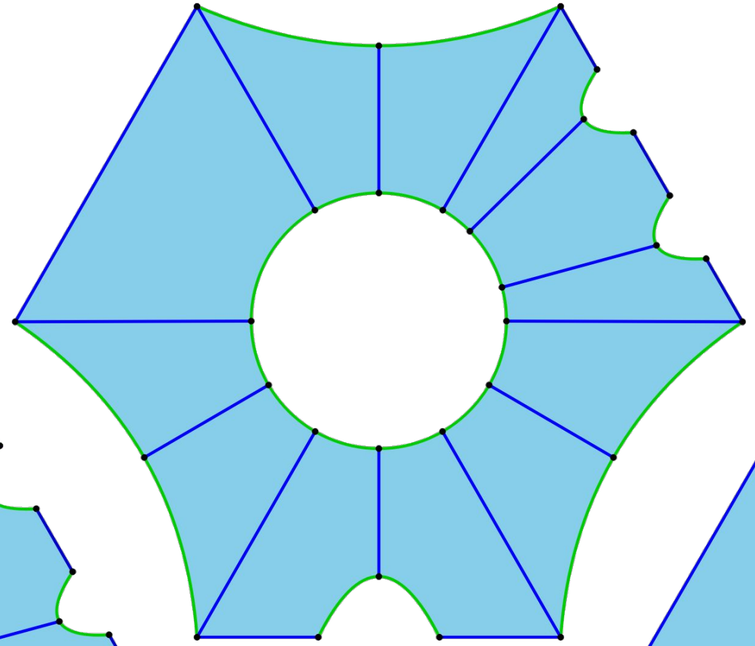


Autres processus de subdivision

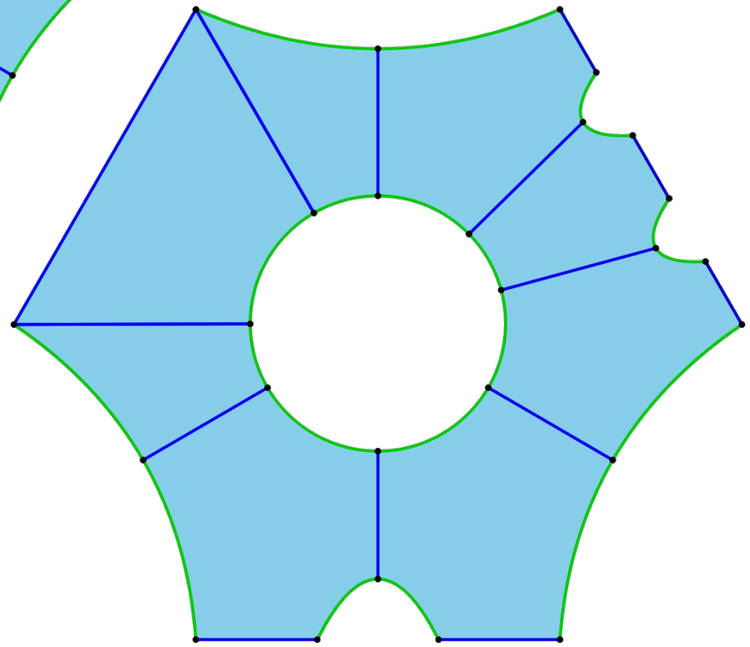




Processus 1

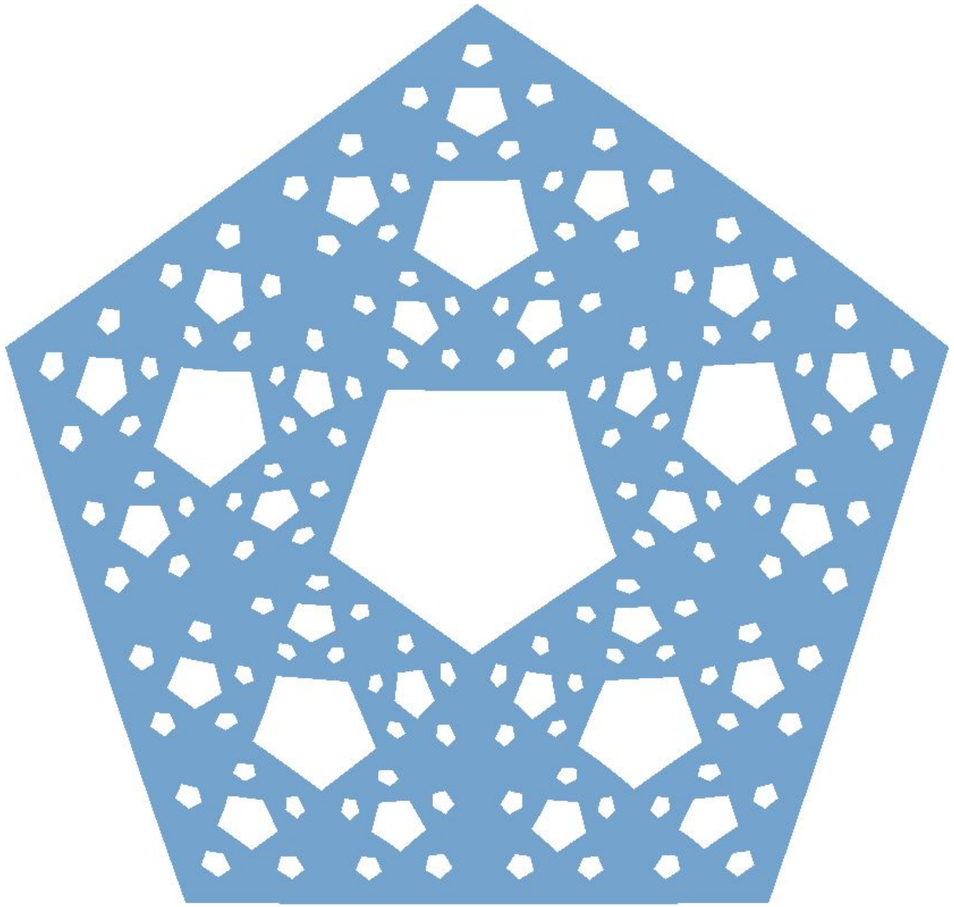


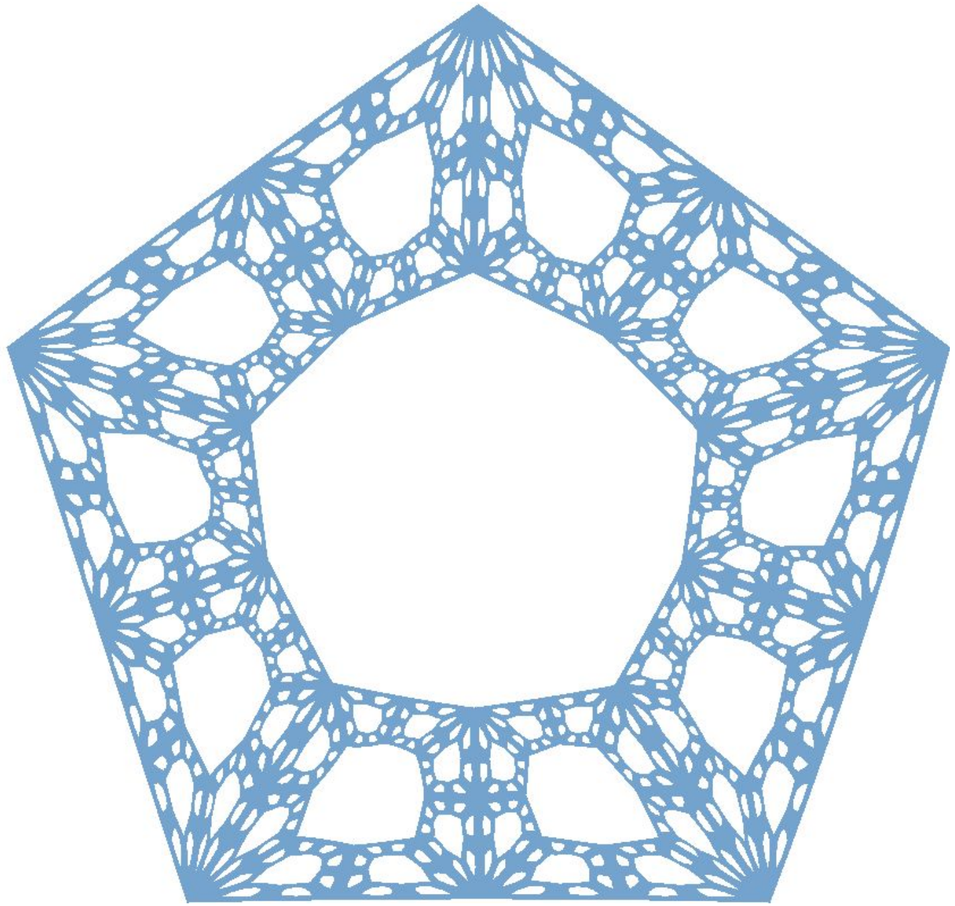
Processus 2



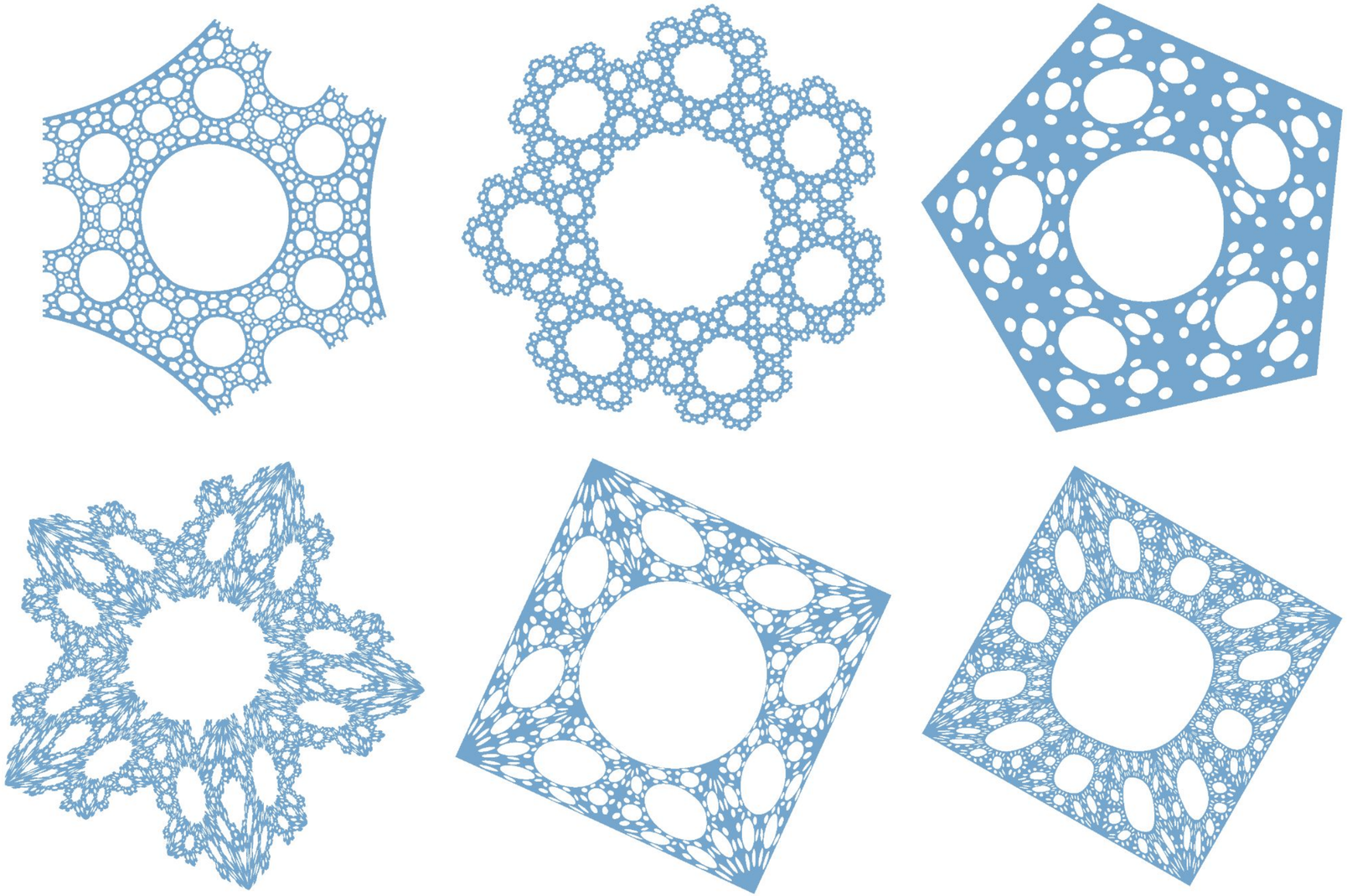
Processus 3

Autres processus de subdivision

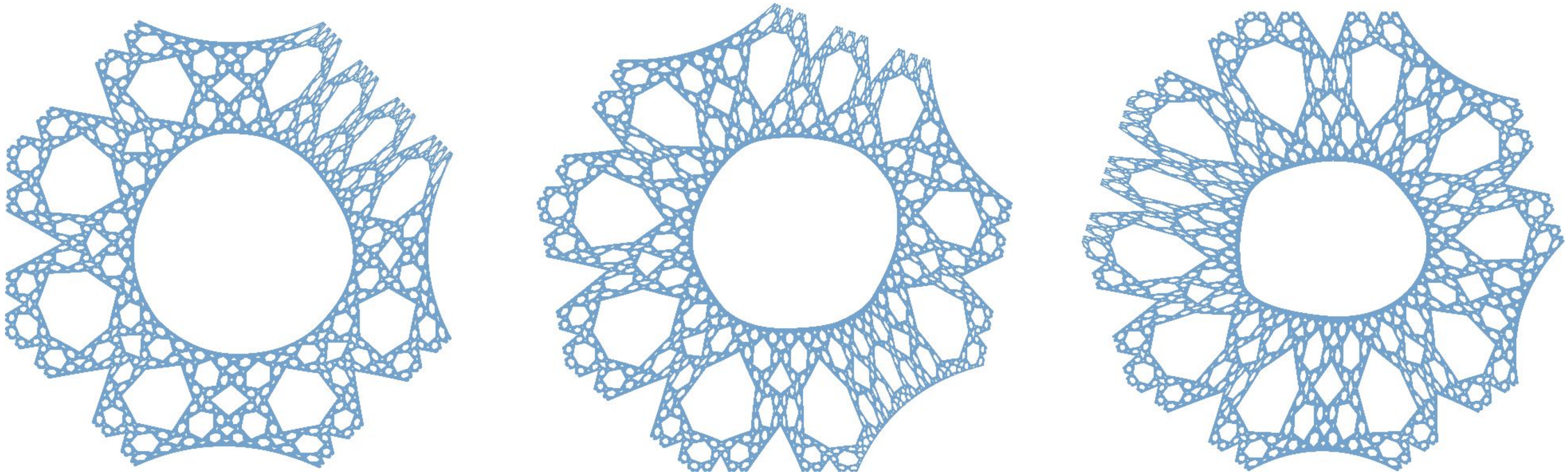




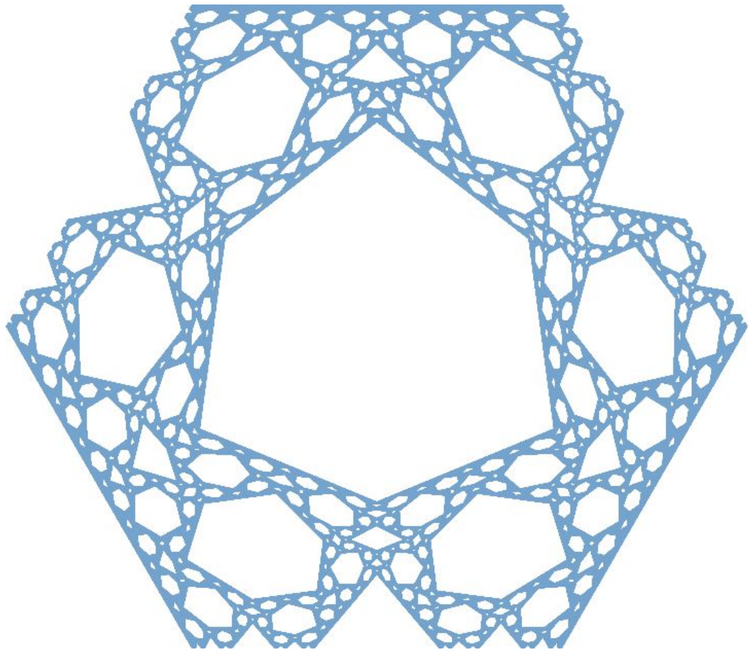
Exemples de formes

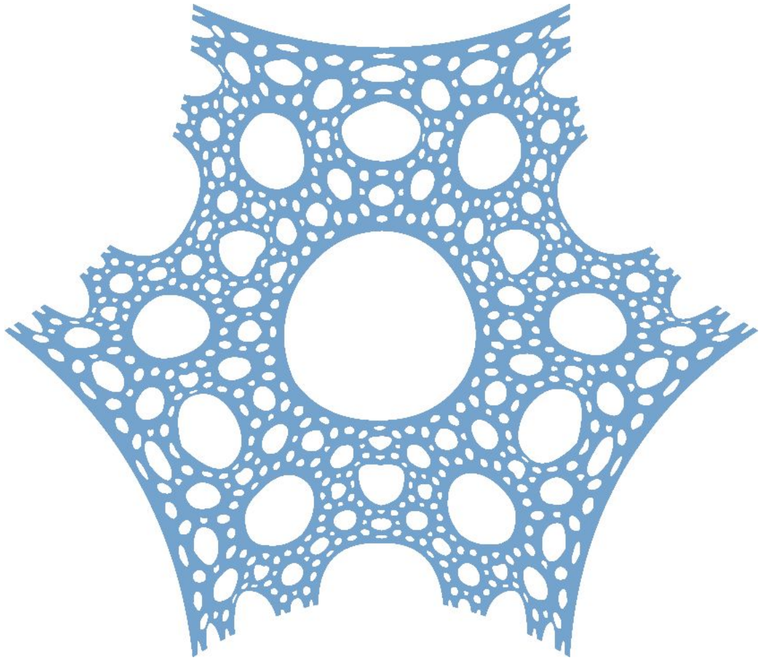


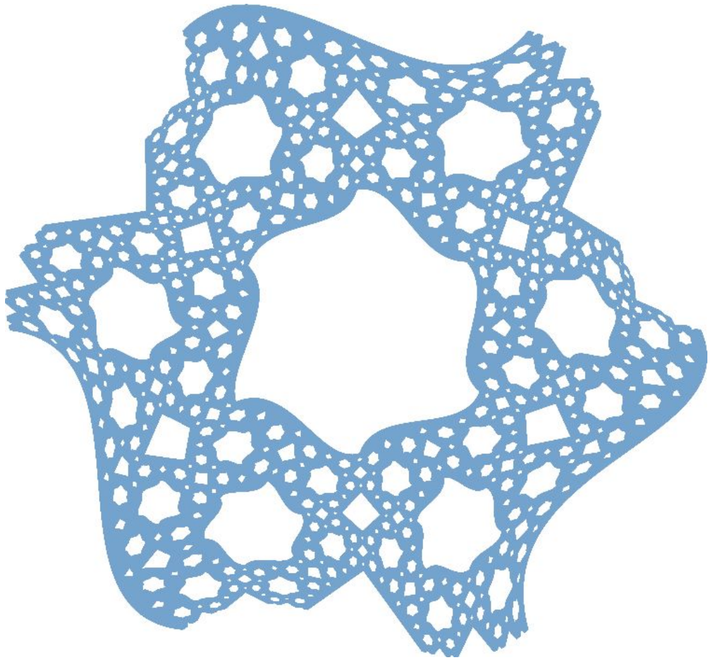
Exemples de formes



Exemples de formes



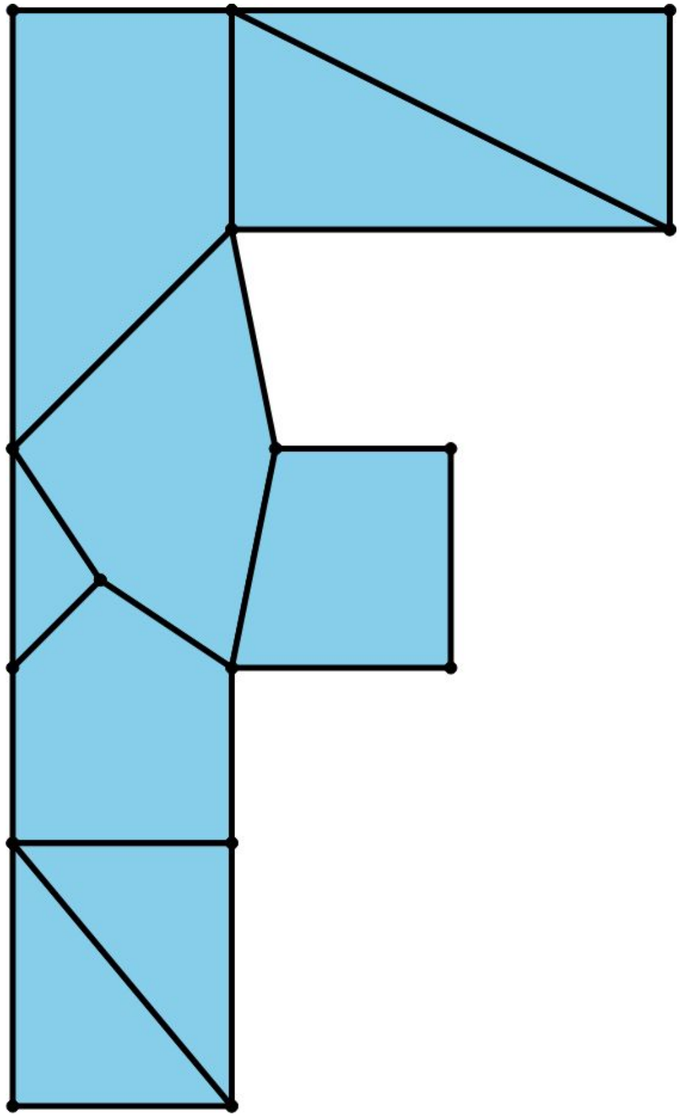




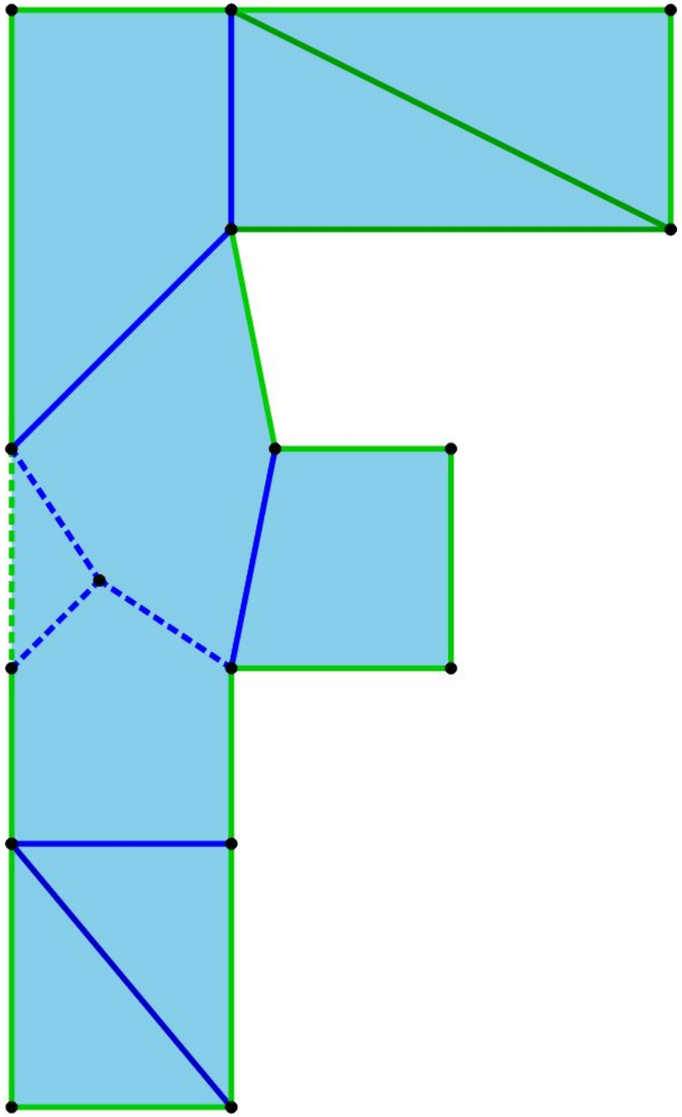
Assemblage



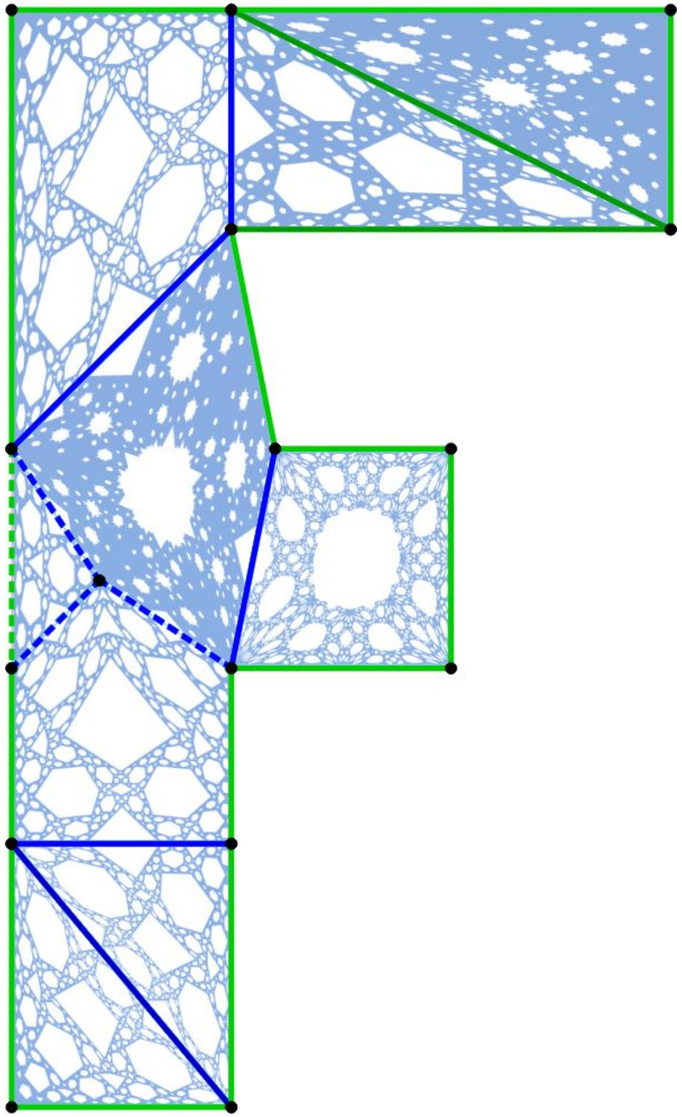
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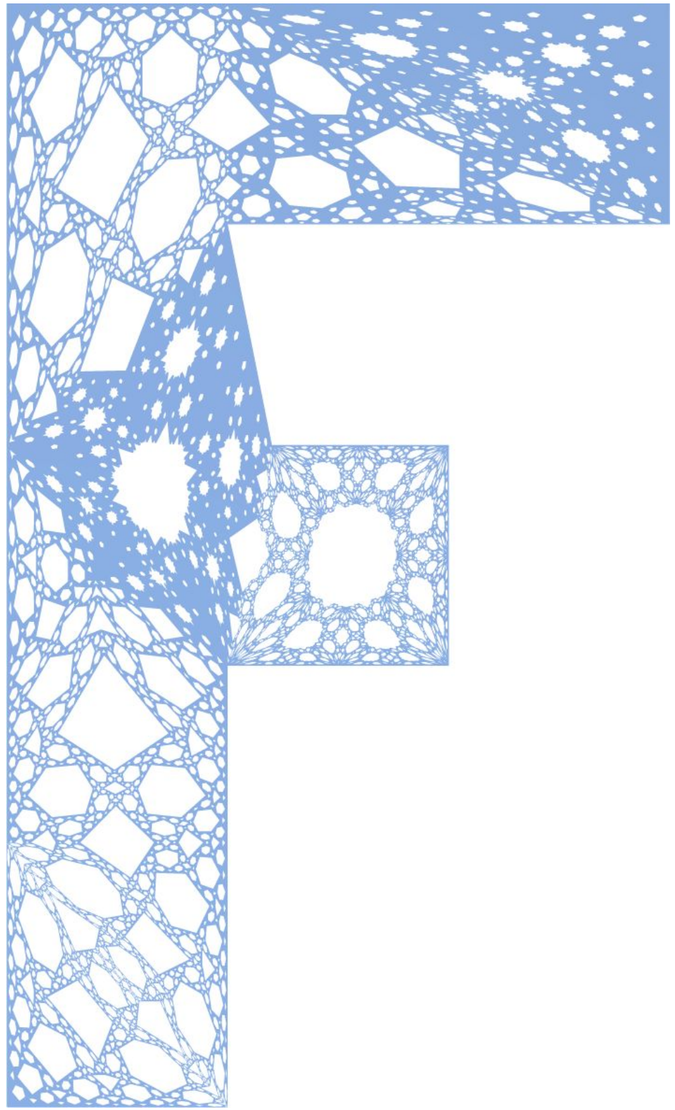
Assemblage



Assemblage

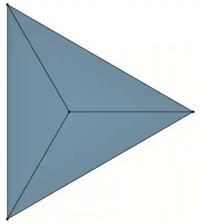


Assemblage

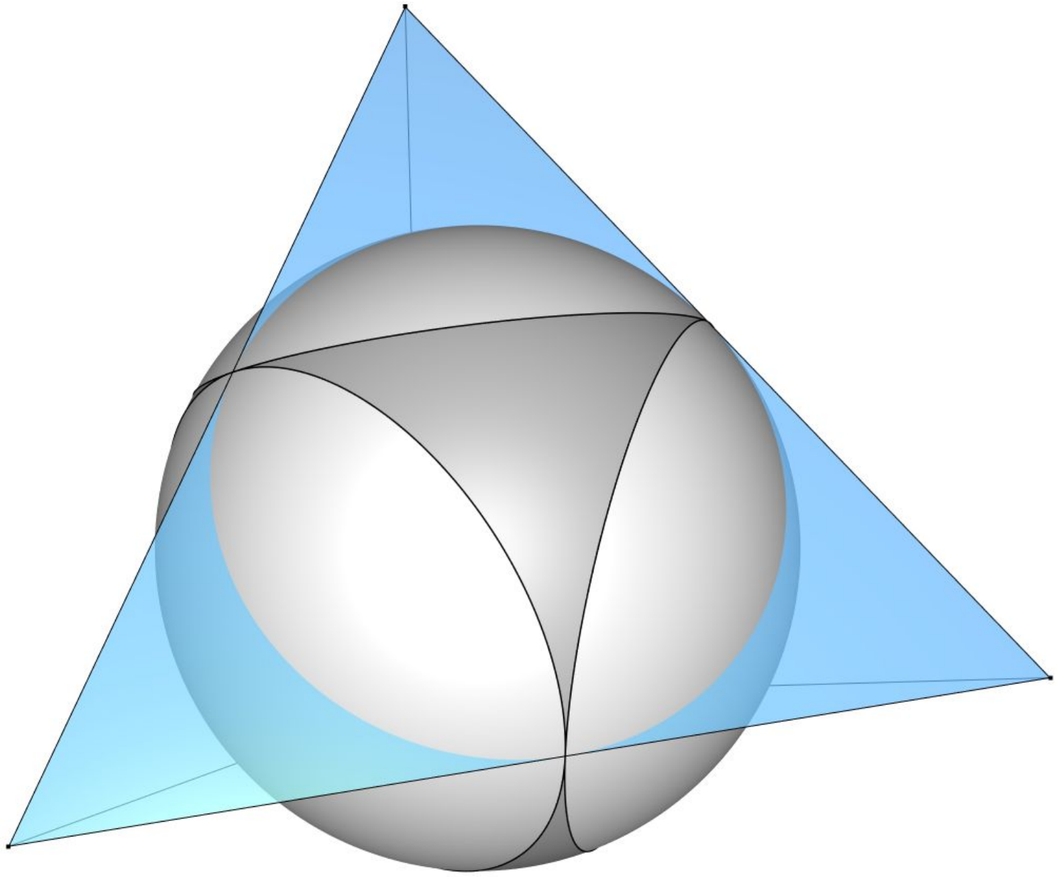


Conception automatique à partir d’un polyèdre

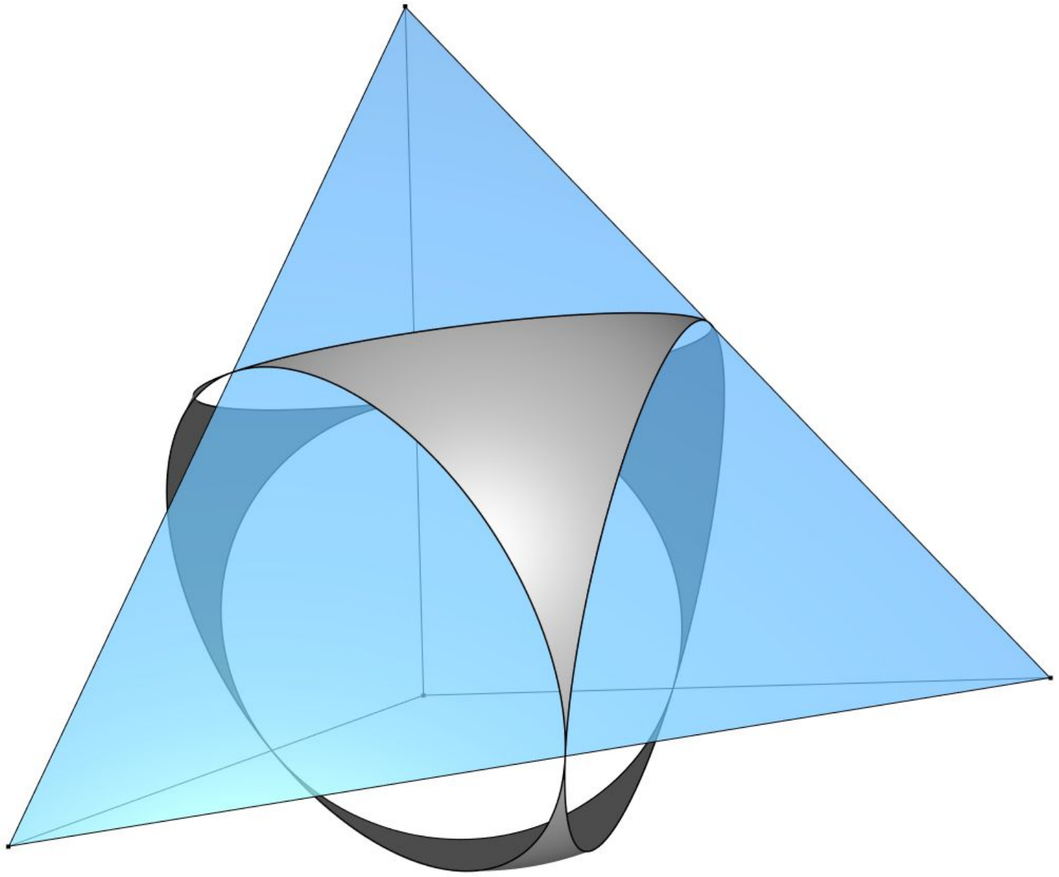
Empilements polyédriques de cercles



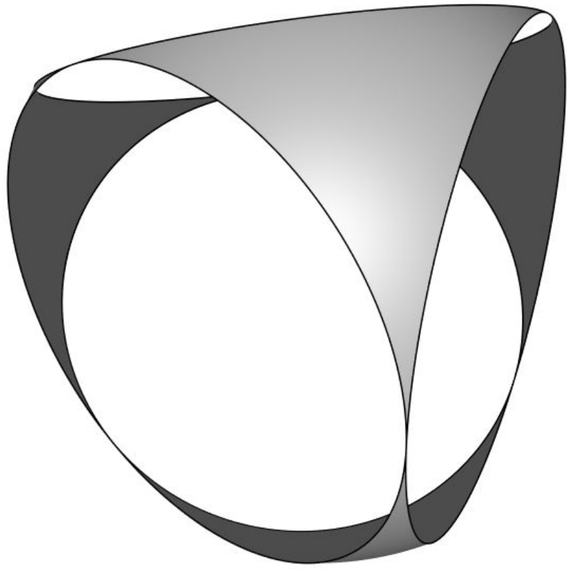
Tétraèdre



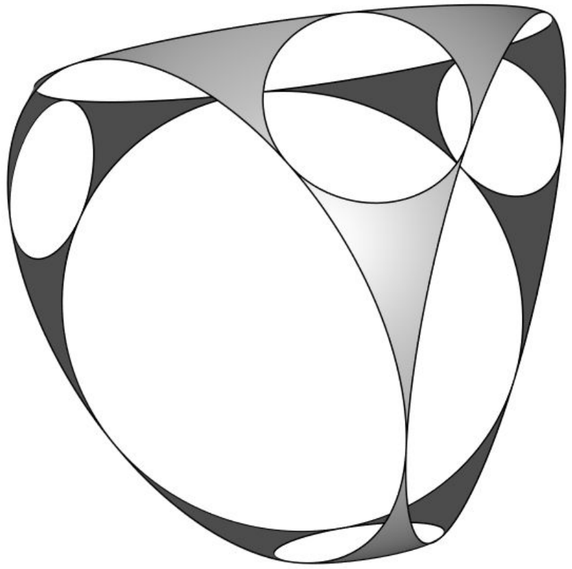
Tétraèdre



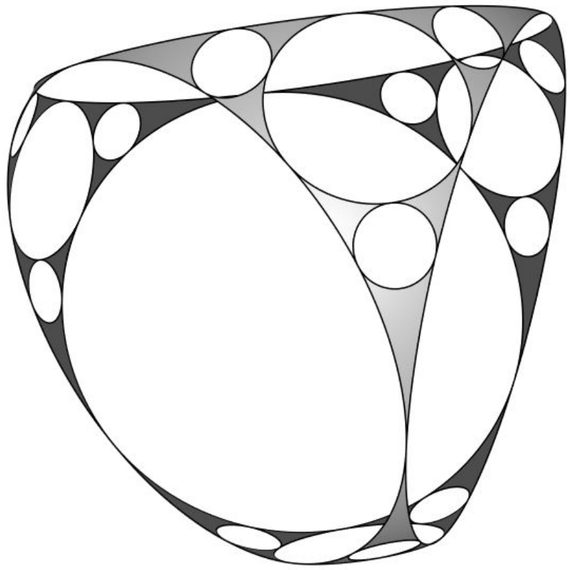
Tétraèdre



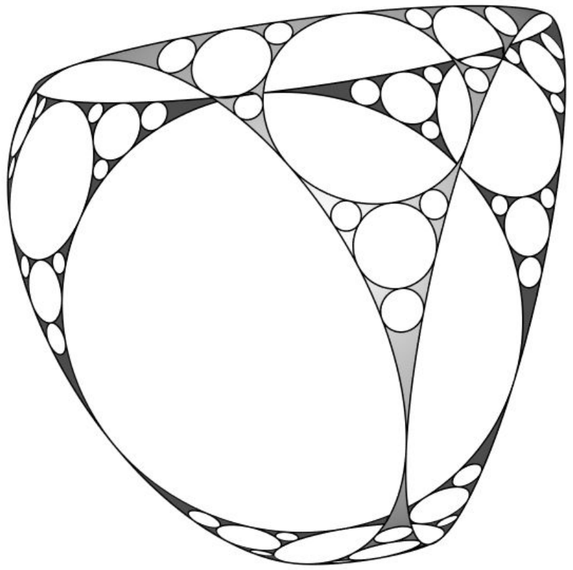
Tétraèdre



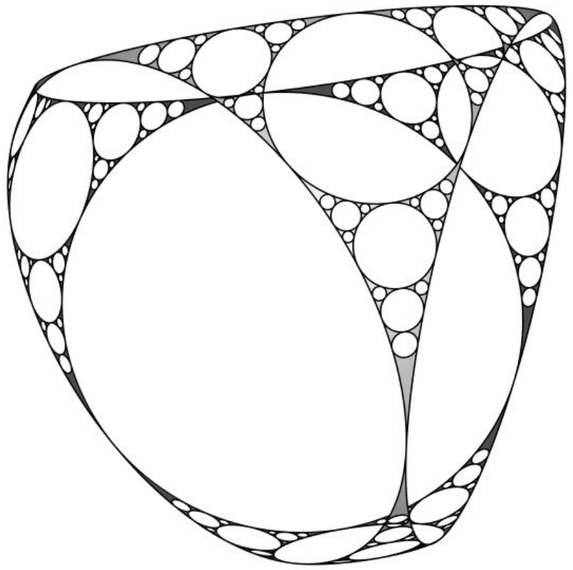
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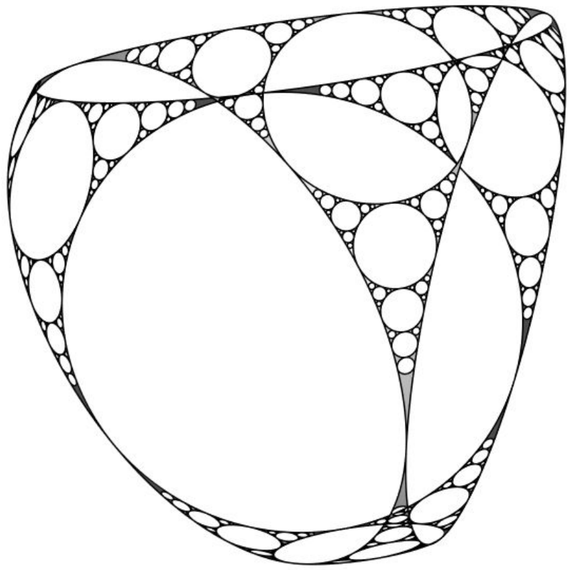
Tétraèdre



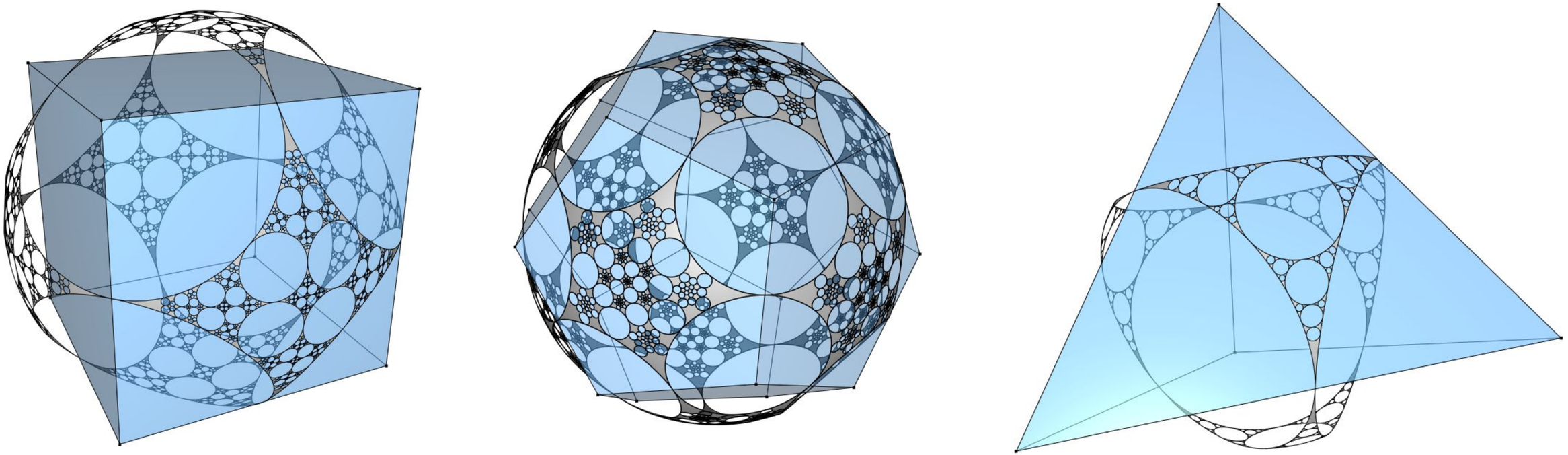
Tétraèdre



Tétraèdre



Processus de subdivision

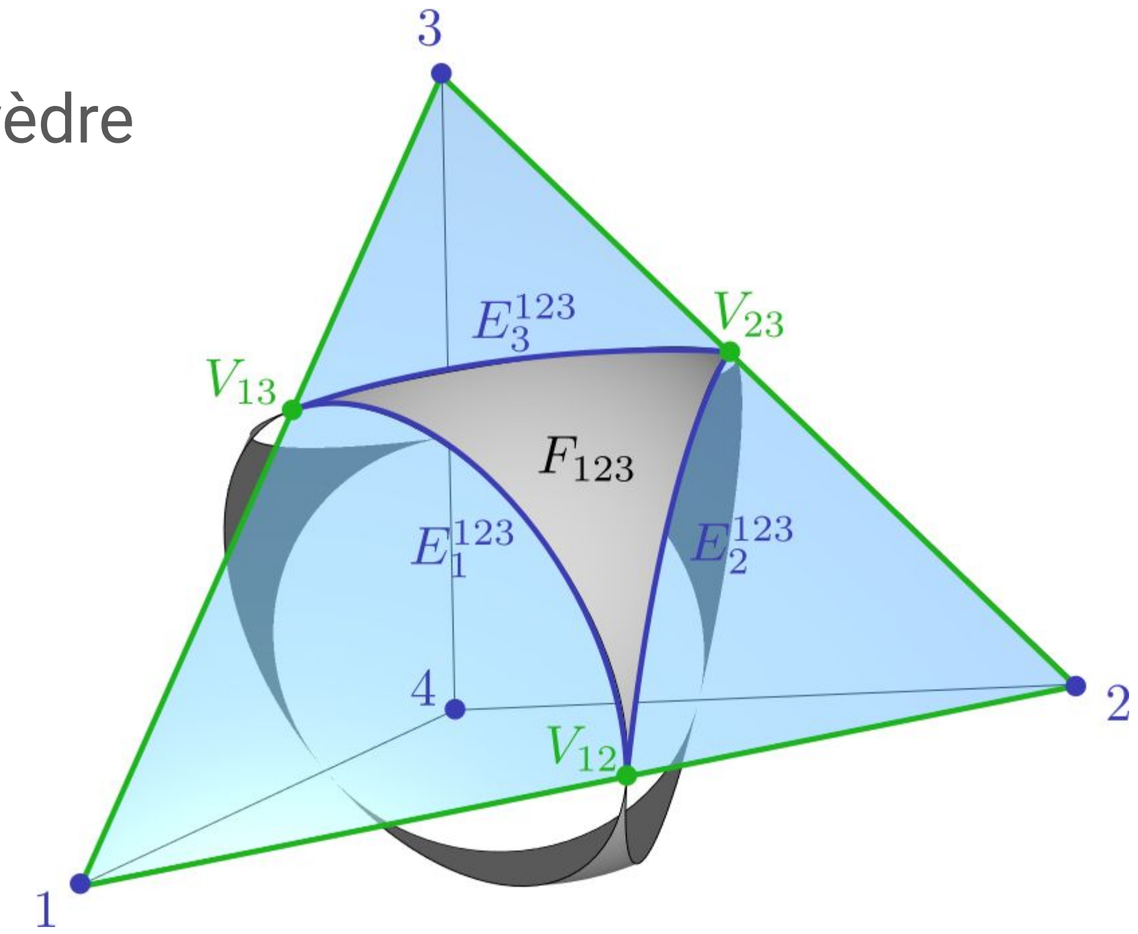


Encodage de la topologie avec un BC-IFS

- Automate
  - États
  - Transitions
    - Opérateurs de bords
    - Opérateurs de subdivision
- Contraintes d’incidence
- Contraintes d’adjacence

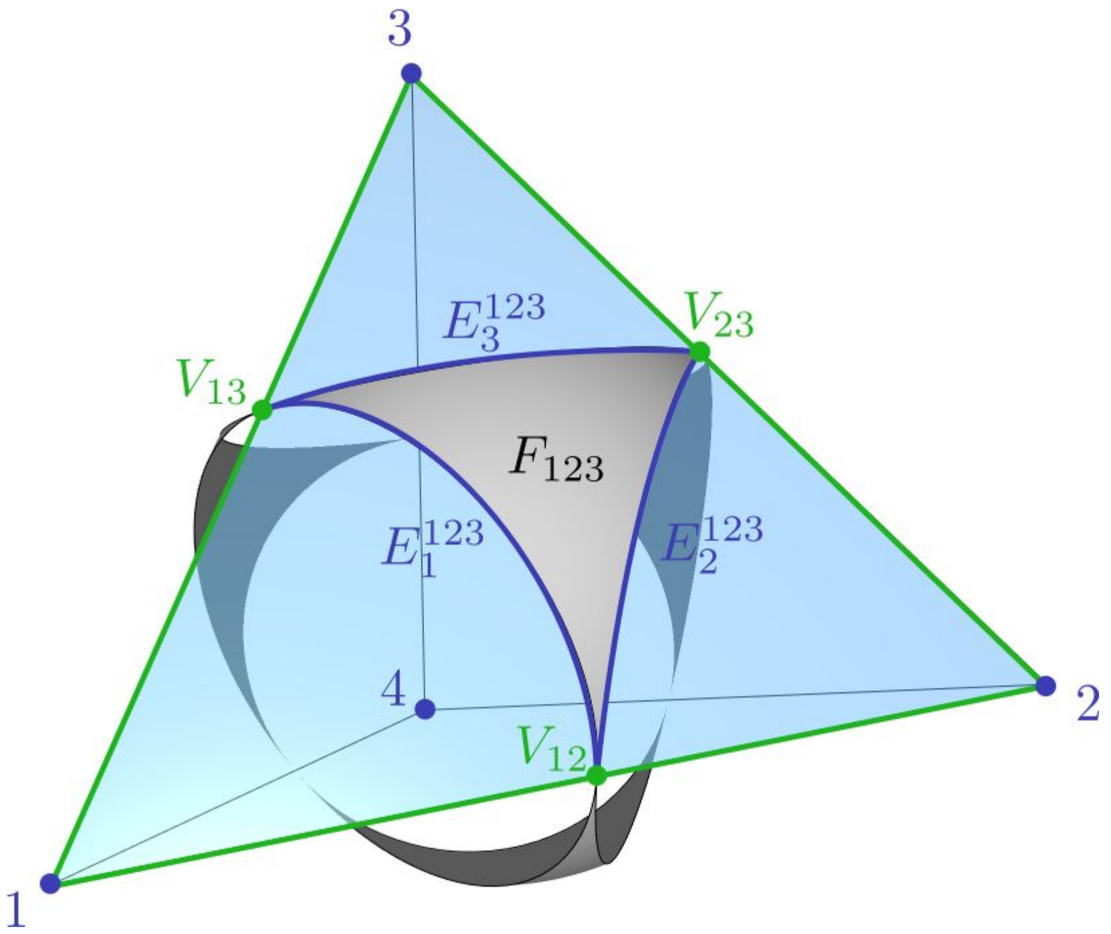
Bijections

- Sommet fractal = arête du polyèdre
- Arête fractale = sommet d'une face du polyèdre
- Face fractale = face du polyèdre



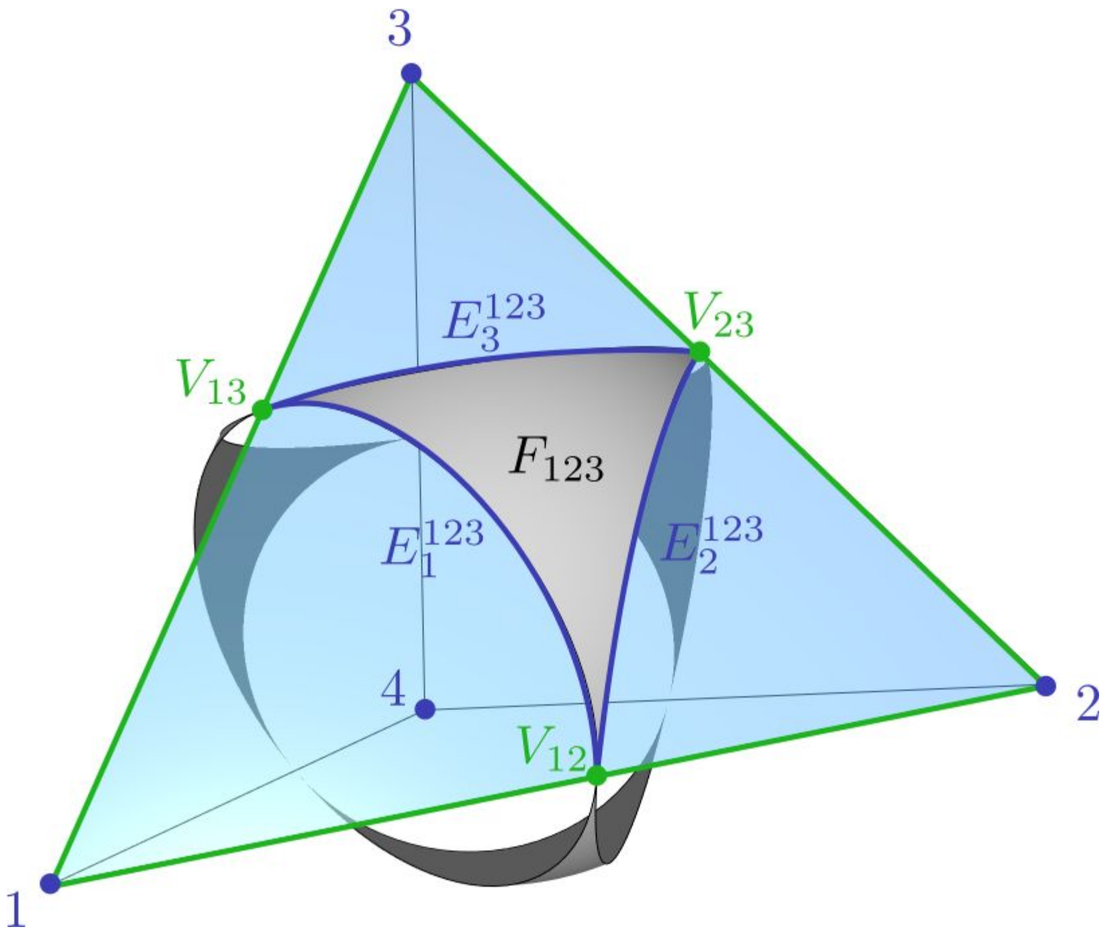
Automate

- États
  - Cellules topologiques



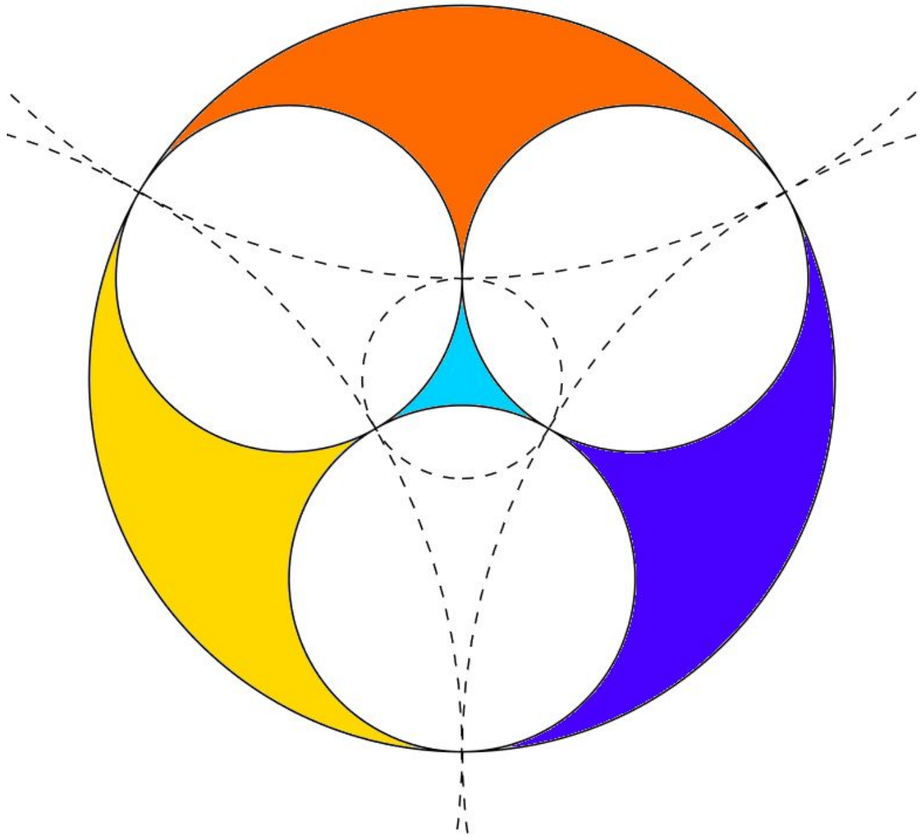
Automate

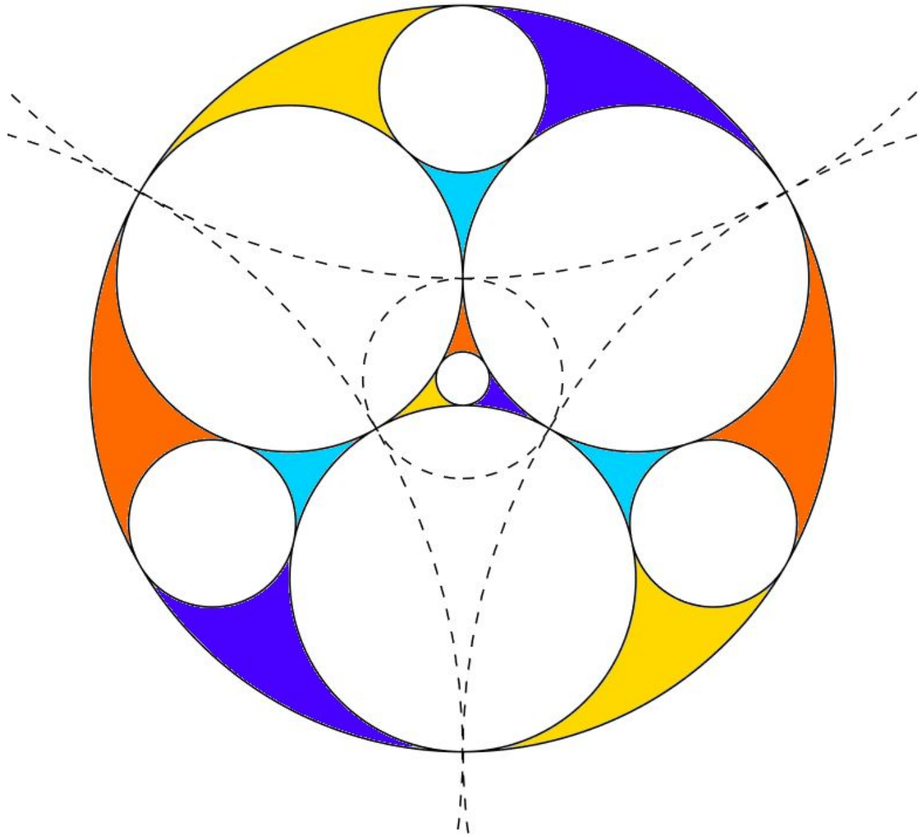
- États
  - Cellules topologiques
- Opérateurs de bord
  - Déduits à partir de la notation



Automate

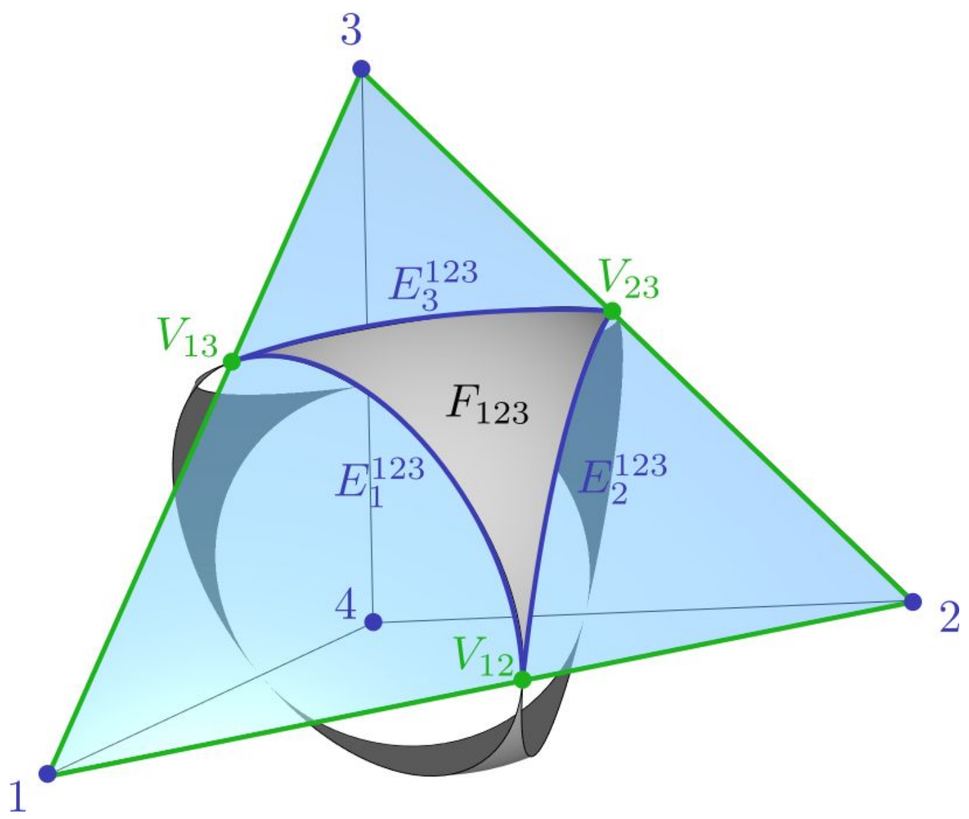
- Opérateurs de subdivision

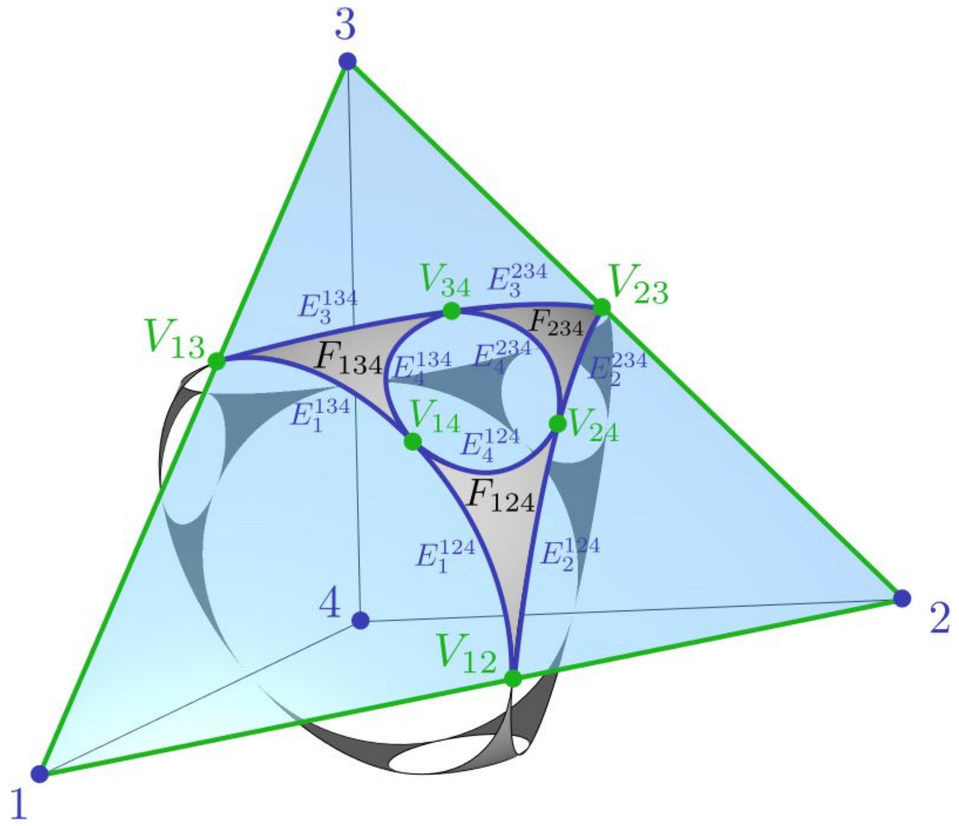




Automate

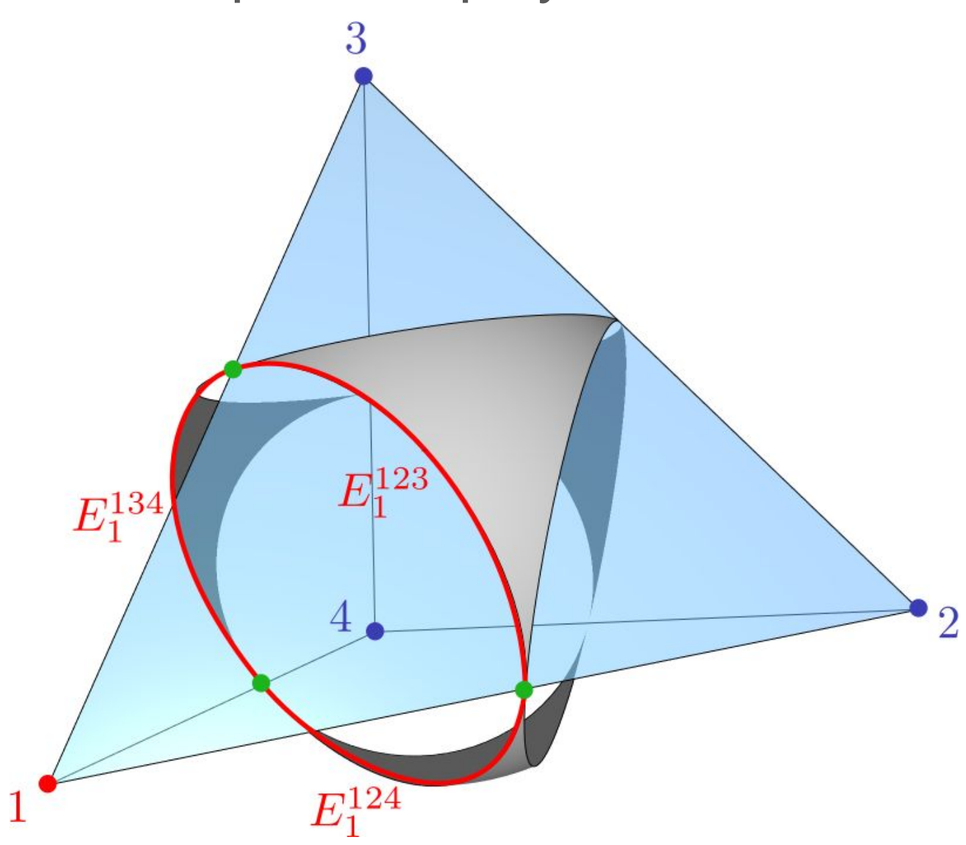
- Opérateurs de subdivision des faces
  - Déduits à partir du polyèdre

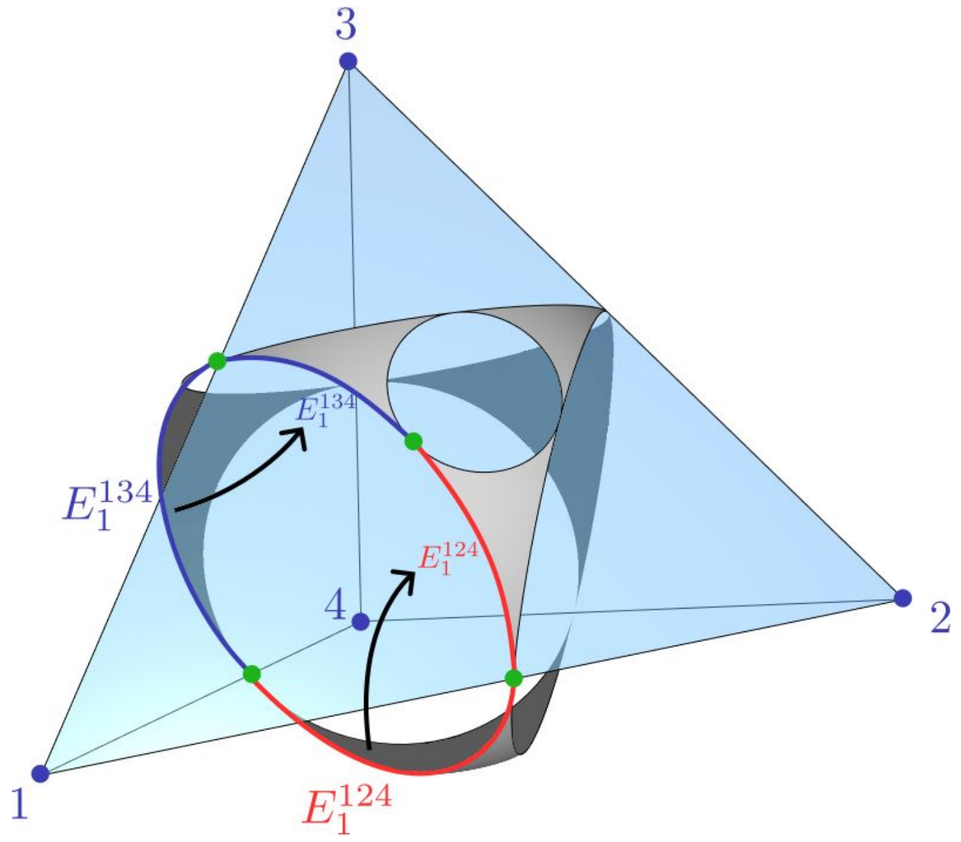




Automate

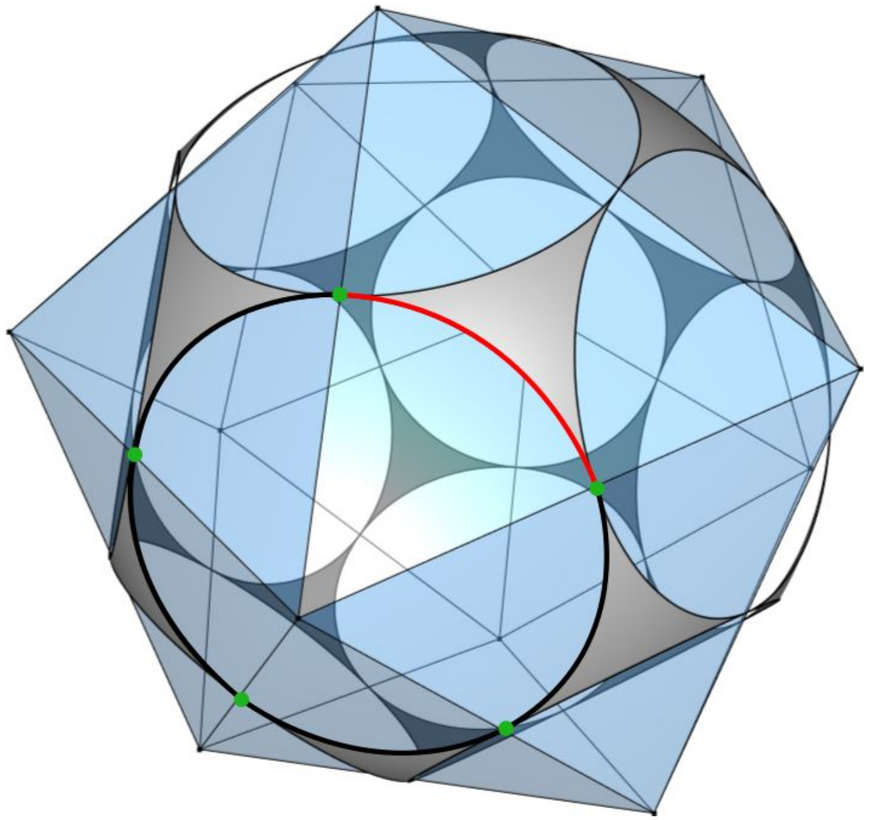
- Opérateurs de subdivision des arêtes
  - Déduits à partir du polyèdre

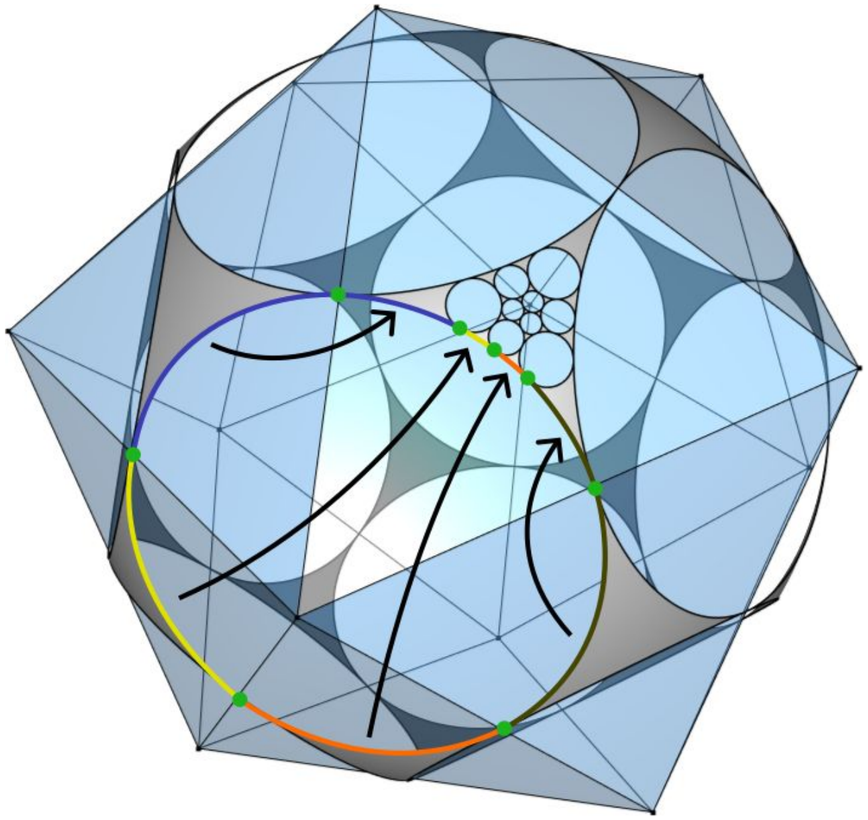




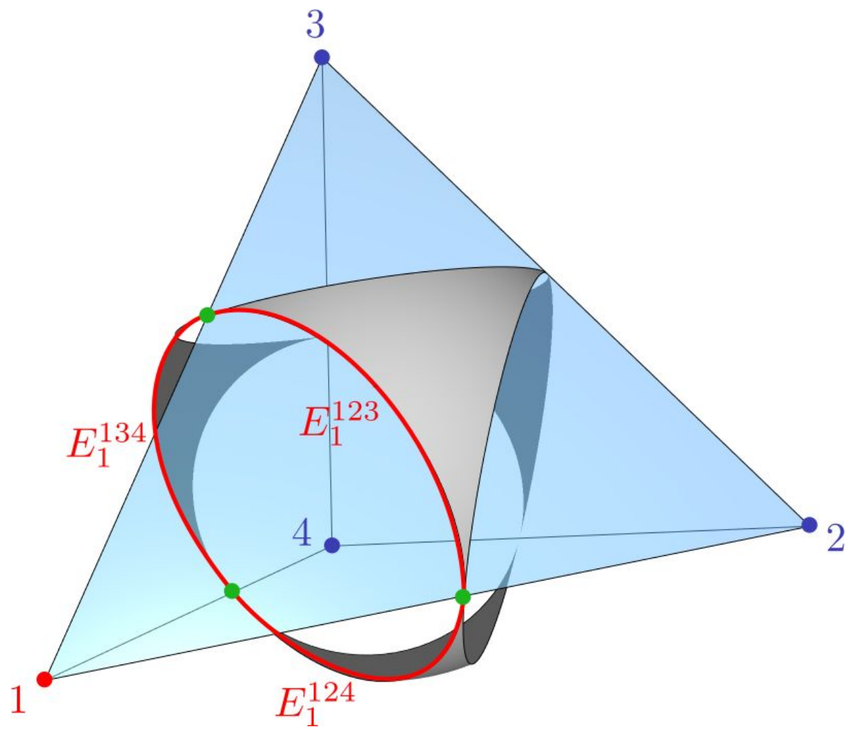
Automate

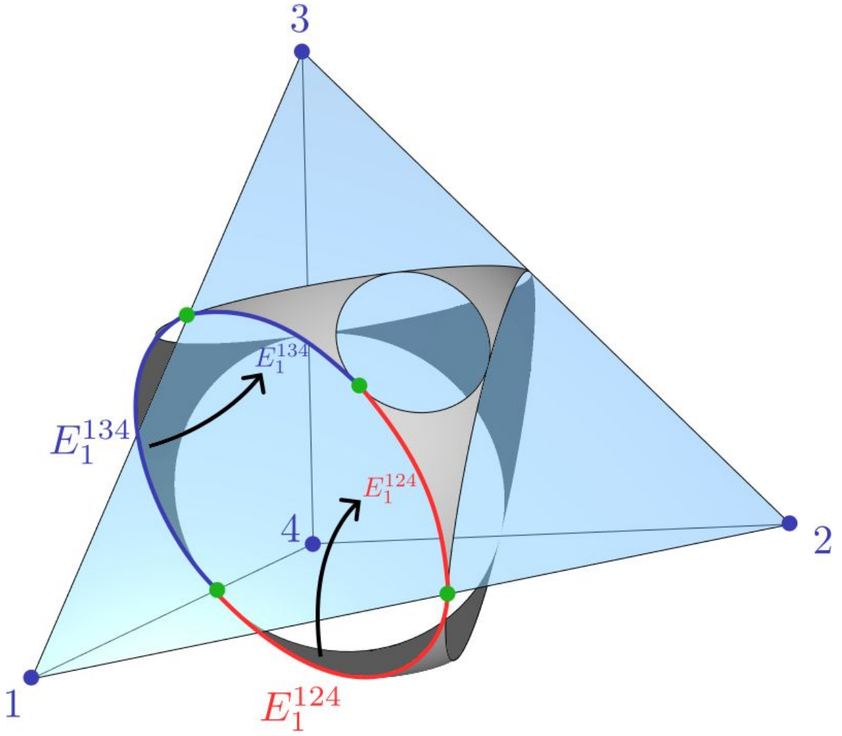
- Opérateurs de subdivision des arêtes
  - Déduits à partir du polyèdre

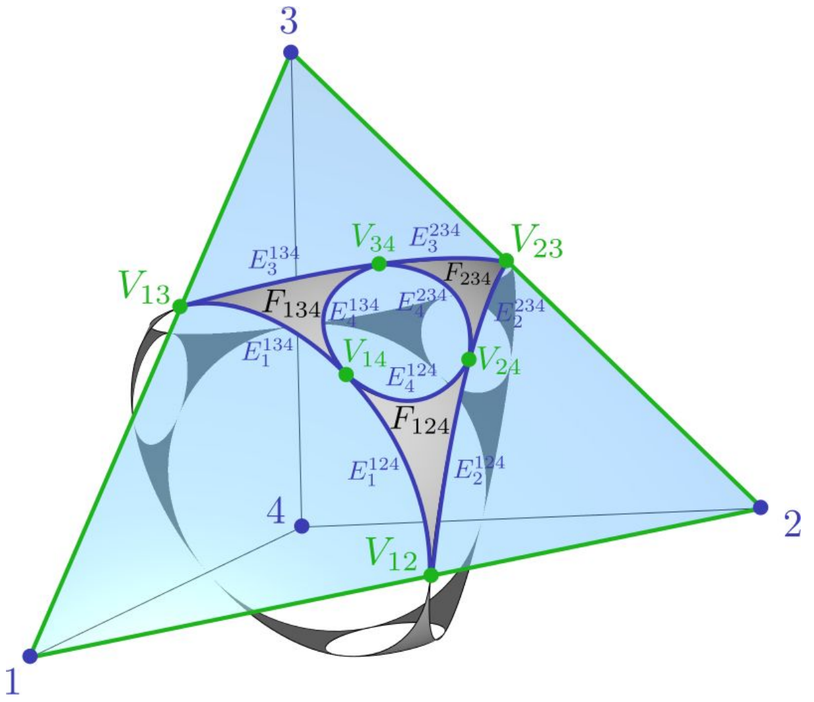




Contraintes d'incidence

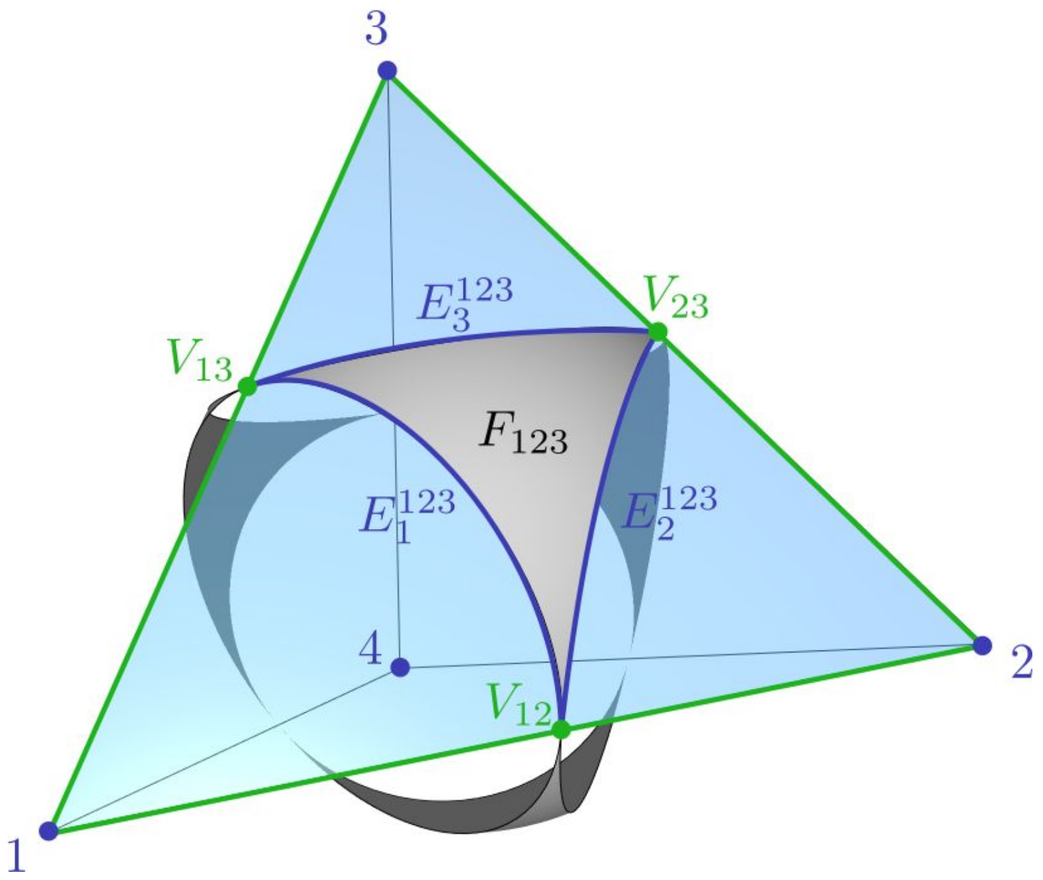


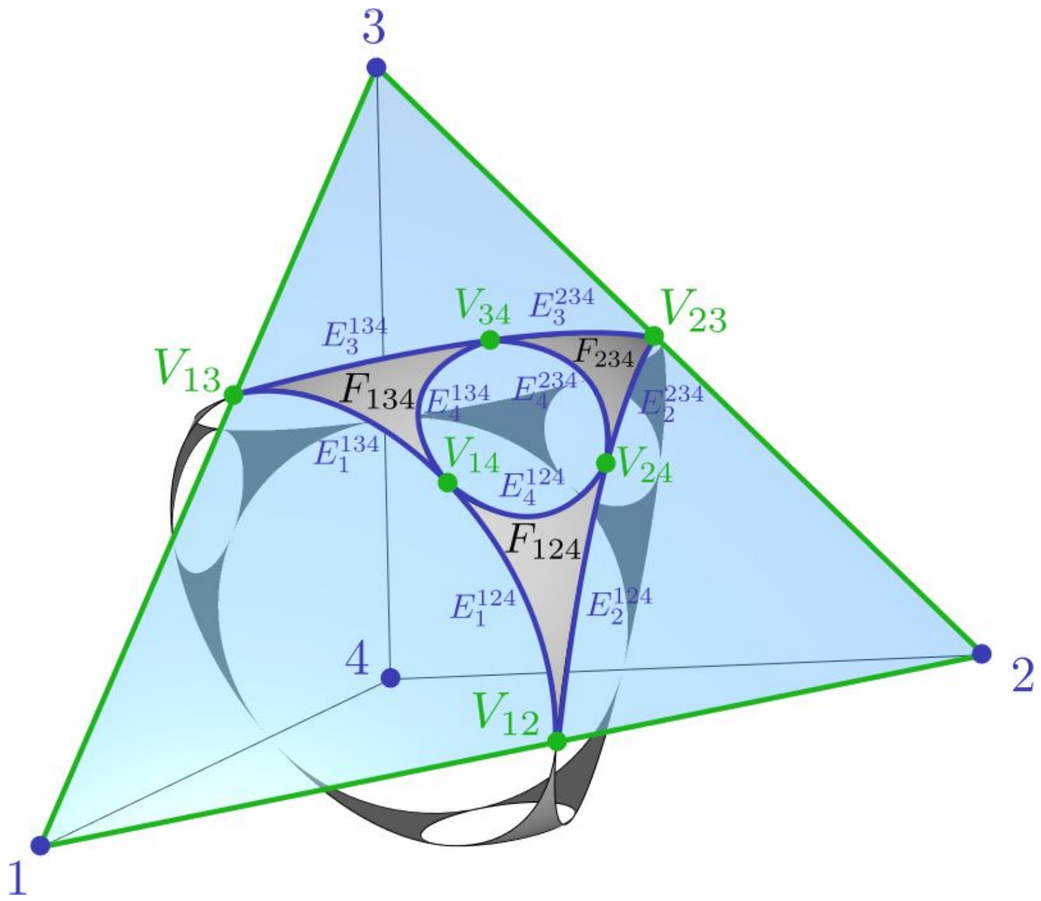




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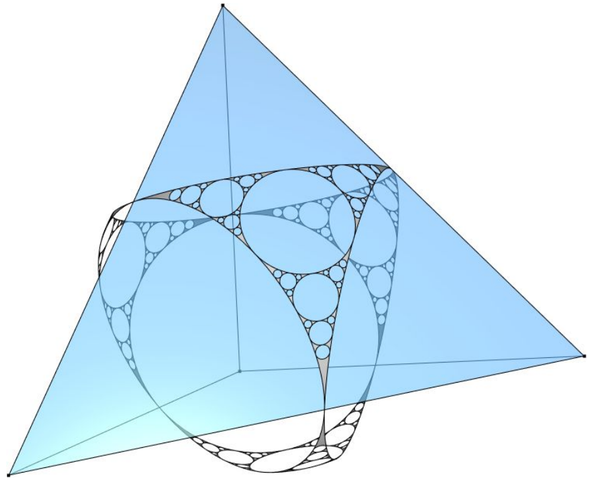
Contraintes d’adjacence

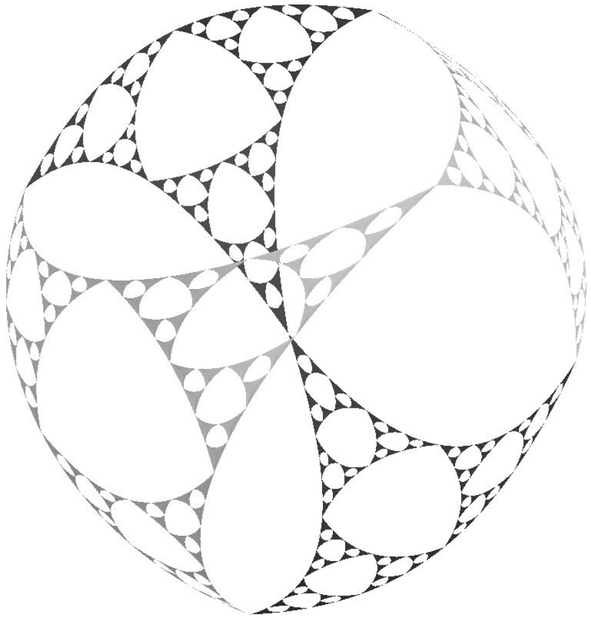


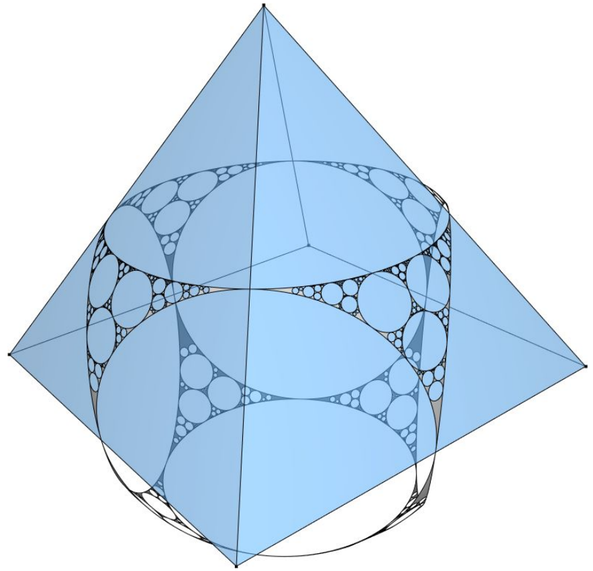


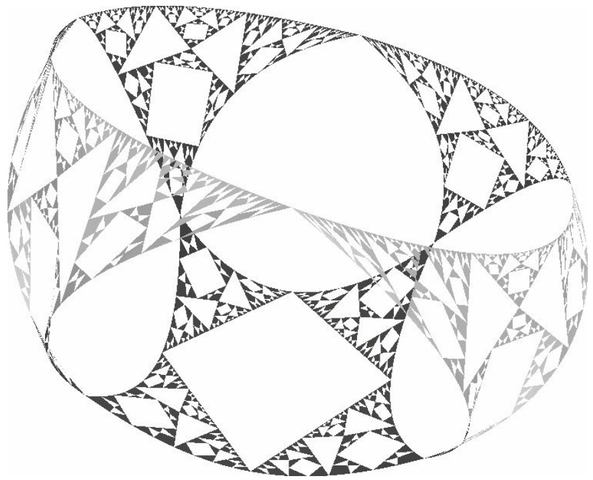
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Extraction à partir d’un polyèdre

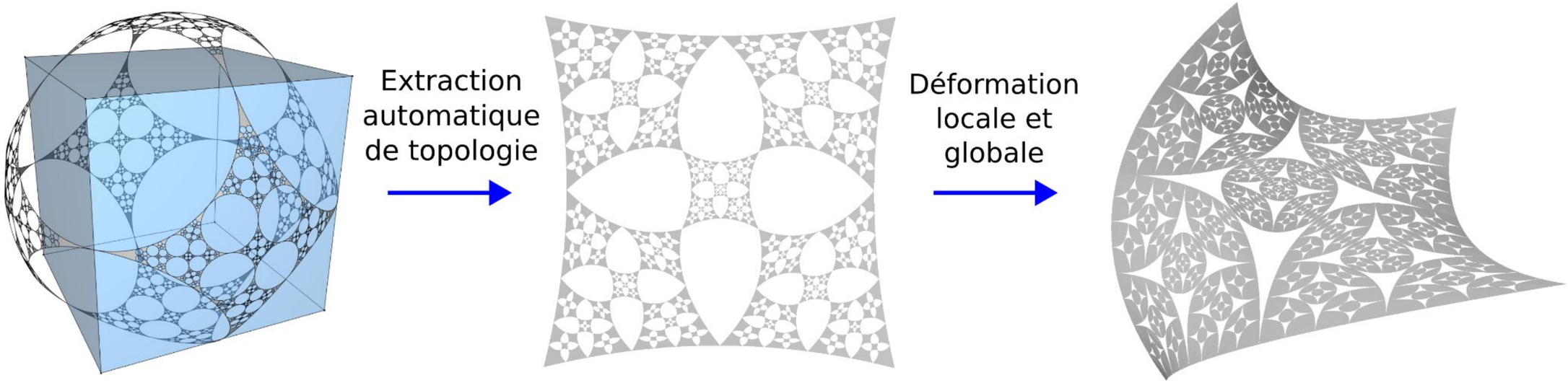




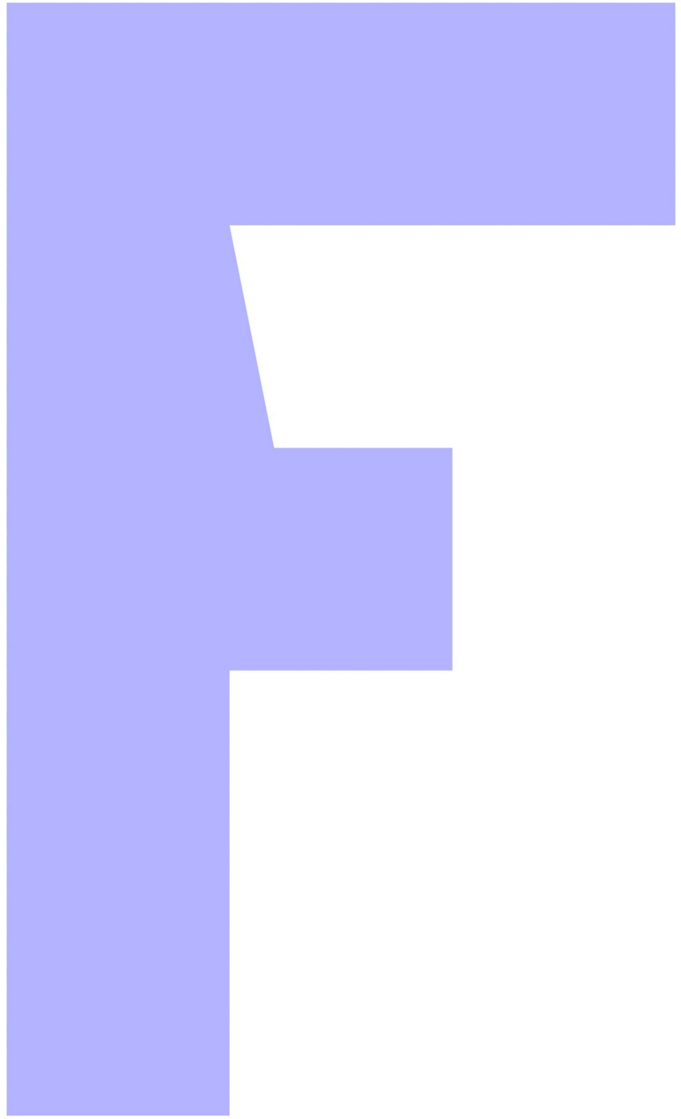




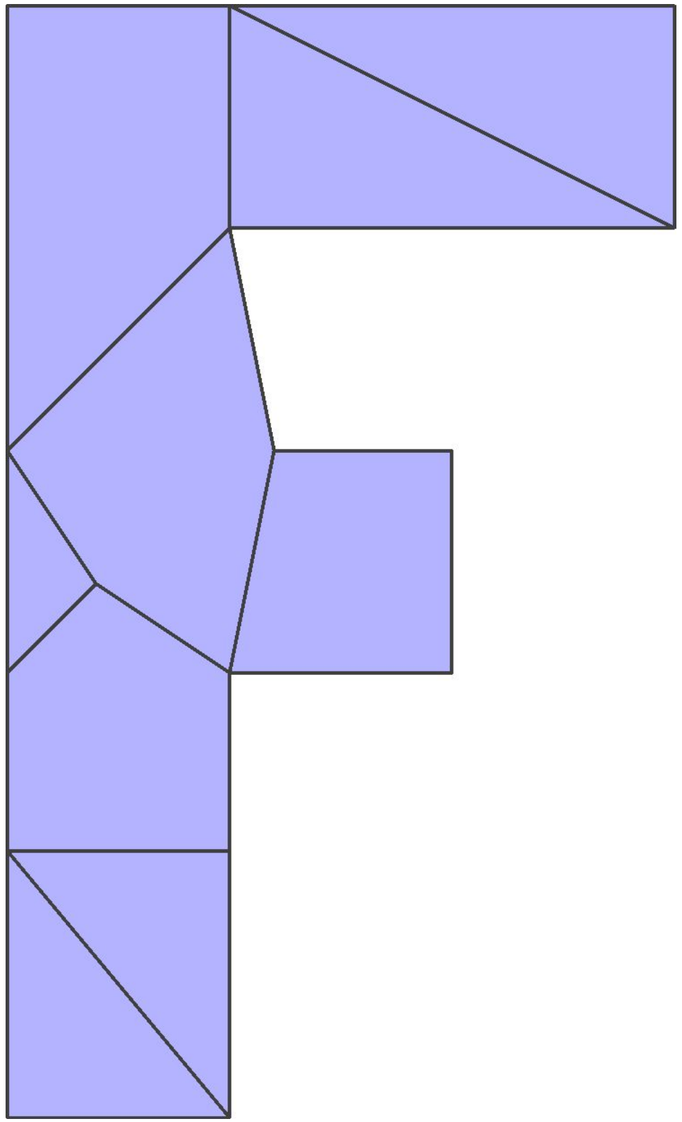
Extraction à partir d’un polyèdre



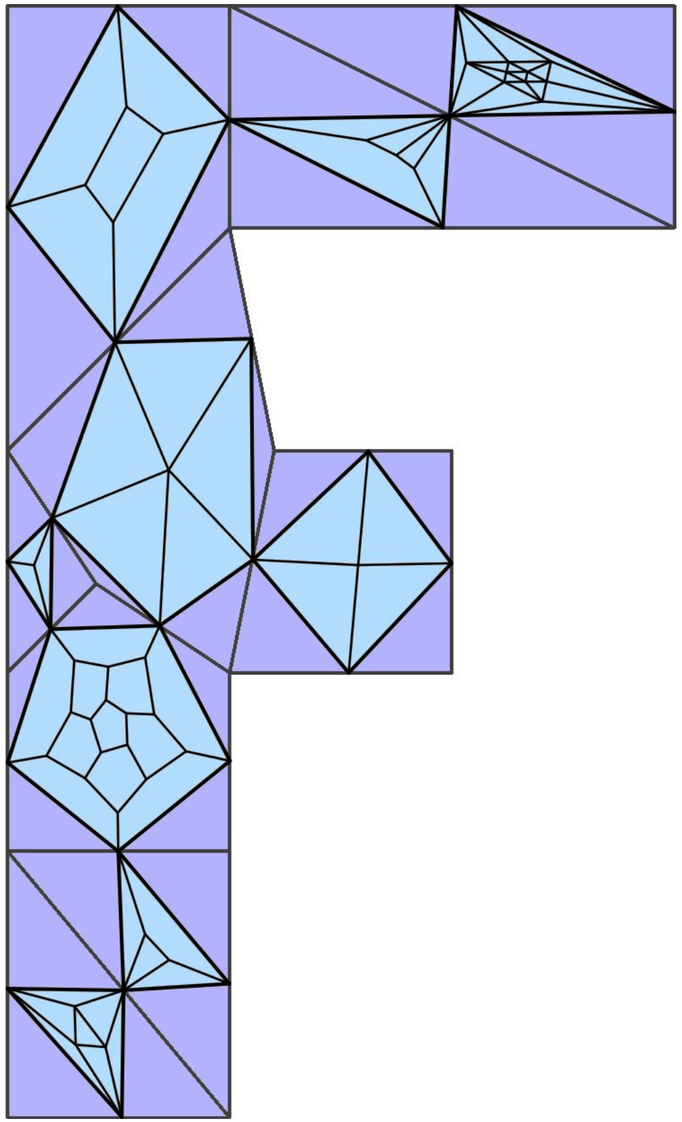
Assemblage



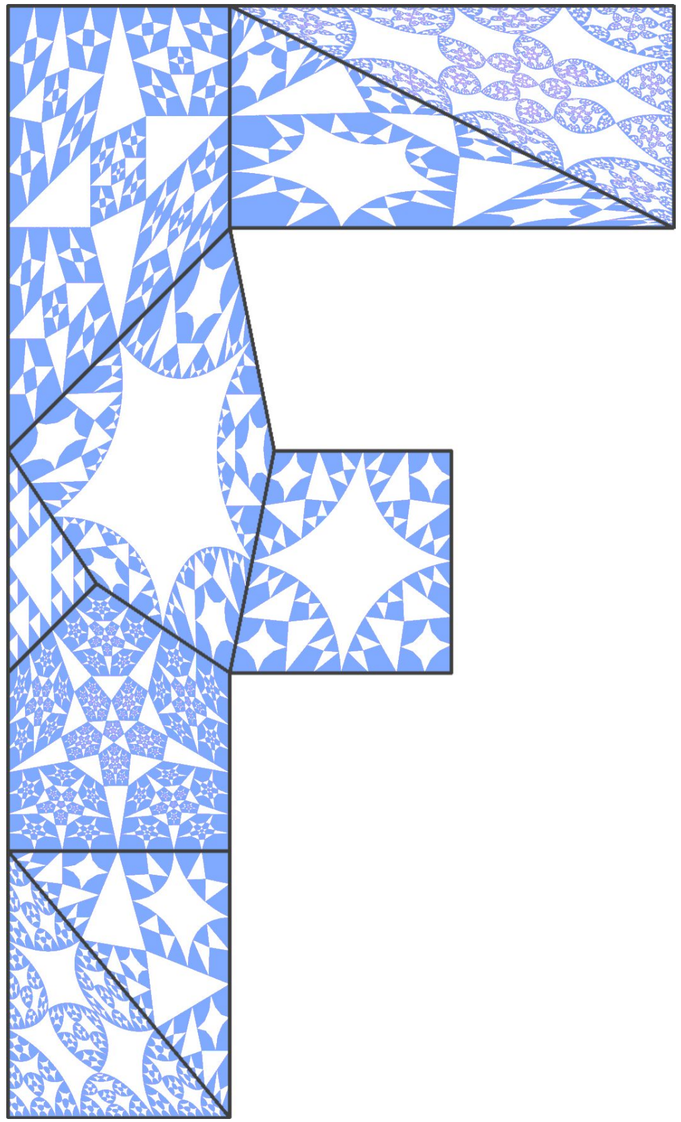
Assemblage



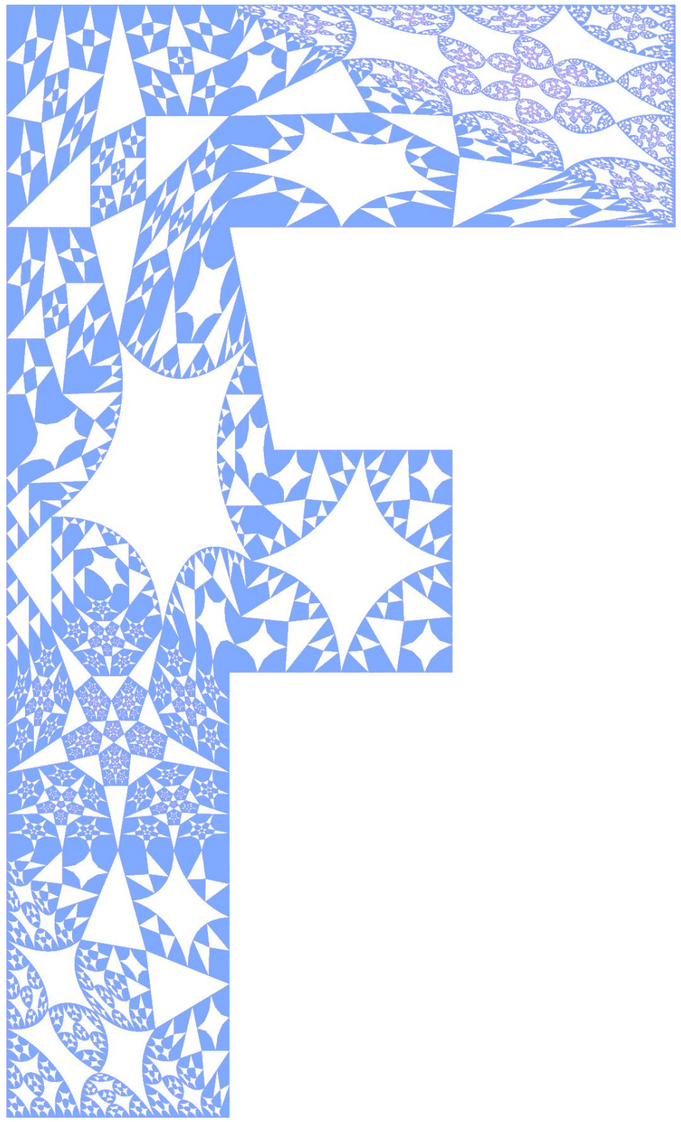
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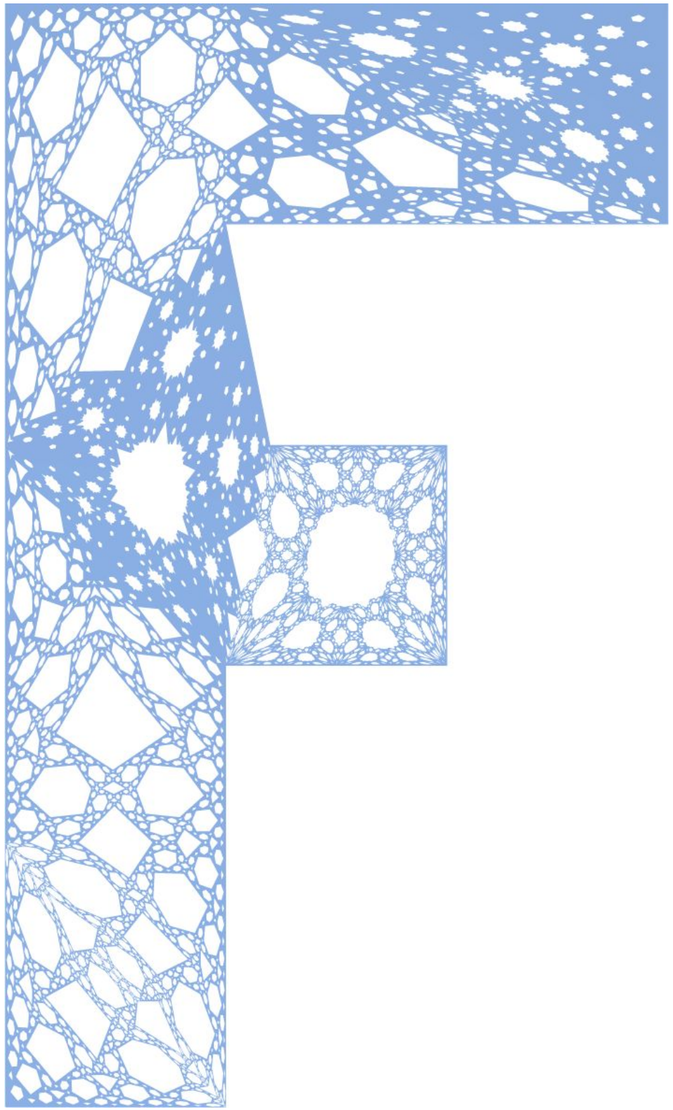
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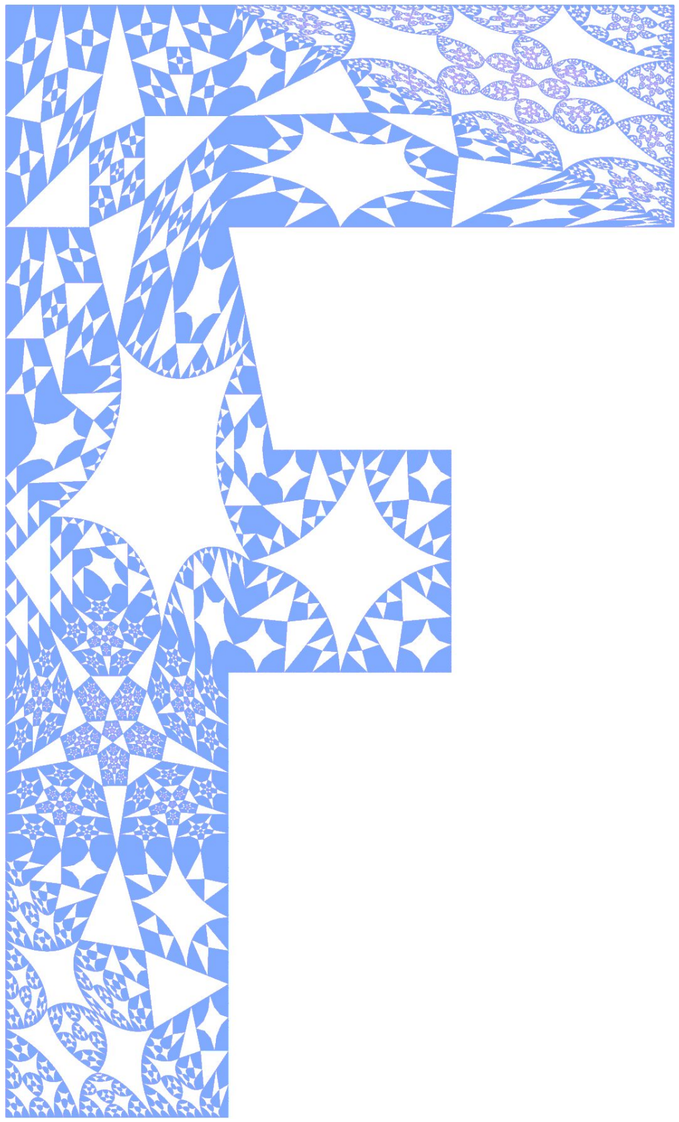


Assemblage



Assemblage

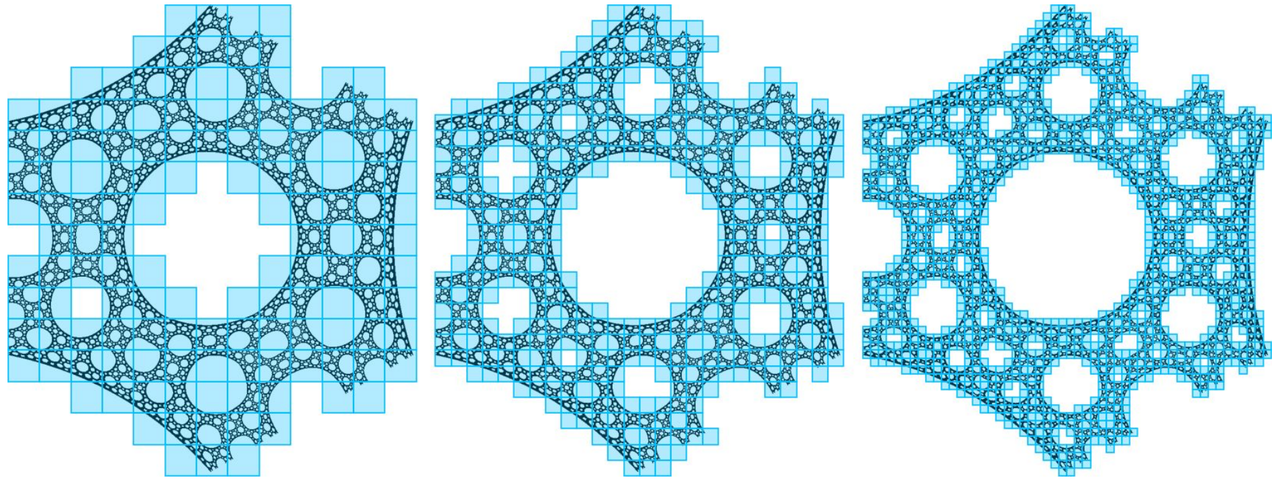




Mesures sur les structures fractales

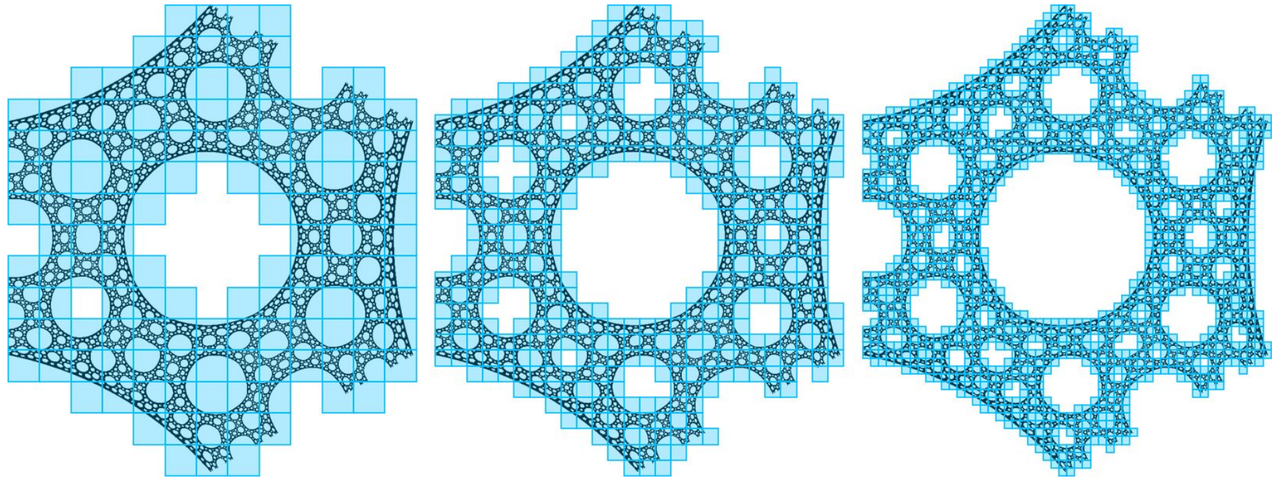
Mesures sur les fractales

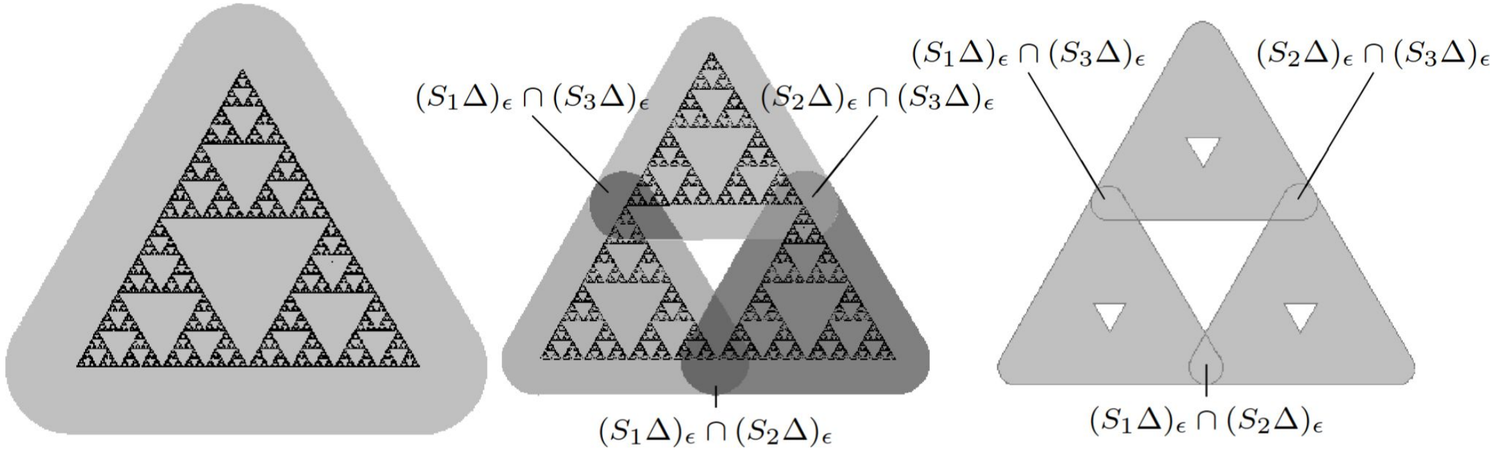
- Dimension fractale
  - Dimension de Minkowski-Bouligand



Mesures sur les fractales

- Dimension fractale
  - Dimension de Minkowski-Bouligand
- Caractéristique d'Euler
  - Adaptée aux fractales avec un voisinage

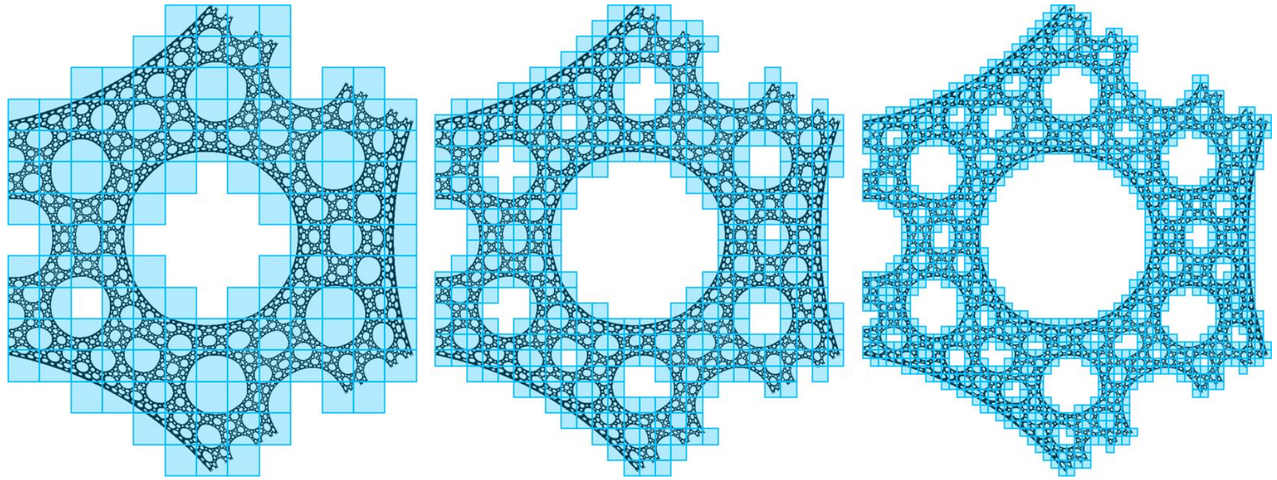


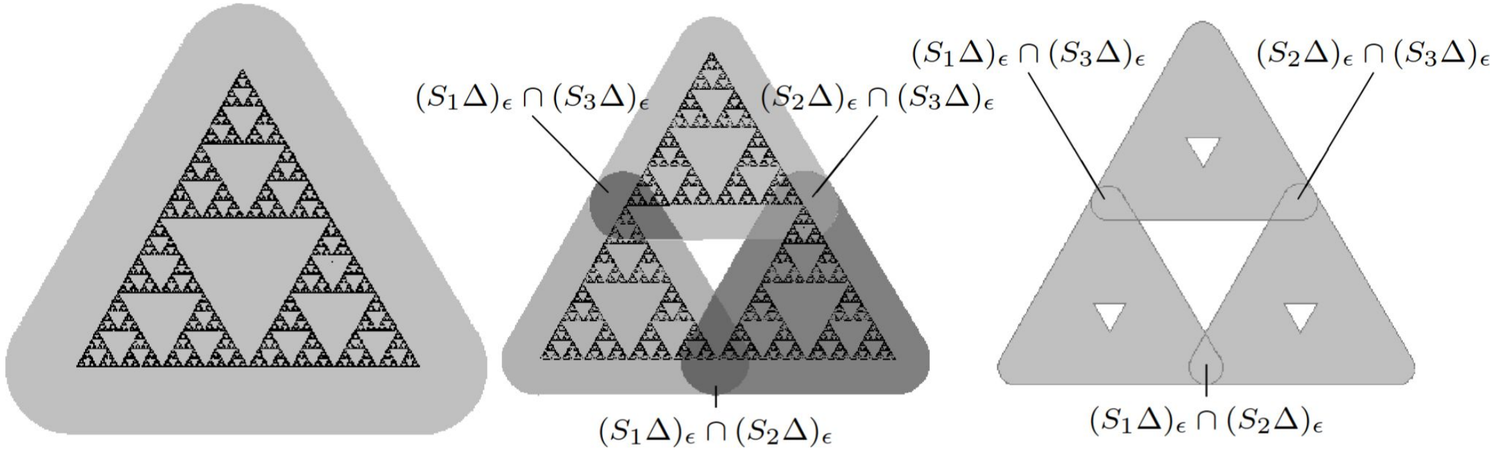


[LW07]

Mesures sur les fractales

- Dimension fractale
  - Dimension de Minkowski-Bouligand
- Caractéristique d'Euler
  - Adaptée aux fractales avec un voisinage
- Surface et périmètre
  - Évolution au fil des itérations



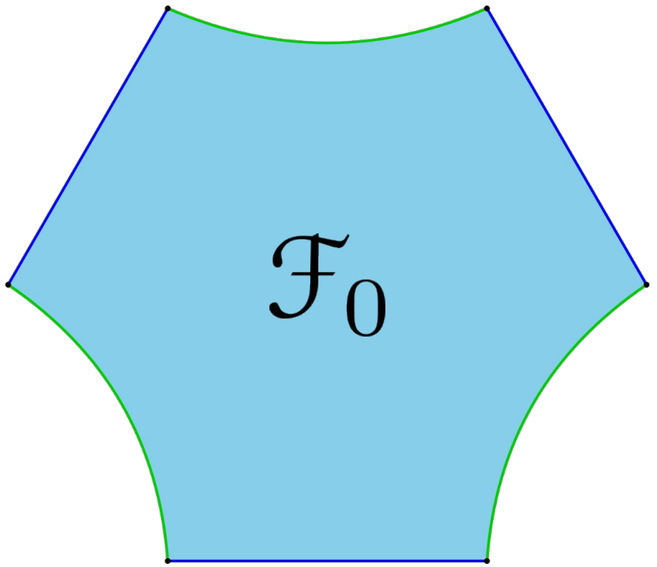


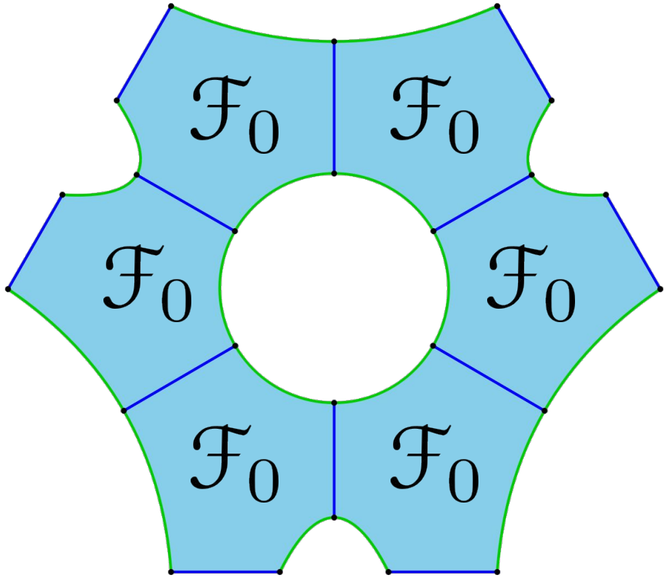
[LW07]

Mesure purement topologique

- Nombre de sous-faces

$$N_i^C = n^i$$





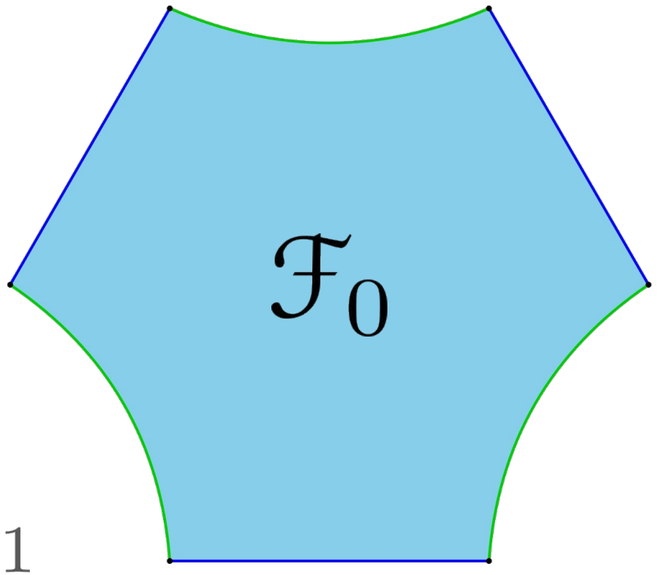
Mesure purement topologique

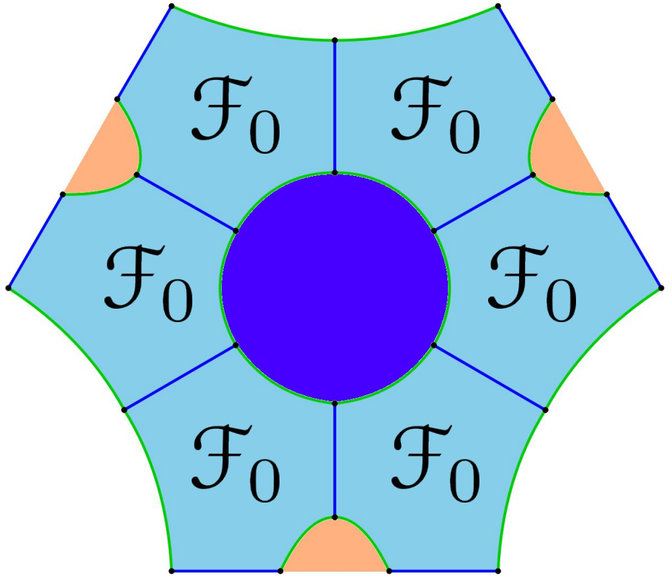
- Nombre de sous-faces

$$N_i^C = n^i$$

- Nombre de lacunes

$$N_i^L = N_1^L + n \times N_{i-1}^L$$





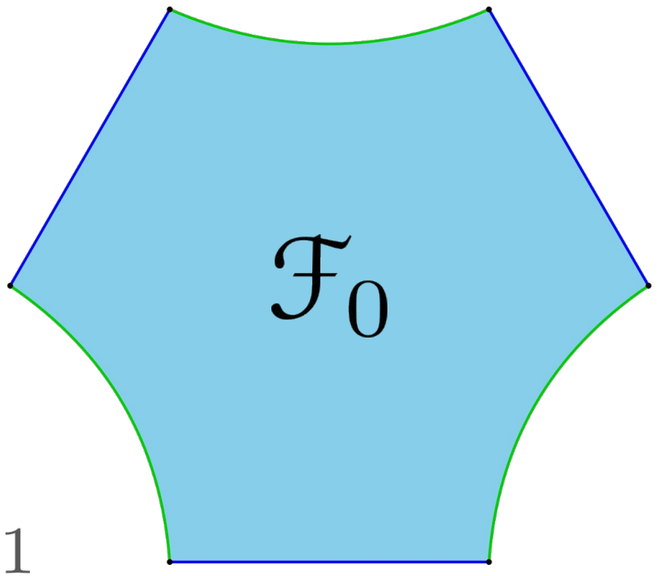
Mesure purement topologique

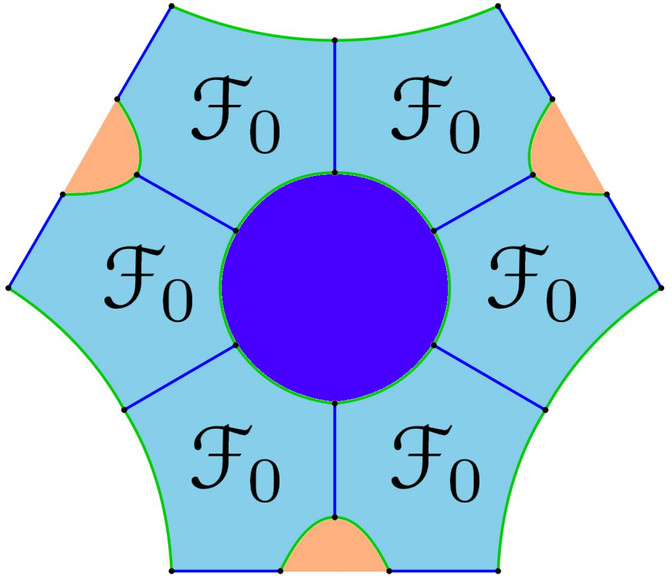
- Nombre de sous-faces

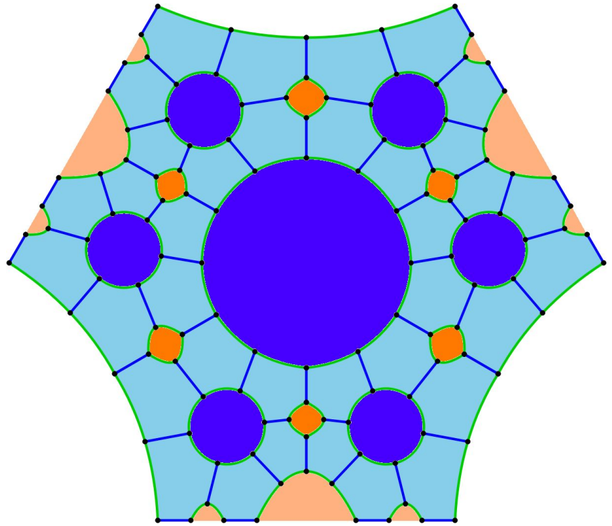
$$N_i^C = n^i$$

- Nombre de lacunes

$$N_i^L = N_1^L + n \times N_{i-1}^L$$







Mesure purement topologique

- Nombre de sous-faces

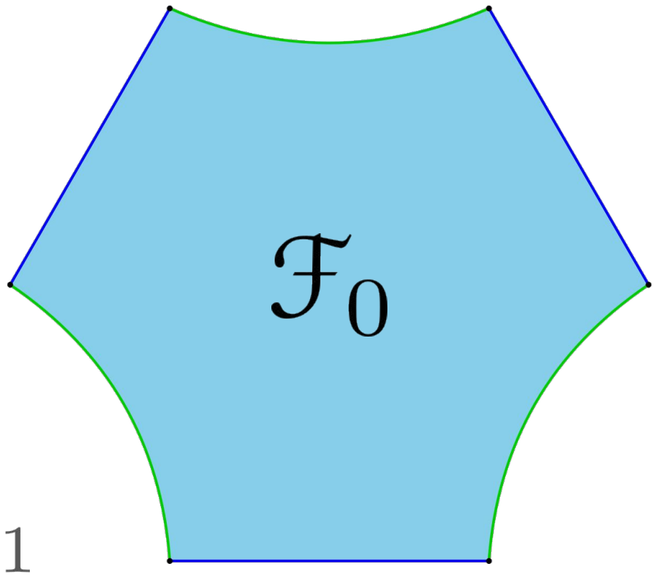
$$N_i^C = n^i$$

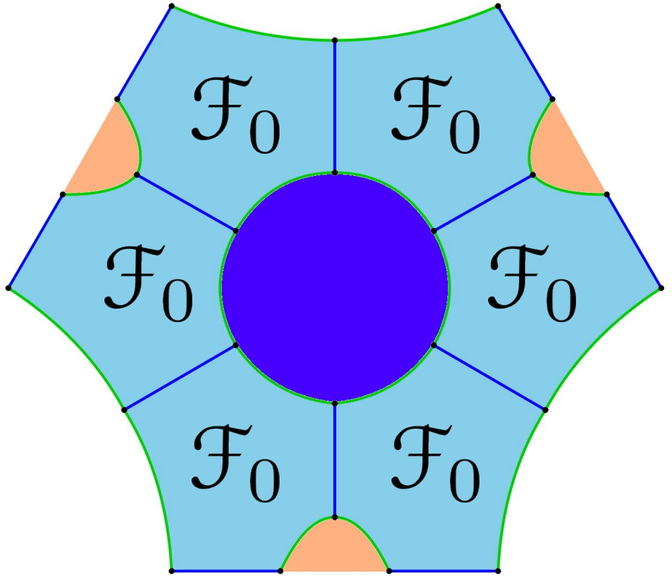
- Nombre de lacunes

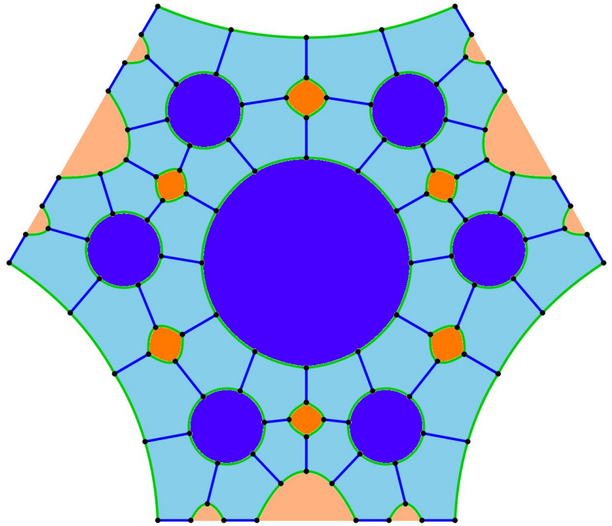
$$N_i^L = N_1^L + n \times N_{i-1}^L$$

- Ratio

$$R_i = \frac{N_i^L}{N_i^C + N_i^L}$$

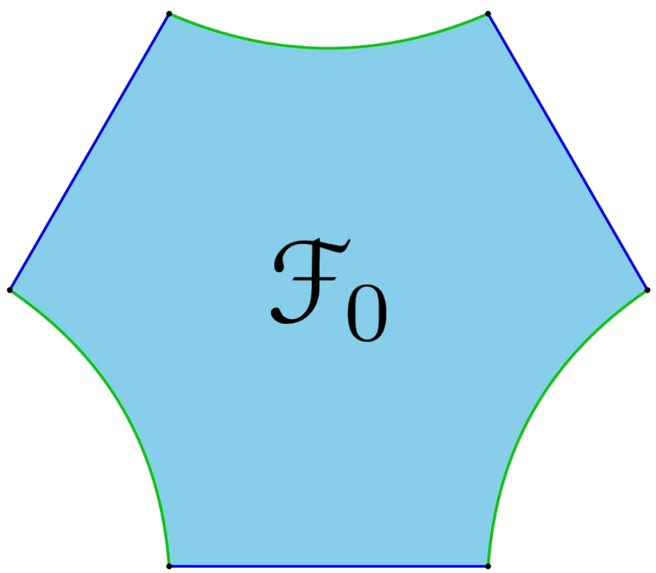


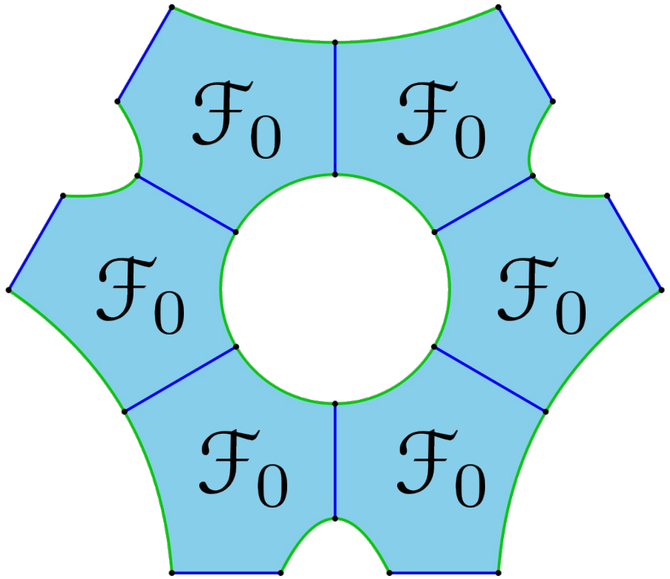


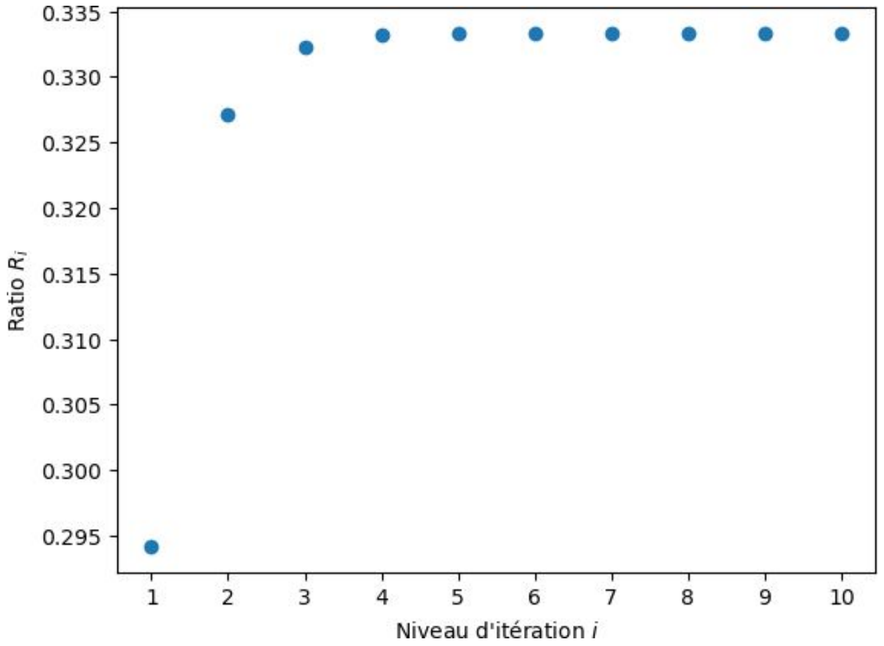


Limite du ratio

$$R = \lim_{i \to +\infty} R_i = \frac{N_1^L}{N_1^L + N_1^C - 1}$$



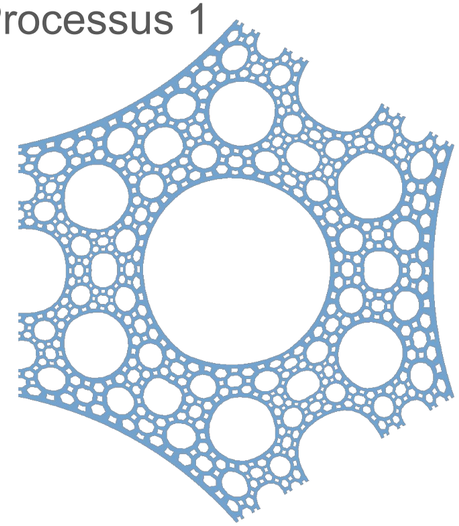




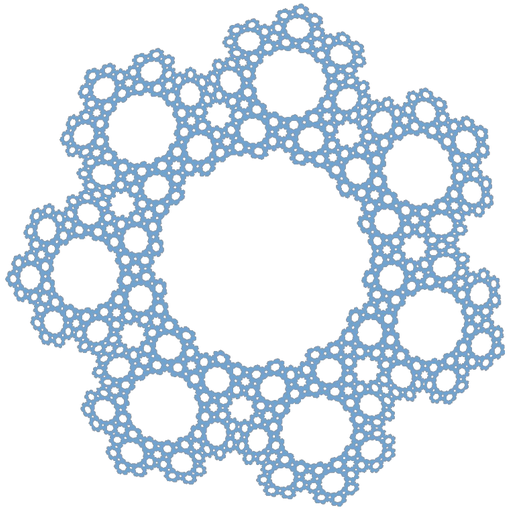
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Limite du ratio

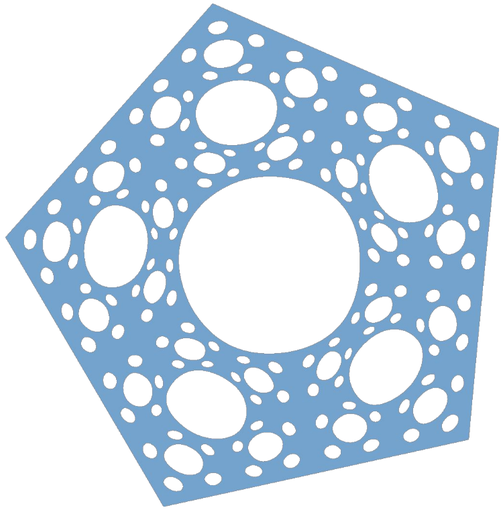
Processus 1



$R \approx 0{,}33$

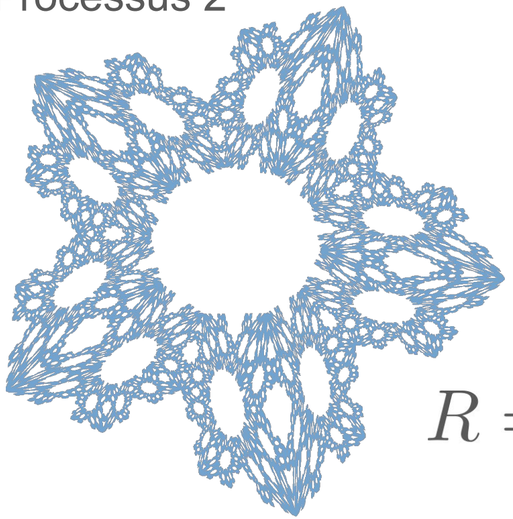


$R \approx 0{,}43$

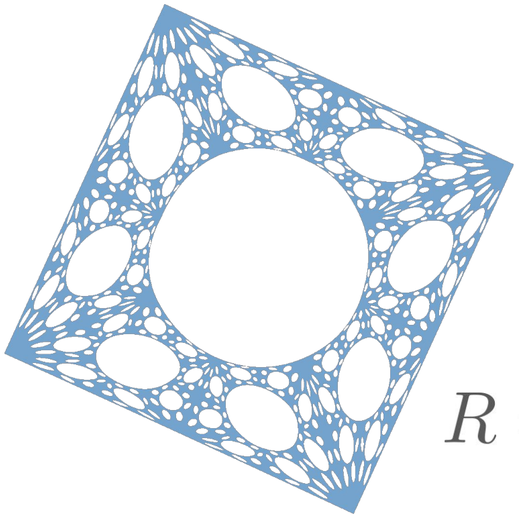


$R = 0{,}20$

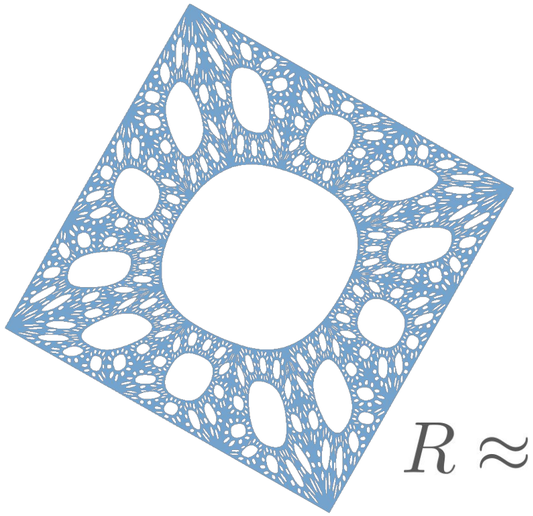
Processus 2



$R = 0{,}28$



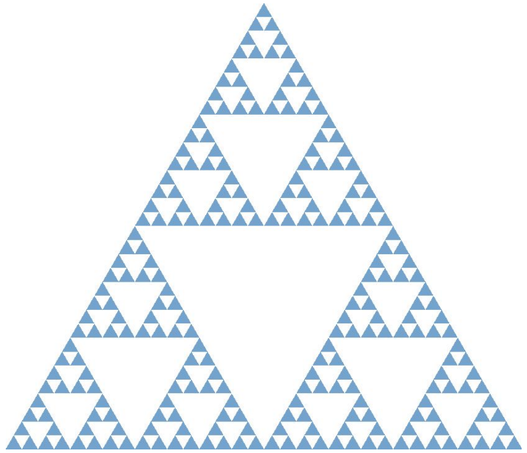
$R = 0{,}125$



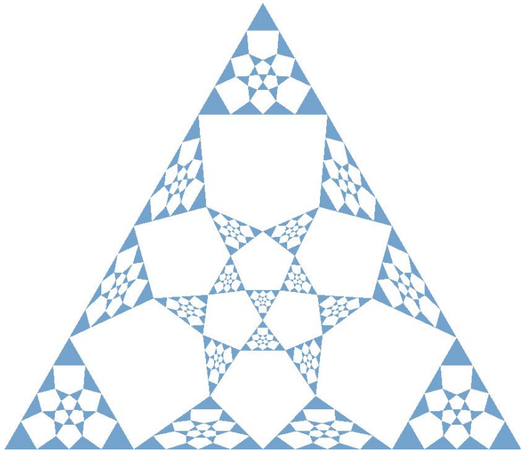
$R \approx 0{,}08$

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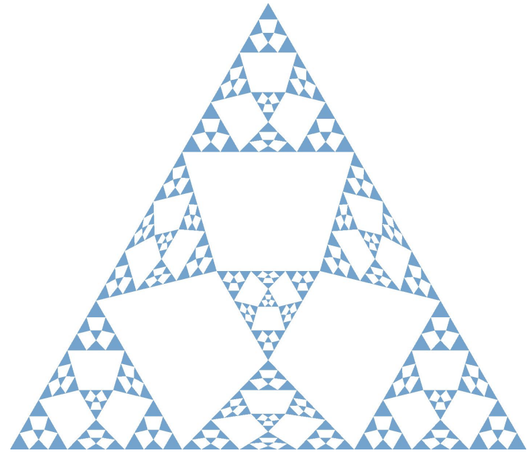
Limite du ratio



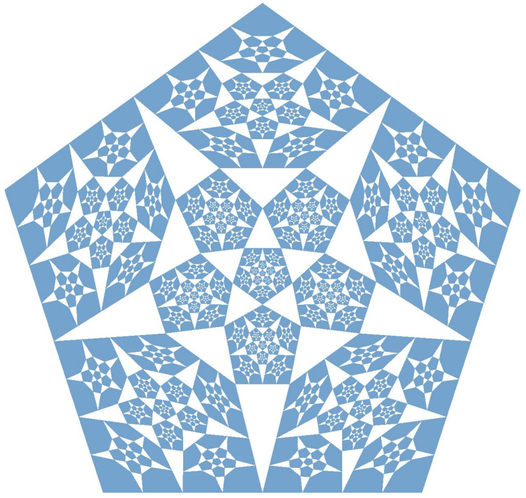
$R \approx 0{,}33$



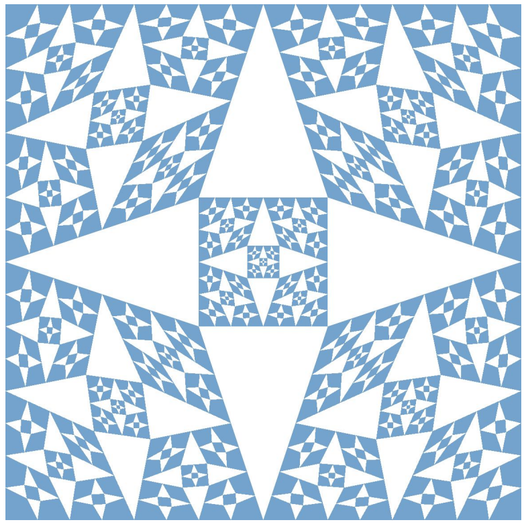
$R \approx 0{,}33$



$R \approx 0{,}33$



$R = 0{,}60$



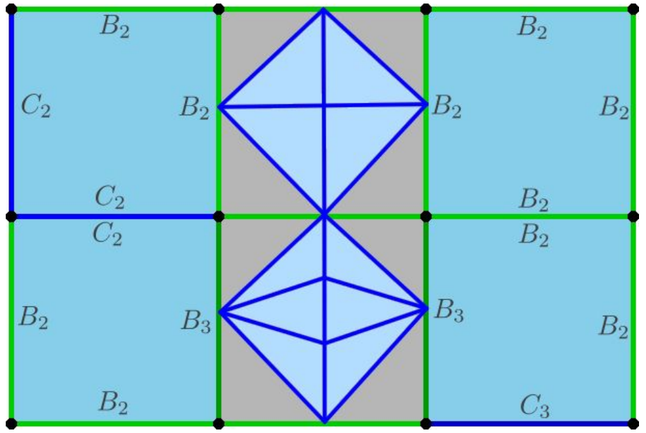
$R = 0{,}50$

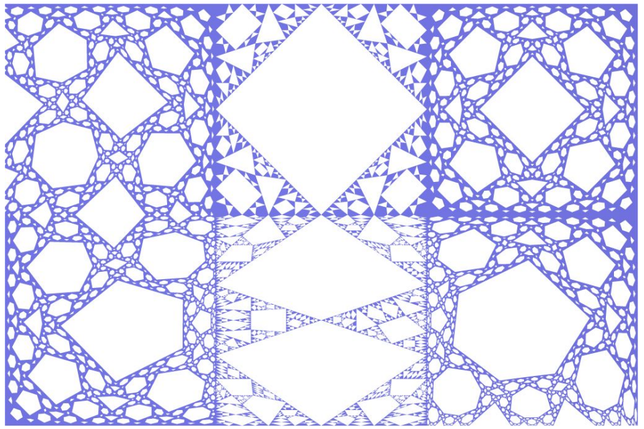
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Conclusion

Contributions

- 2 approches de conception automatique de structures lacunaires fractales
  - Processus de subdivision générique
  - Algorithmes pour traduction avec BC-IFS
  - Compatibles pour l'assemblage

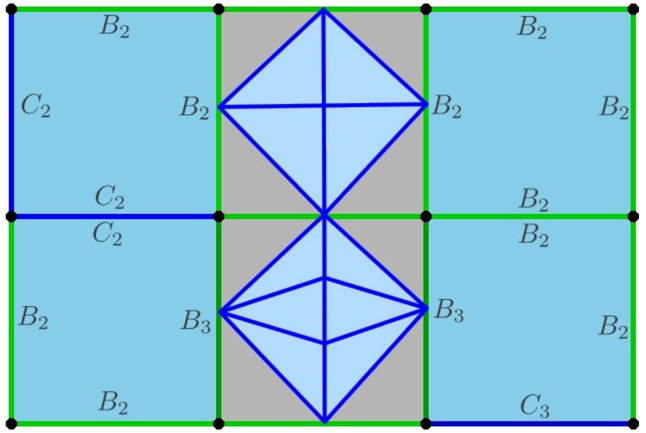


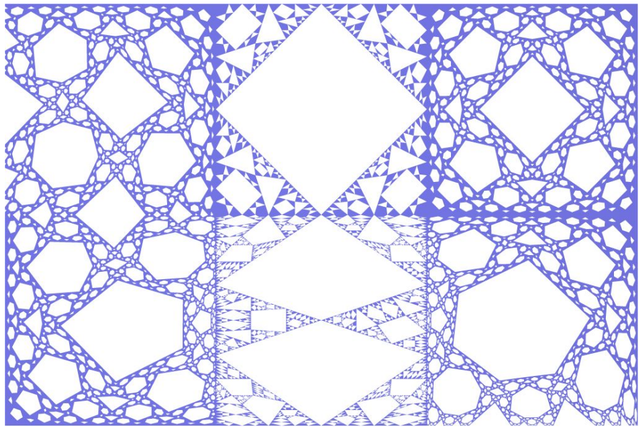


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Contributions

- 2 approches de conception automatique de structures lacunaires fractales
  - Processus de subdivision générique
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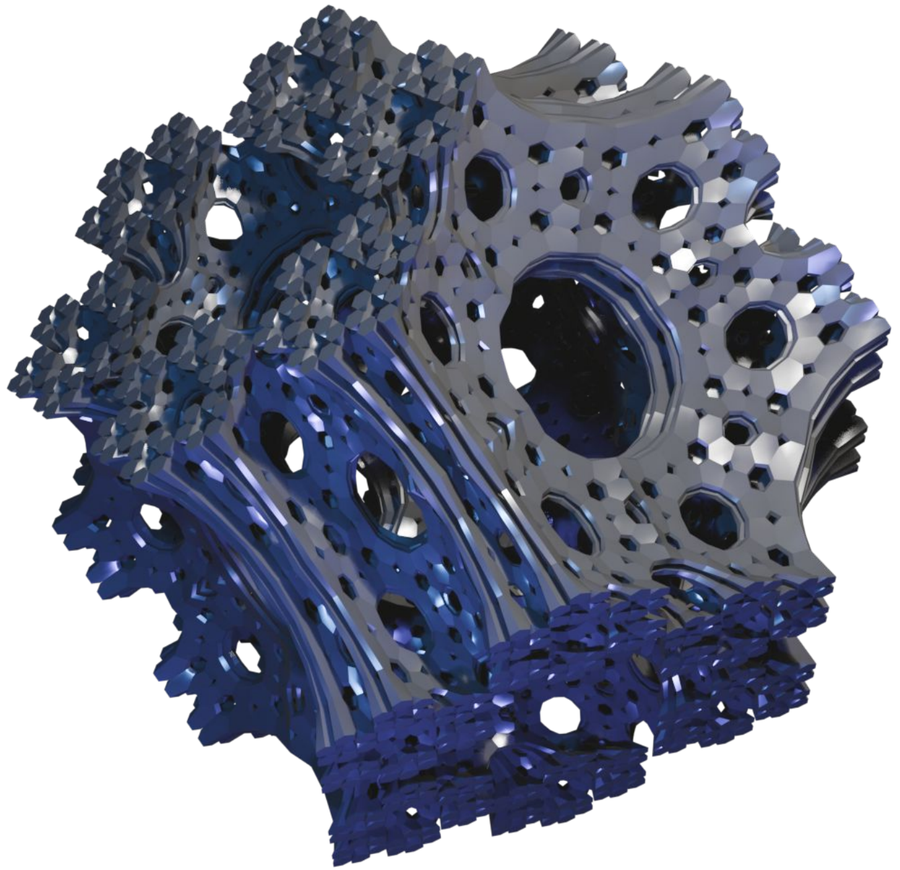


- Mesure purement topologique
  - Calcul à partir du niveau 1 d'itération
  - Calcul à partir de l'automate sinon

$$R = \lim_{i \to +\infty} R_i = \frac{N_1^L}{N_1^L + N_1^C - 1}$$

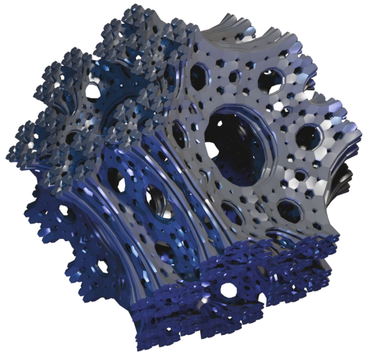
Travaux futurs

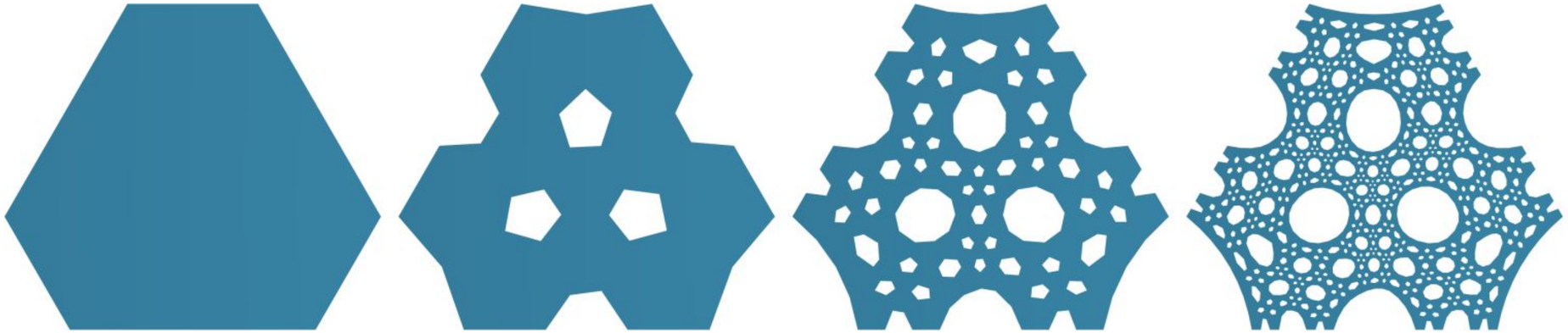
- Extension à la 3D



Travaux futurs

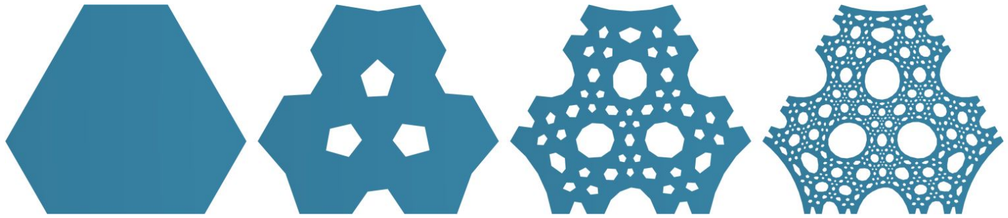
- Extension à la 3D
- Autres processus en 2D
  - Plusieurs lacunes centrales

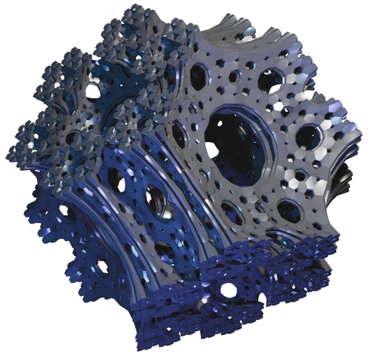


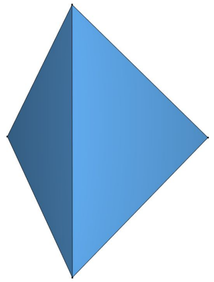


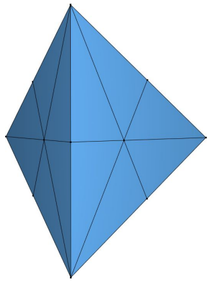
Travaux futurs

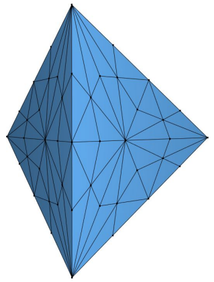
- Extension à la 3D
- Autres processus en 2D
  - Plusieurs lacunes centrales
  - Autres types d’empilements

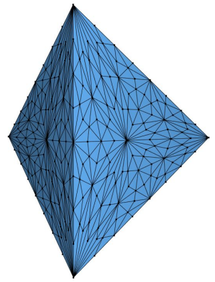


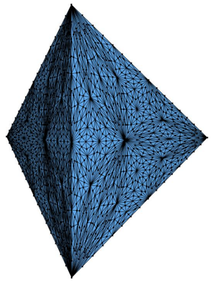


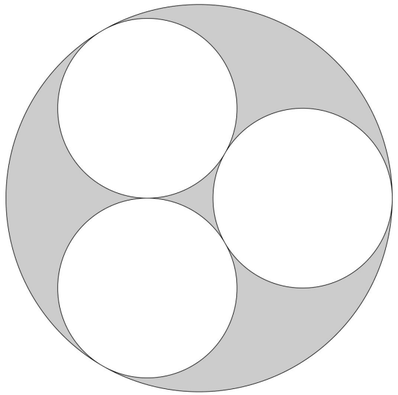


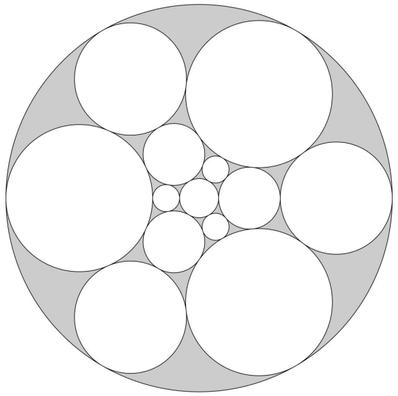


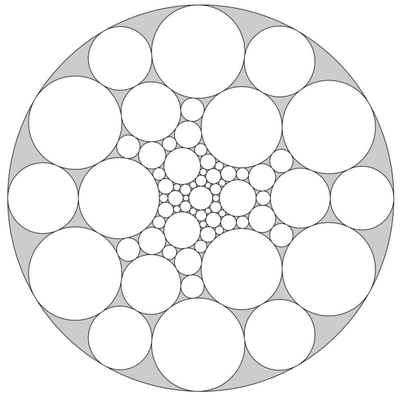


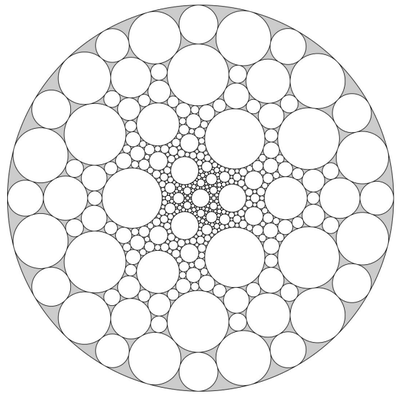


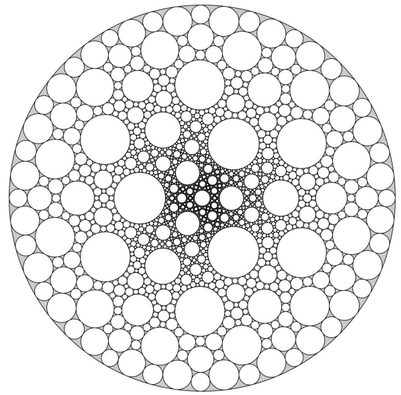






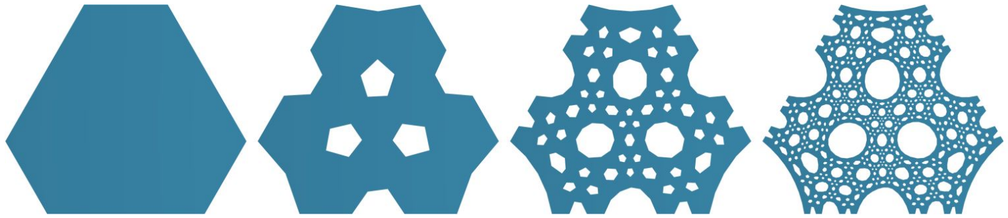


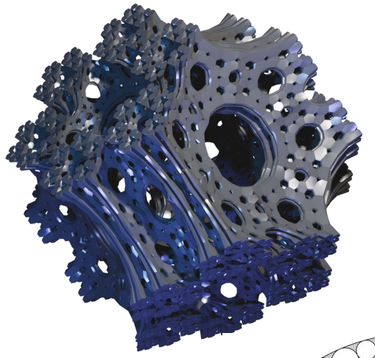


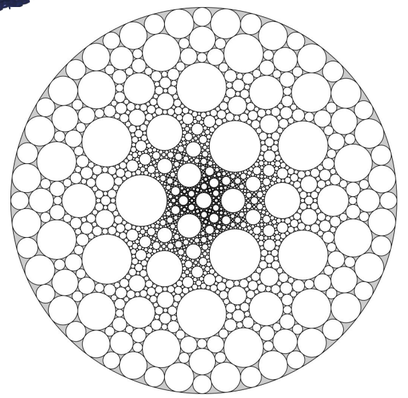


Travaux futurs

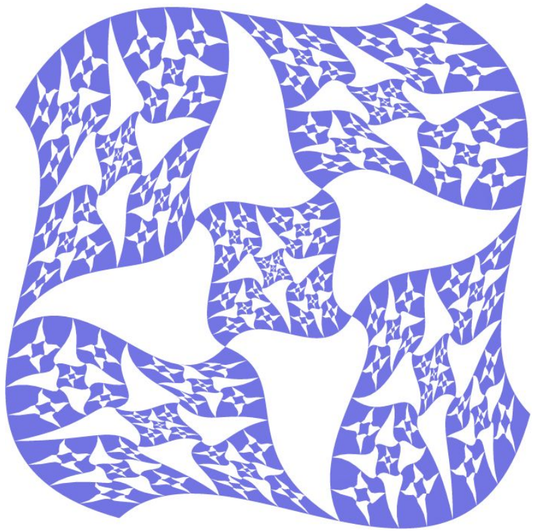
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- Autres processus en 2D
  - Plusieurs lacunes centrales
  - Autres types d'empilements





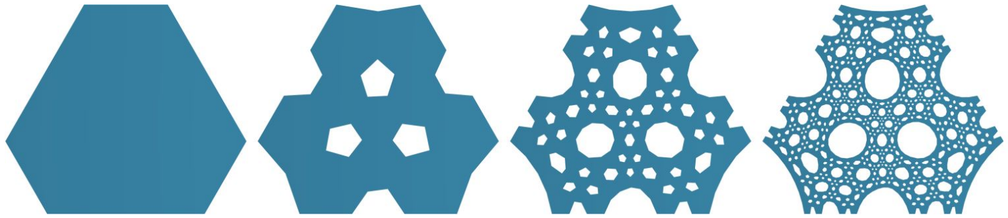


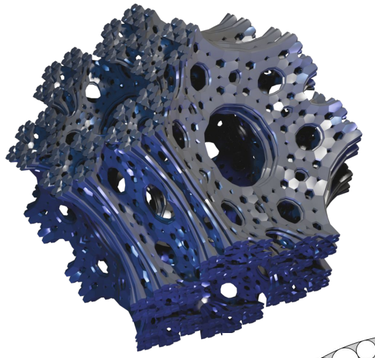
- Impression 3D des structures issues d'empilements
  - Épaississement des raccords

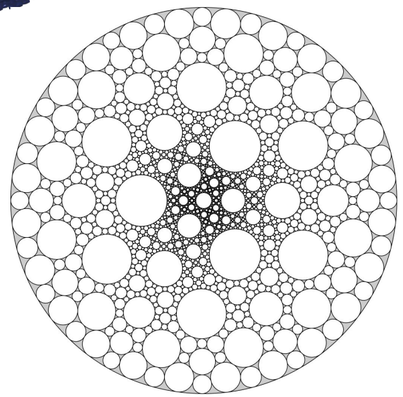


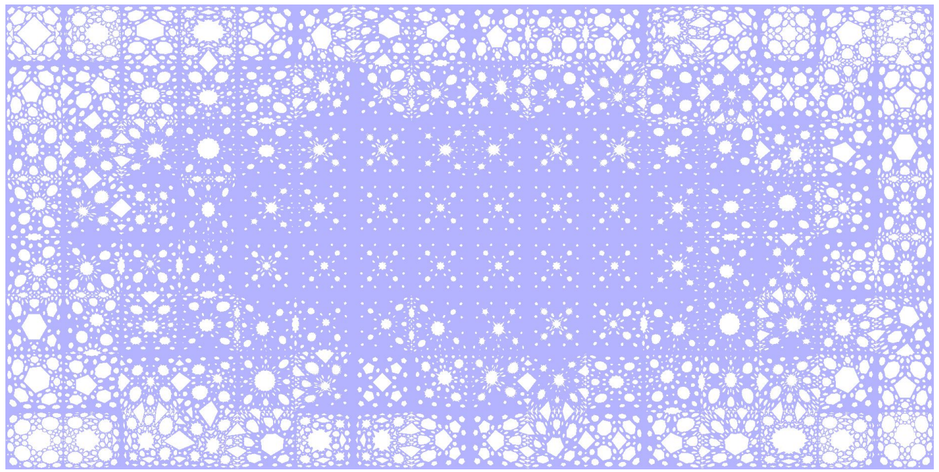
Travaux futurs

- Extension à la 3D
- Autres processus en 2D
  - Plusieurs lacunes centrales
  - Autres types d'empilements
- Impression 3D des structures issues d'empilements
  - Épaississement des raccords
- Optimisation topologique
  - Adaptation automatique de la topologie







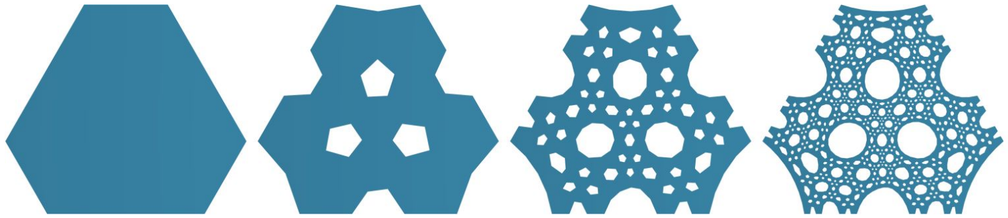


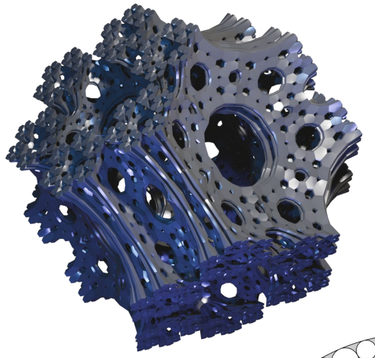


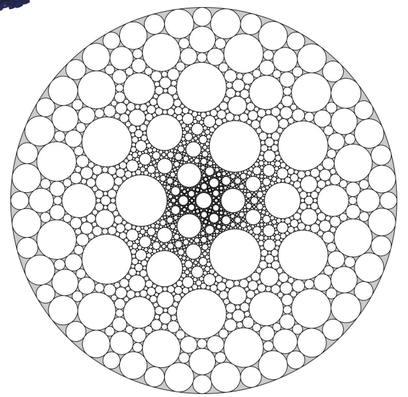
Nathan Jacquinet

Travaux futurs

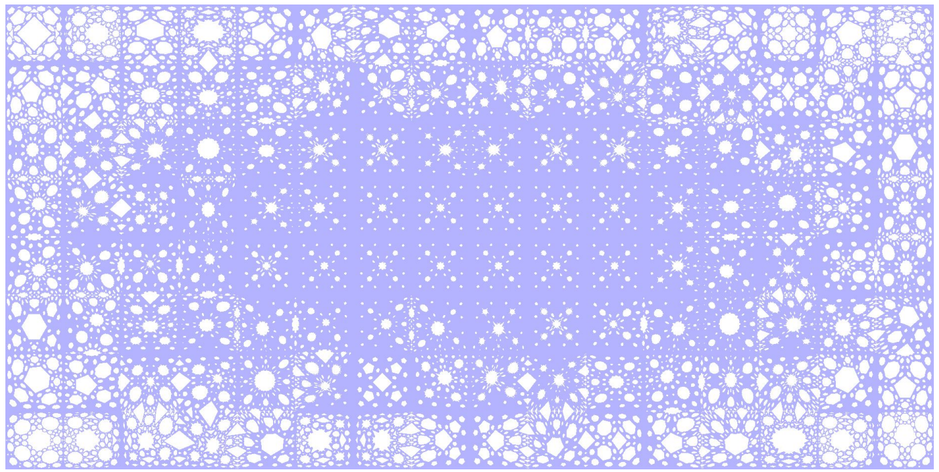
- Extension à la 3D
- Autres processus en 2D
  - Plusieurs lacunes centrales
  - Autres types d'empilements







- Impression 3D des structures issues d'empilements
  - Épaississement des raccords
- Optimisation topologique
  - Adaptation automatique de la topologie
- Autres mesures sur fractales
  - Homologie persistante

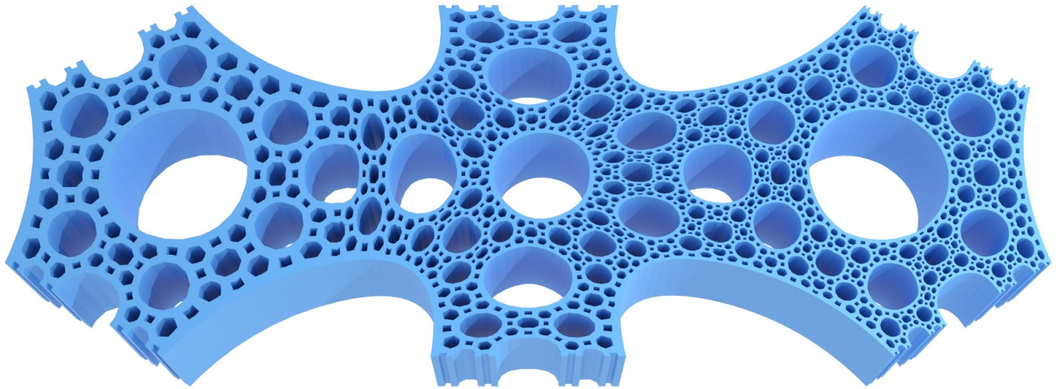


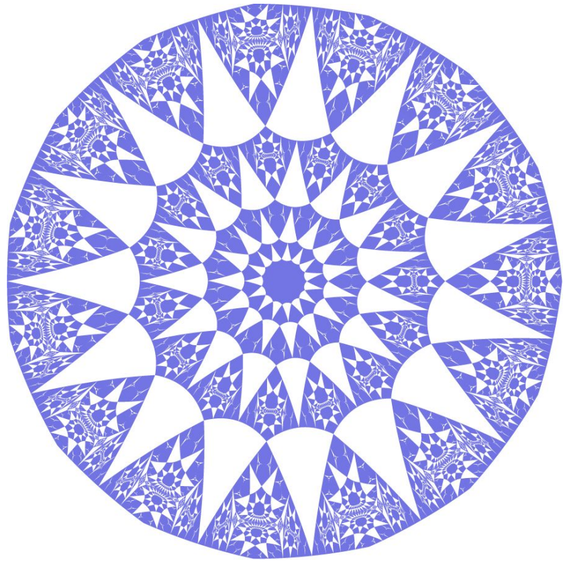
Nathan Jacquinet

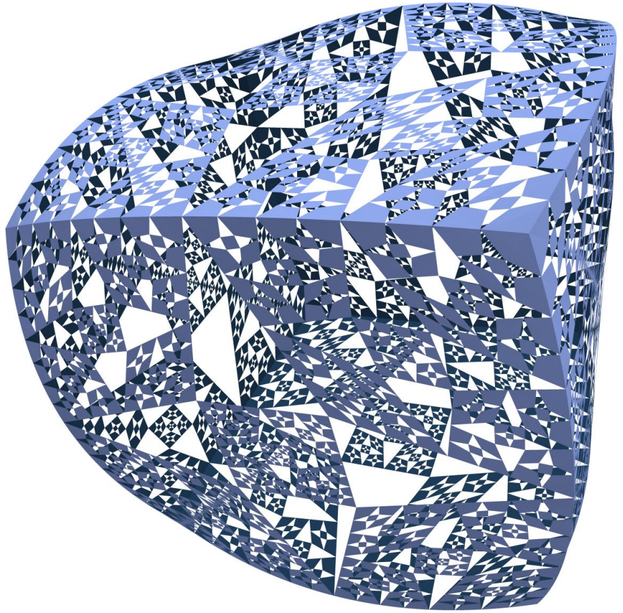


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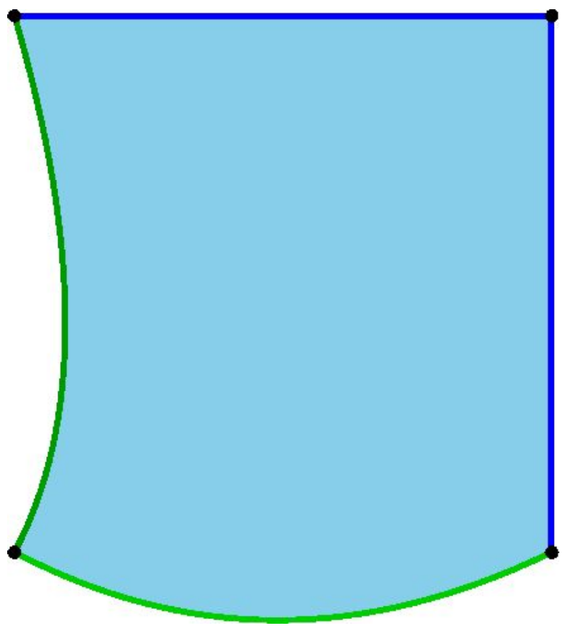


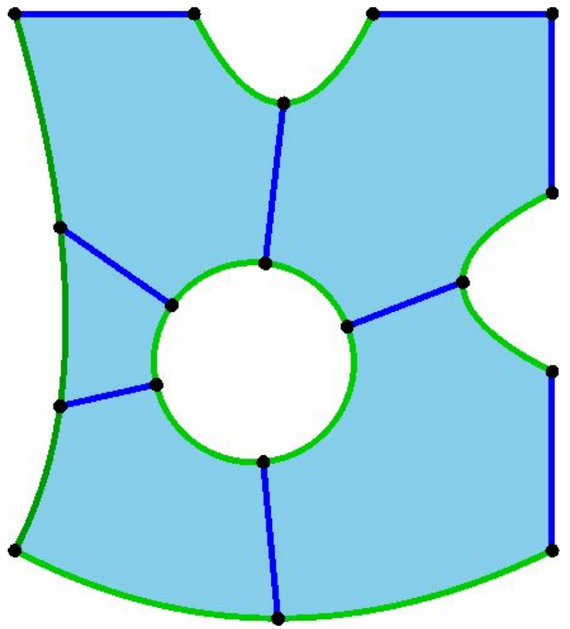




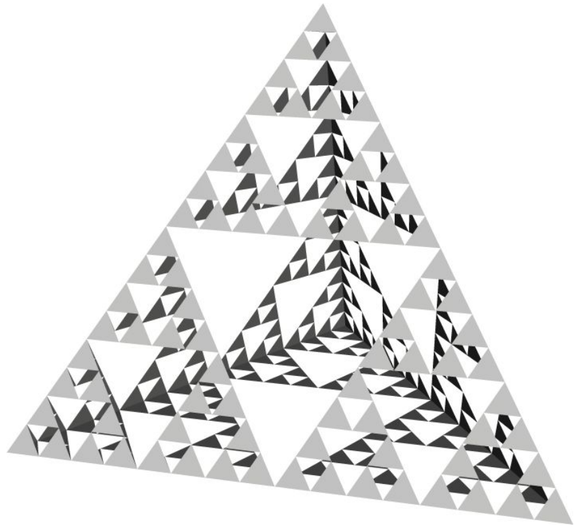
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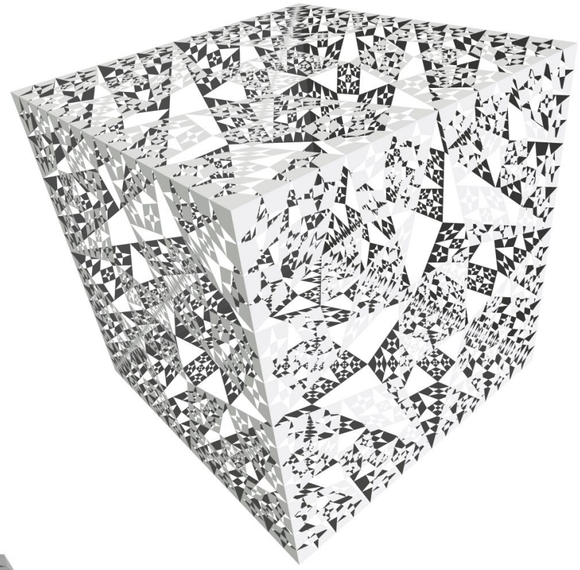
Bonus : Convergence du processus de subdivision

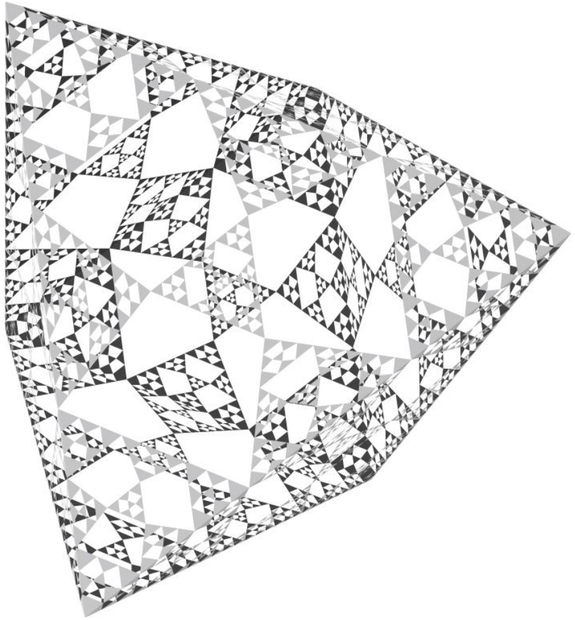


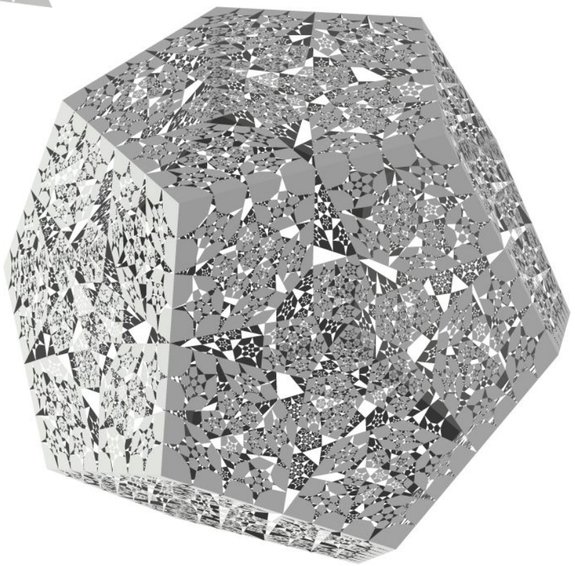


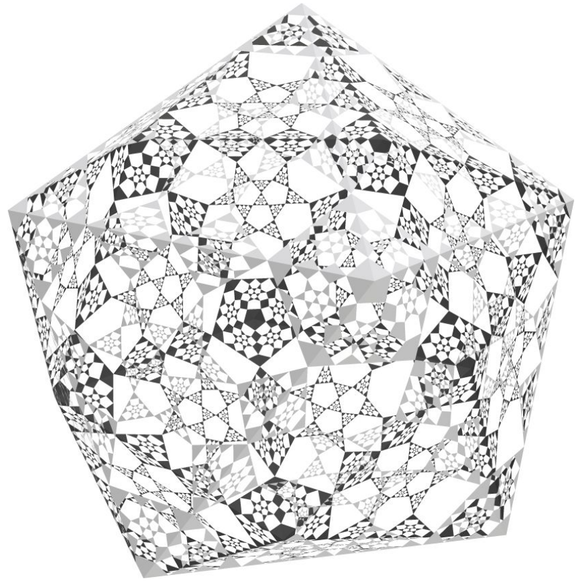
Bonus : Polyèdres fractals



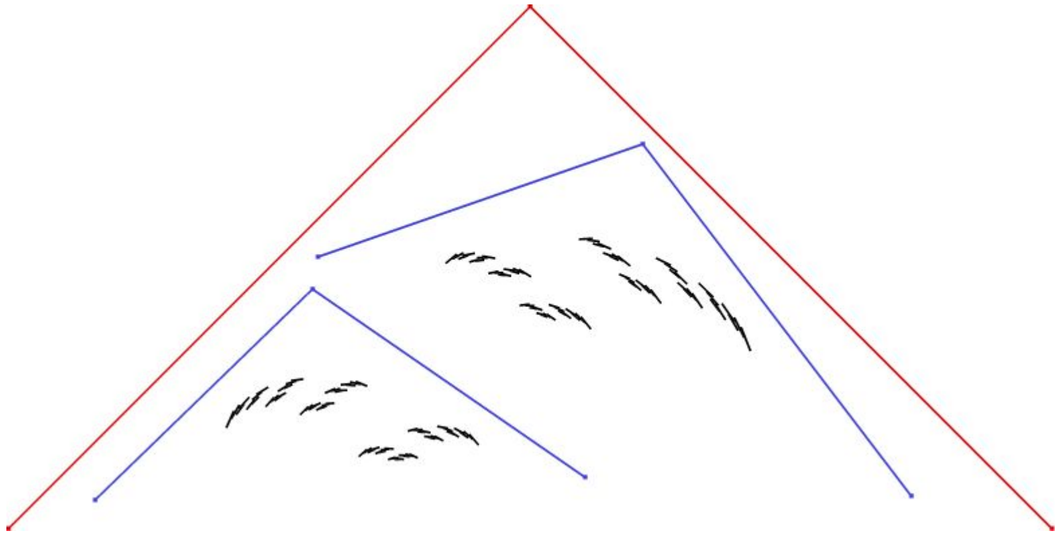


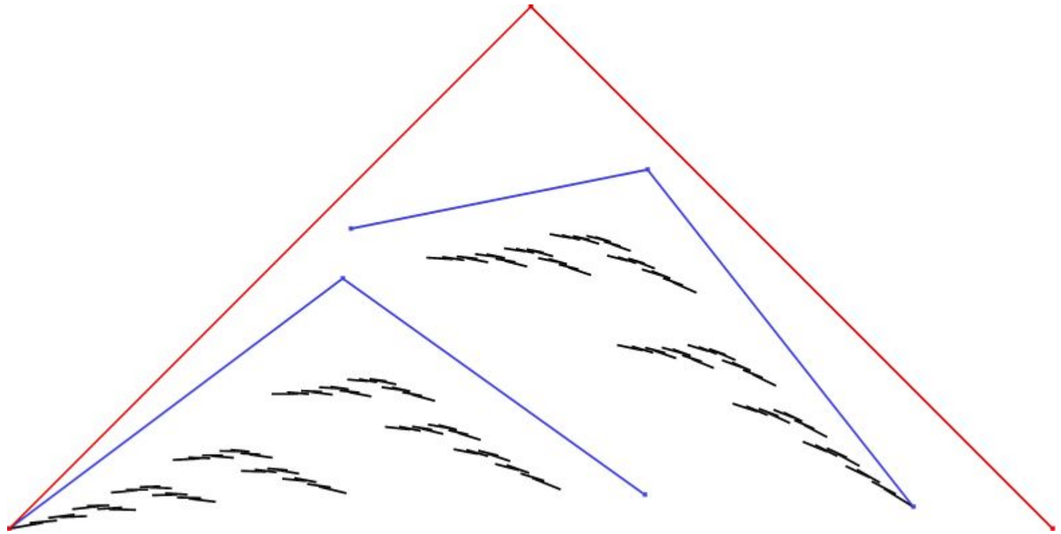


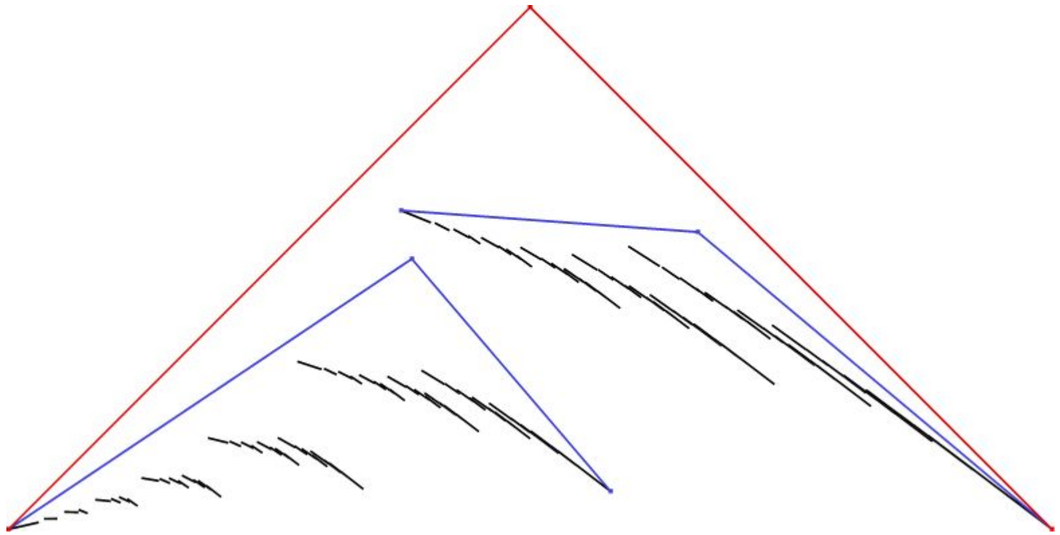


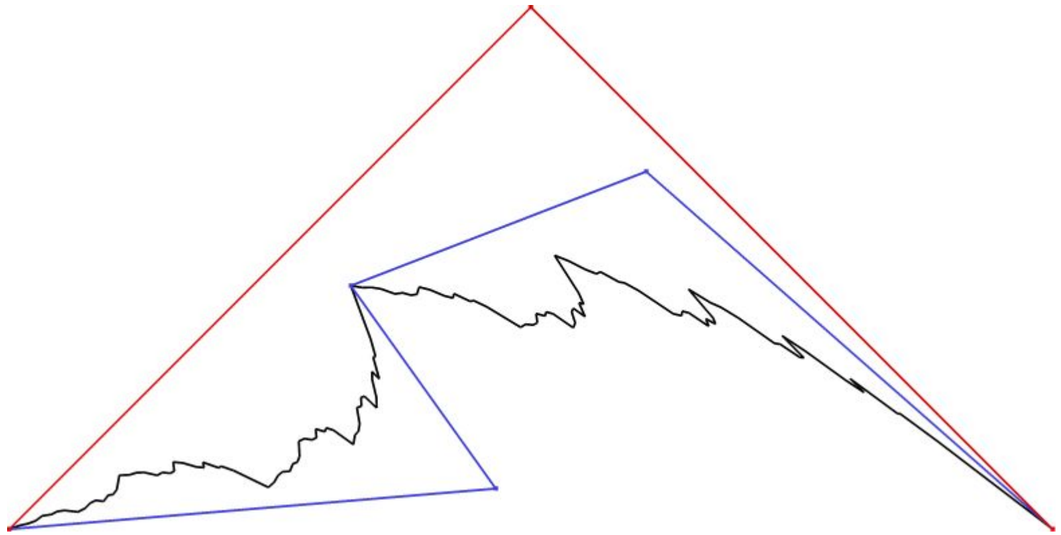


Bonus : Illustration des contraintes

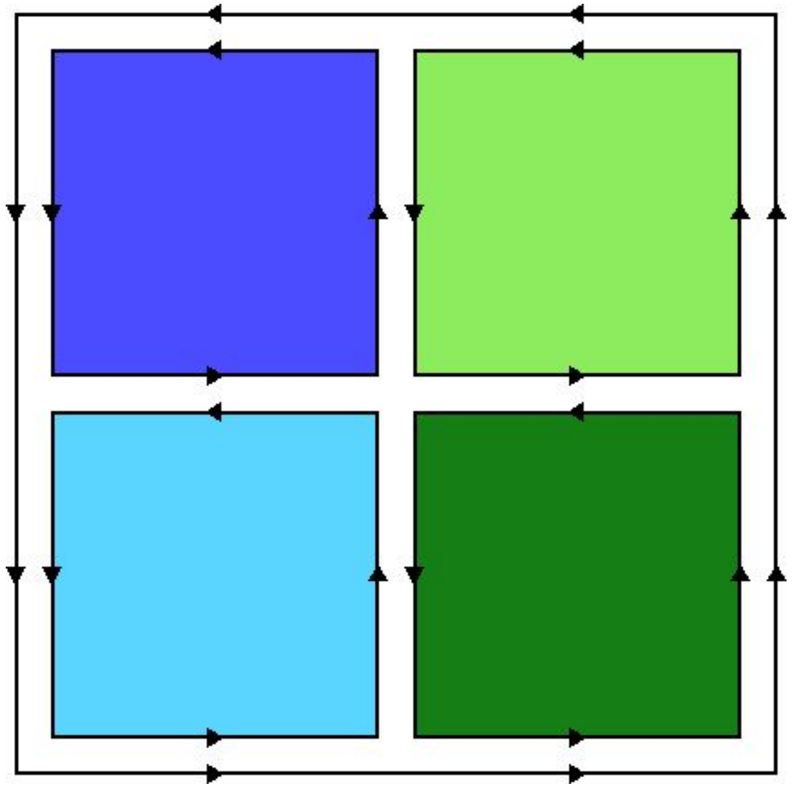


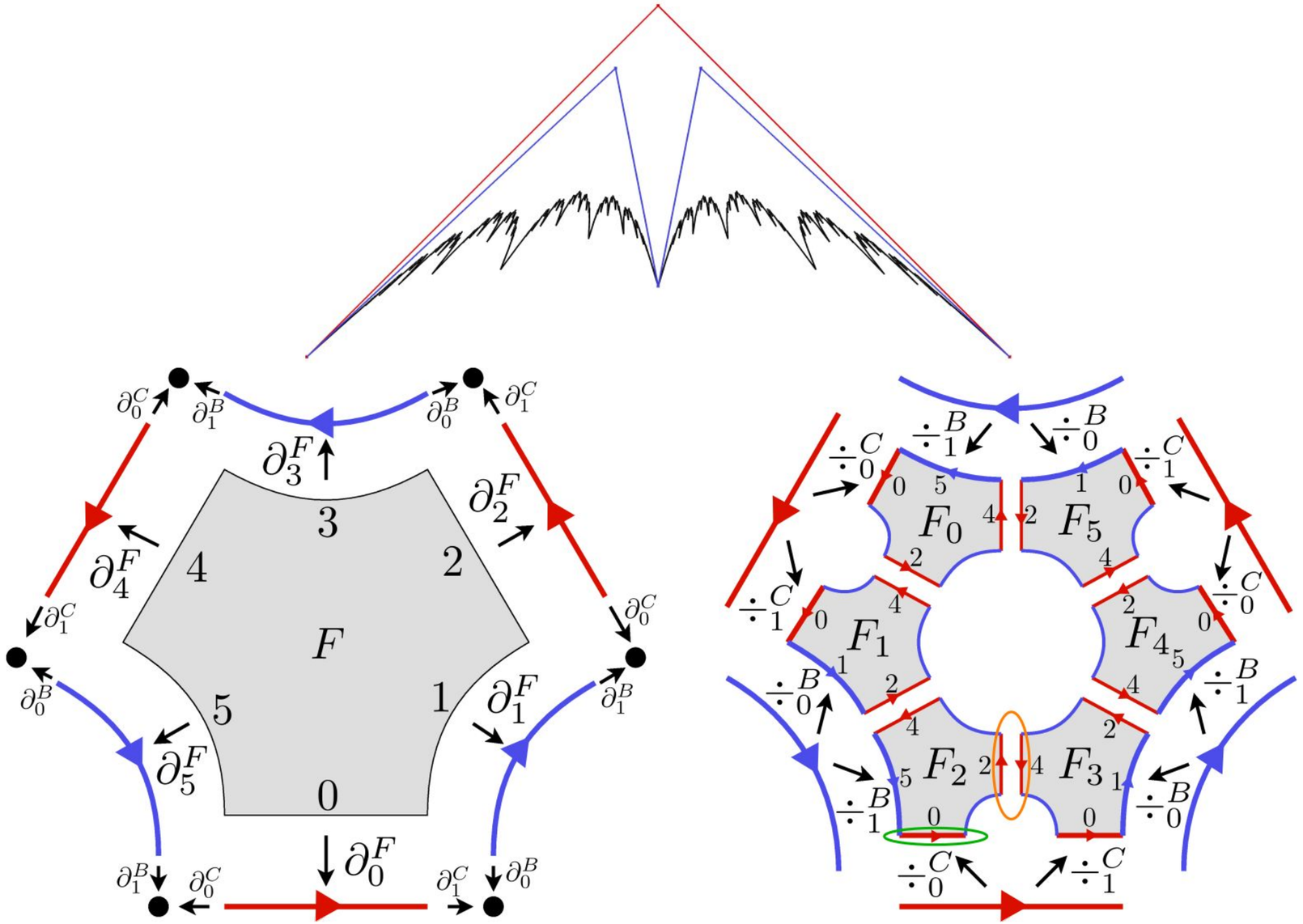






Bonus : Orientation et symétrie





Bonus : Lacunes de primitive

